New York City Interscholastic Mathematics League

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division

CONTEST NUMBER 1

PART I ***FALL, 2007*** ***CONTEST 1*** ***TIME: 10 MINUTES***

F07SF1 If $2^{12}+2^{12}+2^{12}+2^{12}=2^x$, compute x.

F07SF2 Compute the sum of the first 200 positive even integers.

PART II ***FALL, 2007*** ***CONTEST 1*** ***TIME: 10 MINUTES***

F07SF3 Compute the number that exceeds its square by the greatest amount.

F07SF4 10 lines are drawn in a plane; no 2 of which are parallel, and no 3 of which are collinear. Compute the number of regions that these ten lines divide the plane?

PART III ***FALL, 2007*** ***CONTEST 1*** ***TIME: 10 MINUTES***

F07SF5 Two circles of radii 6 and 10 are externally tangent. Compute the length of their external tangent.

F07SF6 Three fair dice are thrown, and the sum of their faces is 8. Compute the probability that the three faces are different numbers.

ANSWERS:

	F07SF1	**14**
	F07SF2	**40200**
	F07SF3	**1/2**
	F07SF4	**56**
	F07SF5	$4\sqrt{15}$
	F07SF6	**4/7**

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Sophomore-Freshman Division CONTEST NUMBER 1

Fall 2007 Solutions

F07SF1 $2^{12}+2^{12}+2^{12}+2^{12}=4\cdot 2^{12}=2^{14}\rightarrow x=\mathbf{14}.$

F07SF2 The sum is double the sum of the first 200 integers. $2\frac{(200\times 201)}{2}=\mathbf{40{,}200}$.

F07SF3 $A=x-x^2=x(1-x)$. This will be a maximum when the 2 factors, x and $1-x$ are equal.
$x=1-x,\ x=\mathbf{\frac{1}{2}}$.

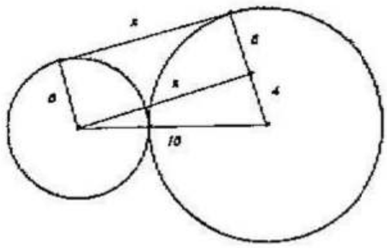
F07SF4 One line divides it into 2 regions, 2 into 4 regions and 3 into 7. Continuing this pattern, 10 divides it into 1+1+2+3+...+10 = **56**.

F07SF5

$16^2-4^2=x^2$

$\sqrt{240}=x$

$\mathbf{4\sqrt{15}}=x$



F07SF6 The ways that 8 can be the sum:

(1,1,6) (3 ways), (1,3,4) (6 ways), (1,2,5) (6 ways), (2,2,4) (3 ways), (2,3,3) (3 ways)

Probability the faces are different= $\frac{12}{21}=\mathbf{\frac{4}{7}}$.

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CONTEST NUMBER 2

PART I ***FALL, 2007*** ***CONTEST 2*** ***TIME: 10 MINUTES***

F07SF7 A positive integer is added to the sum of its digits and the result is 98. Compute the positive integer.

F07SF8 Two concentric circles form a ring whose area is 40π. A chord of the larger circle is tangent to the smaller circle. Compute the length of this chord.

PART II ***FALL, 2007*** ***CONTEST 2*** ***TIME: 10 MINUTES***

F07SF9 If $x^2+7x+n=0$, compute the number of integral values of n, $-50<n<50$, such that the roots of the equation are integers.

F07SF10 Compute the coordinates (x, y) of all points of the intersection of the graphs of $x^2+y^2=16$ and $y=x^2-4$.

PART III ***FALL, 2007*** ***CONTEST 2*** ***TIME: 10 MINUTES***

F07SF11 Compute the length of the shortest altitude in a triangle with sides of length 5, 12, and 13.

F07SF12 If $x+\dfrac{1}{x}=5$, compute $x^3+\dfrac{1}{x^3}$.

ANSWERS:

F07SF7	**85**
F07SF8	$4\sqrt{10}$
F07SF9	**8**
F07SF10	$(\sqrt{7},3)$ $(-\sqrt{7},3)$ $(0,-4)$
F07SF11	**60/13** or $4\frac{8}{13}$
F07SF12	**110**

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Sophomore-Freshman Division CONTEST NUMBER 2

Fall 2007 Solutions

F07SF7 $10t+u+t+u=98 \to 11t+2u=98$. Thus t must be 8, and u is 5. **Answer is 85**

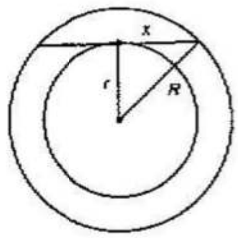
F07SF8

$\pi R^2-\pi r^2=40\pi$

$\pi(R^2-r^2)=40\pi$

$R^2-r^2=40 \qquad x=\sqrt{40}$

chord $=2x=2\sqrt{40}=\mathbf{4\sqrt{10}}$



F07SF9 If n is positive, n could be 12, 10, or 6. If n is negative, n could be -8. -18, -30, or -44. Including 0, we have **8** values.

F07SF10 Substituting $y+4$ for x^2, $y^2+y-12=0$, thus
$y=+3$, $y=-4$ $\quad$ $\mathbf{(\sqrt{7},3)\ (-\sqrt{7},3)\ (0,-4)}$
Note: Graphing, we get a circle and a parabola which are tangent at the bottom.

F07SF11 The shortest altitude is the one to the hypotenuse. Since the area is
$\frac{1}{2}\cdot 5\cdot 12=30,\quad \frac{1}{2}h\cdot 13=30\to \quad h=\mathbf{\frac{60}{13}\ \text{or}\ 4\frac{8}{13}}$

F07SF12

$$x+\frac{1}{x}=5\to x^2+2+\frac{1}{x^2}=25\to x^2+\frac{1}{x^2}=23.$$

$$\left(x^2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=23\times 5.$$

$$x^3+x+\frac{1}{x}+\frac{1}{x^3}=115\to x^3+\frac{1}{x^3}=\mathbf{110}$$

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CONTEST NUMBER 3

PART I ***FALL, 2007*** ***CONTEST 3*** ***TIME: 10 MINUTES***

F07SF13 If $\frac{1}{x}$ is the average of $\frac{1}{6}$ and $\frac{1}{10}$, compute x.

F07SF14 Compute the number of 3 digit integers that are not divisible by either 2 or 5.

PART II ***FALL, 2007*** ***CONTEST 3*** ***TIME: 10 MINUTES***

F07SF15 Compute the area of a regular hexagon whose side is 10.

F07SF16 Compute the number of positive integers less than 1000 with exactly 3 factors.

PART III ***FALL, 2007*** ***CONTEST 3*** ***TIME: 10 MINUTES***

F07SF17 Five men can plow a square field whose side is 60 feet in 4 hours. At the same rate of work, compute how many hours 10 men could plow a square field whose side is 180 feet?

F07SF18 A right triangle has one leg of length 15. Compute all ordered pairs of integers (b,c) where b is the length of the other leg and c is the length of the hypotenuse.

ANSWERS:

F07SF13	**7.5 or $7\frac{1}{2}$ or $\frac{15}{2}$**
F07SF14	**360**
F07SF15	**$150\sqrt{3}$**
F07SF16	**11**
F07SF17	18
F07SF18	**$(112,113),(36,39),(8,17),(20,25)$**

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Sophomore-Freshman Division   CONTEST NUMBER 3

Fall 2007 Solutions

F07SF13 $\frac{1}{6}+\frac{1}{10}=2\cdot\frac{1}{x}$ Multiplying by $30x$, $x(5+3)=60$ $x=\frac{60}{8}=7.5$.

F07SF14   All numbers whose units digit is 1,3,7, or 9 fit the description. There are 40 of these per hundred, and 900 three digit numbers. $40\times 9=\mathbf{360}$.

F07SF15   There are 6 equilateral triangles with side 10. The area of an equilateral triangle is $\frac{s^2}{4}\sqrt{3}$.

$6\cdot\frac{100}{4}\sqrt{3}\rightarrow \mathbf{150\sqrt{3}}=A$

F07SF16   The numbers that have exactly 3 factors are the squares of the prime numbers. The primes 2,3,5,7,11,13,17,19,23,29,and 31 have squares less than 1000. **11** numbers.

F07SF17   A side of the second field is three times a side of the first, so the area of the second field is nine times the area of the first. It would take 5 men 4(9) = 36 hours to plow this second field, so it will take 10 men 18 hours.

F07SF18 $c^2-b^2=225$. $(c+b)(c-b)=225$. Factoring 225, we get $225\times1, 75\times3, 25\times9, 45\times5$.
(Obviously 15×15 will not produce an answer.)
Each will produce 1 answer.

For example:

$c+b=225$
$c-b=1$
$c=113 \quad b=112$

(112,113) (36,39) (8,17) (20,25)