NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 1

PART I	*SPRING 2007*	*CONTEST 1*	*TIME: 10 MINUTES*

S07A1 If $\frac{n}{1}-\frac{n}{3}+\frac{n}{9}-\frac{n}{27}+\cdots=2007$, compute n.

S07A2 Solve for x: $\log_2\left(x^2-9\right)+\log_2(x)-\log_2(x+3)=2$.

PART II	*SPRING 2007*	*CONTEST 1*	*TIME: 10 MINUTES*

S07A3 Compute the number of positive integers less than 2007 that are multiples of 2 or 7, but not multiples of both 2 and 7.

S07A4 Oleg is biking from New York to Vermont. The probability that he will see a car within the next fifteen minutes is $\frac{63}{64}$. Assume that the probability of seeing a car during any time interval depends only on the length of the interval and does not depend on the starting and ending points of the interval. If $\frac{a}{b}$ is the probability that Oleg will see a car within the next five minutes, compute $\frac{a}{b}$.

PART III	*SPRING 2007*	*CONTEST 1*	*TIME: 10 MINUTES*

S07A5 The graph of the function $f(x)=3x^2+13x+4$ is reflected over the line $y=x$. Compute the minimum distance between a point on $y=f(x)$ and its reflection.

S07A6 Compute the sum of all integral values of x such that $\frac{(x+2)^3}{x+7}$ is an integer.

ANSWERS:

S07A1	**2676**
S07A2	**4**
S07A3	**1003**
S07A4	$\frac{3}{4}$
S07A5	**0**
S07A6	**-56**

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 1

SPRING 2007 Solutions

S07A1 **2676.** $n-\frac{n}{3}+\frac{n}{9}-\frac{n}{27}+\ldots=2007$ The sum of an infinite geometric series is $S=\frac{a}{1-r}$, where a = the first term = n and r = the common ratio= $\frac{-1}{3}$. $\therefore S=\frac{n}{1+\frac{1}{3}}=\frac{3n}{4}=2007$.

Therefore $n = 2676$.

S07A2 **4.** $\log_2(x^2-9)+\log_2(x)-\log_2(x+3)=\log_2\left(\frac{(x^2-9)x}{(x+3)}\right)=\log_2((x-3)x)=2$ or

$x^2-3x=4 \therefore x=-1,4$ Reject $x = -1$ because we cannot take the log of a negative number.

S07A3 **1003.** To find the number of multiples of a given number n less than or equal to 2007, find the greatest integer less than or equal to the quotient of 2007 and n.
$\left[\frac{2007}{2}\right]=1003$, $\left[\frac{2007}{7}\right]=286$, $\left[\frac{2007}{14}\right]=143$. The multiples of 2 and 7 include multiples of 14 (both 2 and 7). Since we want only multiples of 2 and 7 and not both, we must subtract the multiples of 14 twice. $1003 + 286 - 143 - 143 = 1003$.

S07A4 $\mathbf{\frac{3}{4}}$. Let the probability that Oleg sees a car in 5 minutes $= p$. Then the probability he does not see a car in 5 minutes $= 1-p$. In fifteen minutes, the probability he doesn't see a car is $(1-p)^3$. The probability that he sees a car in 15 minutes is

$1-(1-p)^3=\frac{63}{64}\rightarrow(1-p)^3=\frac{1}{64}\rightarrow 1-p=\frac{1}{4}\rightarrow p=\frac{3}{4}$. (Note: p^3 is not the probability of seeing a car in 15 minutes but rather the probability of seeing a car in each 5 minute interval.)

S07A5 **0.** Let d = the distance between (x,y) and (y,x), then

$d=\sqrt{(x-y)^2+(y-x)^2}=\sqrt{2(y-x)^2}=\sqrt{2}\sqrt{(3x^2+13x+4-x)^2}=\sqrt{2}\sqrt{(3x^2+12x+4)^2}$.

Since $|3x^2+12x+4|$ has a minimum when it is 0 (it has real roots), $d=0$.

S07A6 **-56.** $\frac{x^3+6x^2+12x+8}{x+7}=x^2-x+19-\frac{125}{x+7}$. The factors of 125 are: $\pm1,\pm5,\pm25,\pm125$.
The sum of these factors is zero. Since $x + 7$ must equal each factor to divide 125 evenly, the sum of all such x and 7 (eight times) = 0 therefore the sum of all x = **-56**.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 2

PART I	*SPRING 2007*	*CONTEST 2*	*TIME: 10 MINUTES*

S07A7 A regular hexagon with side of length s has a perimeter that is numerically equal to its area. Compute s.

S07A8 If a and b are the complex roots of the quadratic equation $x^2+3kx+k=0$, where k is a real number, compute the minimum value of a^2+b^2.

PART II	*SPRING 2007*	*CONTEST 2*	*TIME: 10 MINUTES*

S07A9 A *day* is divided into 24 *hours*. Each *hour* has 60 *minutes*. Each *minute* has 60 *seconds*. Using a new system of measurement, each *day* has 20 *naps*. Each *nap* has 40 *winks*. Compute the number of *seconds* in each *wink*.

S07A10 The sum of twice the arithmetic mean of x and y and the square of the geometric mean of x and y is -4. If x and y are real numbers such that $x^2+y^2=32$, compute the maximum value of xy.

PART III	*SPRING 2007*	*CONTEST 2*	*TIME: 10 MINUTES*

S07A11 In baseball, a player earns a walk when the pitcher throws four balls before three strikes, but strikes out when the pitcher throws three strikes before four balls. If every pitch is either a ball or a strike and is equally likely to be either a ball or a strike, compute the probability that the player walks.

S07A12 If $\overline{AD}$ is a median in triangle ABC, and $(AC)^2+(AB)^2=\frac{9}{2}(BC)^2$, compute $\frac{AD}{BC}$.

ANSWERS:		
	S07A7	$\frac{4\sqrt{3}}{3}$
	S07A8	$\frac{-1}{9}$
	S07A9	**108**
	S07A10	**2**
	S07A11	$\frac{11}{32}$
	S07A12	$\sqrt{2}$

New York City Interscholastic Mathematics League

Senior A Division CONTEST NUMBER 2

SPRING 2007 Solutions

S07A7 $\frac{4\sqrt{3}}{3}$. Area of the hexagon is $\frac{6s^2\sqrt{3}}{4}$ and the perimeter is $6s$ $\therefore \frac{6s^2\sqrt{3}}{4}=6s$.

Solving for s yields 0, which we reject, or $\frac{4\sqrt{3}}{3}$.

S07A8 $-\frac{1}{9}$ If a and b are the roots of $x^2+3kx+k=0$, $a+b=-3k$ and $ab=k$. Hence

$(a+b)^2=9k^2 \therefore a^2+b^2+2ab=9k^2 \rightarrow a^2+b^2=9k^2-2k=\left(3k-\frac{1}{3}\right)^2-\frac{1}{9}$

The minimum value of a^2+b^2 occurs when

$\left(3k-\frac{1}{3}\right)=0 \therefore k=\frac{1}{9}$ and $a^2+b^2=9\cdot\frac{1}{81}-2\cdot\frac{1}{9}=-\frac{1}{9}$.

S07A9 **108.** $\frac{24hours}{1day}\times\frac{60\min}{1hour}\times\frac{60\sec}{1\min}=\frac{20naps}{1day}\times\frac{40winks}{1nap}$ or $24\times60\times60\sec=20\times40winks$

Dividing both sides by 800, simplifies the equality to

$3(6)(6)\sec=1wink \therefore 108\sec=1wink$.

S07A10 **2.** $2\left(\frac{x+y}{2}\right)+\left(\sqrt{xy}\right)^2=x+y+xy=-4$ and $x^2+y^2=32$. So

$x+y=-(4+xy) \therefore (x+y)^2=(4+xy)^2$ yields $x^2+y^2+2xy=16+8xy+x^2y^2$

Therefore $32=16+6xy+x^2y^2$. Let $A = xy$, then $A^2+6A-16=0 \therefore A=-8$ or 2. Reject $A=-8$ and we get $xy=2$.

S07A11 $\frac{11}{32}$. After 6 pitches are thrown, the batter must have walked or struck out. If we expand $(B+S)^6$, the exponents of B and S will tell us how many balls and strikes the batter has and the numerical coefficient will tell the number of ways that outcome may occur. The sixth row of Pascal's triangle is 1, 6, 15, 20, 15, 6, and 1. The batter walks on the first three possibilities and strikes out on the rest. 1+6+15=22 and 1+6+15+20+15+6+1 = 64.

Therefore the probability the batter walks is $\frac{22}{64}=\frac{11}{32}$. Or:

Because P(4B and 0S) = 1/16, P(4B and 1S)= 4/32, and P(4B and 2S)= 10/64, the probability of walking is 1/16 + 4/32 +10/64 which is 11/32.

S07A12 $\sqrt{2}$. From Stewart's Theorem,

$$(AD)^2=\frac{1}{2}\left((AC)^2+(AB)^2\right)-\left(\frac{(BC)}{2}\right)^2=\frac{1}{2}\left(\frac{9}{2}(BC)^2\right)-\left(\frac{(BC)}{2}\right)^2=2(BC)^2$$

$\therefore (AD)=\sqrt{2}(BC)$ or $\frac{AD}{BC}=\sqrt{2}$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division CONTEST NUMBER 3

PART I	*SPRING 2007*	*CONTEST 3*	*TIME: 10 MINUTES*

S07A13 If $x^2+2x+1=0$, compute:
$$\left(x+\frac{1}{x}\right)+\left(x^2+\frac{1}{x^2}\right)+\left(x^3+\frac{1}{x^3}\right)+\left(x^4+\frac{1}{x^4}\right)+\cdots+\left(x^{2007}+\frac{1}{x^{2007}}\right).$$

S07A14 $(\log_2 4-1)+(\log_2 6-1)+(\log_2 8-1)+\cdots+(\log_2 2004-1)+(\log_2 2006-1)=\log_2(k!)$. Compute k.

PART II	*SPRING 2007*	*CONTEST 3*	*TIME: 10 MINUTES*

S07A15 Compute the number of positive integers n, where $n \le 2007$, for which $(n+1)!\ +n!$ is divisible by 2007.

S07A16 Compute the number of three-element subsets of $\{1,2,3,\ldots,29,30\}$ such that the sum of the three elements is divisible by 3.

PART III	*SPRING 2007*	*CONTEST 3*	*TIME: 10 MINUTES*

S07A17 The base 10 fraction $\frac{a}{b}$ expressed in base 8 is $.\overline{2}$. Compute $\frac{a}{b}$ in base 10.

S07A18 In right triangle ABC, the sides have integral lengths. The perimeter of the triangle is numerically equal to its area. If $a<b<c$, compute the number of distinct ordered triples (a,b,c).

ANSWERS:

S07A13	**-2**
S07A14	**1003**
S07A15	**1786**
S07A16	**1360**
S07A17	$\mathbf{\frac{2}{7}}$
S07A18	**2**

New York City Interscholastic Mathematics League

Senior A Division CONTEST NUMBER 3

SPRING 2007 Solutions

S07A13 **-2.** $x^2+2x+1=0 \rightarrow x=-1$ is the only solution. Thus each odd place term is -2 and each even place term gives 2. This pattern continues so the first 2006 terms cancel to 0 and the 2007[th] term is –2.

S07A14

$\log_2 4-1=\log_2 4-\log_2 2=\log_2 2$

$\log_2 6-1=\log_2 6-\log_2 2=\log_2 3$

$\log_2 8-1=\log_2 8-\log_2 2=\log_2 4$, etc.

Now the left side of the equation is:

$\log_2 2+\log_2 3+\log_2 4+\ldots+\log_2 1003=\log_2(2\cdot 3\cdot 4\cdot\ldots\cdot 1003)=\log_2 1003!\rightarrow k=1003.$

(note $\log_2 1=0$.)

S07A15 **1786.** $(n+1)!+n!=(n+1)n!+n!=(n+2)n!$ The prime factorization of 2007 is $3^2\times 223$. Therefore $223\le n\le 2007$, or $n+2=223$ and $n=221$. Thus n can be 1785 + 1 =1786 numbers.

S07A16 **1360.** We can either choose three numbers with the same remainder mod 3 (three cases) or three numbers with a different remainder (fourth case). Each of the first three cases can be done in ${}_{10}C_3=120$ ways and the fourth in $10^3=1000$ ways. $3(120)+1000=1360$.

Or: (without *mod*)

Among our thirty numbers we have ten which leave a remainder of 0 on division by 3, ten which leave a remainder of 1, and ten which leave a remainder of 2. In order to get a final remainder of 0, we must choose either one from each group ($10\cdot 10\cdot 10=1000$ ways) or three from the same group (${}_{10}C_3=120$ ways per group, with three groups gives 360 ways). Thus the answer is 1000 + 360 = 1360.

S07A17 $\mathbf{\frac{2}{7}}$. $.2222\ldots_8=\frac{2}{8}+\frac{2}{8^2}+\frac{2}{8^3}+\cdots=\frac{\frac{1}{4}}{1-\frac{1}{8}}=\frac{2}{7}$.

S07A18 **2.** We are given $P=a+b+c=A=\frac{ab}{2}$. We get $2c=ab-2a-2b$, but we also know that $c=\sqrt{a^2+b^2}$ from the Pythagorean theorem. Substituting for c and squaring both sides we get $\left(2\sqrt{a^2+b^2}\right)^2=(ab-2a-2b)^2 \therefore 4a^2+4b^2=4a^2+4b^2+a^2b^2-4a^2b-4ab^2+8ab$

So $a^2b^2-4a^2b-4ab^2+8ab=0$ or $ab(ab-4a-4b+8)=0$. Since $a\neq 0$ and $b\neq 0 \therefore ab-4a-4b+8=0$. Solving for a yields $a=4+\frac{8}{b-4}$ therefore b = 2, 3, 5, 6, 8, or 12. Since 0< a <b, only $(5,12,13)$ and $(6,8,10)$ work and there are 2 ordered triples.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 4

PART I   ***SPRING 2007***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S07A19   If x is a positive real number such that $3x^{3x^{3x^{3x^{\cdot^{\cdot^{\cdot}}}}}} = 6$, compute x.

S07A20   Concentric circles C_1, C_2, and C_3 have radii of 1, 2, and 4, respectively. One of the bases of a trapezoid is a chord of C_3 and tangent to C_2 and the other base is a chord of C_2 and tangent to C_1, and the center of the circles is in the interior of the trapezoid. Compute the area of the trapezoid.

PART II   ***SPRING 2007***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S07A21   Given: $\log_4 x = \log_9 y = \log_6 24$, compute xy.

S07A22   If $\dfrac{2x^3-5x^2-x+4}{(x-1)^3} = \dfrac{A}{(x-1)^0} + \dfrac{B}{(x-1)^1} + \dfrac{C}{(x-1)^2} + \dfrac{D}{(x-1)^3}$ for real numbers A, B, C, and D, compute $A+B+C+D$.

PART III   ***SPRING 2007***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S07A23   Compute all value(s) of x for which $15+15x^2+15x^4+\ldots=16$.

S07A24 $ABCD$ is a square with side of length 1. Equilateral triangle EFG is inscribed within triangle BCD, with E the midpoint of $\overline{BD}$ and points F and G on $\overline{BC}$ and $\overline{CD}$ respectively. The area of triangle EFG is $\dfrac{a+b\sqrt{3}}{4}$, where a and b are integers. Compute $a+b$.

ANSWERS:

S07A19	$\sqrt[5]{2}$
S07A20	$9\sqrt{3}$
S07A21	576
S07A22	-2
S07A23	$\pm\frac{1}{4}$
S07A24	-1

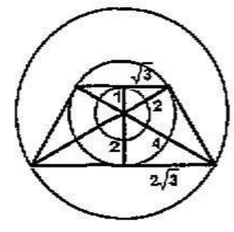
New York City Interscholastic Mathematics League

Senior A Division   CONTEST NUMBER 4

SPRING 2007 Solutions

S07A19 $\sqrt[6]{2}$. $3x^{3x^{3x^{\cdots}}} = 6 \therefore 3x^6 = 6$. $x^6 = 2$ so $x = \sqrt[6]{2}$.

S07A20 $9\sqrt{3}$. (see diagram) The height of the trapezoid is 3 and the bases can be found by forming right triangles as shown, giving bases of $2\sqrt{3}$ and $4\sqrt{3}$, and an area of $3\dfrac{2\sqrt{3}+4\sqrt{3}}{2} = 9\sqrt{3}$.

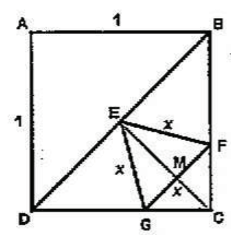


S07A21   **576.** $\log_4(x) = \log_9(y) = \log_6(24) = z$ therefore $x = 4^z, y = 9^z, 6^z = 24$.
$xy = 36^z = (6^z)^2 = 24^2 = 576$.

S07A22   **-2.** Divide the numerator by $x-1$, the remainder (0) is the numerator, *D*, of the $(x-1)^3$ term and the quotient is $2x^2-3x-4$. Divide the new quotient by $x-1$, the remainder (-5) is the numerator, *C*, of the $(x-1)^2$ term and the quotient is $2x-1$. Divide this quotient by $x-1$, the remainder (1) is the numerator, *B*, of the $(x-1)$ term and the quotient is 2 (is the numerator *A*). Or:
Let $x = 2$. We get $A+B+C+D = 2\cdot 2^3 - 5\cdot 2^2 - 2 + 4 = -2$.

S07A23 $\pm\dfrac{1}{4}$. This is the sum of an infinite geometric series and we get:
$15 + 15x^2 + 15x^4 + \ldots = \dfrac{15}{1-x^2} = 16$, therefore $15 = 16 - 16x^2, x^2 = \dfrac{1}{16}$ and $x = \pm\dfrac{1}{4}$.

S07A24   **-1.** Let $FG = x$, and M be the midpoint of $\overline{FG}$. Since FGC is an isosceles right triangle, $MC = \dfrac{x}{2}$. $\overline{ME}$ is the altitude of triangle *EFG*, so $ME = \dfrac{x\sqrt{3}}{2}$. But CE is half the length of a diagonal of a unit square, so $\dfrac{\sqrt{2}}{2} = CE = CM + ME = \dfrac{x}{2} + \dfrac{x\sqrt{3}}{2} \to x = \dfrac{\sqrt{6}-\sqrt{2}}{2}$. The area of ΔEFG is $\dfrac{\sqrt{3}}{4}x^2 = \dfrac{\sqrt{3}}{4}\dfrac{8-4\sqrt{3}}{4} = \dfrac{-3+2\sqrt{3}}{4}$. $-3+2 = -1$.



(Editors note: the proof that triangle *GFC* is isosceles is trivial, but is it unique? If *F* is rotated around *E* by 60 degrees, we get an image line segment that intersects $\overline{CD}$ in only one place, as segments can have at most one point of intersection. Since that intersection must be the position of *G*, *G* is unique and *F* is unique.)

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 5

PART I ***SPRING 2007*** ***CONTEST 5*** ***TIME: 10 MINUTES***

S07A25 If θ is an angle measured in degrees, compute the number of values of $\theta, 0^\circ \le \theta \le 2007^\circ$, such that $\sin\theta = \frac{1}{2}\tan\theta$.

S07A26 $a_1 + a_2 + a_3 + a_4 = 4,\ a_2 + a_3 + a_4 + a_5 = 8,\ a_3 + a_4 + a_5 + a_6 = 12,$ $a_4 + a_5 + a_6 + a_7 = 16,\ \ldots, a_{10} + a_{11} + a_{12} + a_{13} = 40,$ $a_{11} + a_{12} + a_{13} + a_1 = 44,\ a_{12} + a_{13} + a_1 + a_2 = 48,$ and $a_{13} + a_1 + a_2 + a_3 = 52.$ Compute a_7.

PART II ***SPRING 2007*** ***CONTEST 5*** ***TIME: 10 MINUTES***

S07A27 In creating his new cereal, Tony mixes 1 scoop of sugar and 2 scoops of raisins for every x scoops of bran. Sugar costs 5 cents per scoop, raisins cost 10 cents per scoop, and bran costs 15 cents per scoop. The ingredients are then mixed completely and packaged in boxes containing 20 scoops of the mixture. If the ingredients cost \$2.60 per box, compute x.

S07A28 Compute the number of real values of x that satisfy the equation $|x^2 - 3| + x = |3x|$

PART III ***SPRING 2007*** ***CONTEST 5*** ***TIME: 10 MINUTES***

S07A29 Compute the number of terminating zeros in the decimal expansion of 2007!

S07A30 Let $S = \log_3\left(3^{\frac{1}{3}}\right) + \log_3\left(9^{\frac{2}{9}}\right) + \log_3\left(27^{\frac{3}{27}}\right) + \ldots + \log_3\left(\left(3^n\right)^{\frac{n}{3^n}}\right) + \ldots$. Compute S.

ANSWERS:		
	S07A25	**23**
	S07A26	**-1**
	S07A27	**7**
	S07A28	**4**
	S07A29	**500**
	S07A30	$\mathbf{\frac{3}{2}}$

New York City Interscholastic Mathematics League

Senior A Division Contest Number 5

SPRING 2007 Solutions

S07A25 **23.** $\sin\theta = \frac{1}{2}\tan\theta = \frac{\sin\theta}{2\cos\theta}$ $\therefore \sin\theta = 0$ or $\cos\theta = \frac{1}{2}$.

$\therefore \sin\theta = 0$ for $\theta = 0, 180, \ldots, 1980$. There are 12 values.

$\cos\theta = \frac{1}{2}$ when $\theta = 60, 420, 780, 1140, 1500, 1860$ or $300, 660, 1020, 1380, 1740$. There are 11 values. The total is 23.

S07A26 **-1.** Adding all the equations we get

$$4\sum_{k=1}^{13} a_k = 4 + 8 + \ldots + 52 = 4\cdot\frac{13\cdot 14}{2} = 91\cdot 4 \to \sum_{k=1}^{13} a_k = 91.$$

$$a_7 = \sum_{k=1}^{13} a_k - (a_8 + a_9 + a_{10} + a_{11}) - (a_{12} + a_{13} + a_1 + a_2) - (a_3 + a_4 + a_5 + a_6) = 91 - 32 - 48 - 12$$

$= -1$.

S07A27 **7.** Each batch of cereal contains $x + 3$ scoops of cereal. The total cost is $1(5) + 2(10) + x(15) = 15x + 25$. The cost per scoop, $C = \frac{15x+25}{x+3} = \frac{260}{20} = 13 \to x = 7$.

S07A28 **4.** There are four cases, $x^2 - 3 > 0$ and $3x > 0$, $x^2 - 3 > 0$ and $3x < 0$, $x^2 - 3 < 0$ and $3x > 0$, $x^2 - 3 < 0$ and $3x < 0$. The first case yields the equation $x^2 - 3 + x = 3x \therefore x = 3$ or $x = -1$, $x = -1$ is extraneous. The second case yields the equation $x^2 - 3 + x = -3x \therefore x = -2 \pm \sqrt{7}$, $x = -2 + \sqrt{7}$ is extraneous. The third case yields the equation $-x^2 + 3 + x = 3x \therefore x = -3$ or $x = 1$, $x = -3$ is extraneous. The fourth case yields the equation $-x^2 + 3 + x = -3x \therefore x = 2 \pm \sqrt{7}$, $x = 2 + \sqrt{7}$ is extraneous. Thus there are 4 real values of x that satisfy the equation.

S07A29 **500.** We get a zero from every factor of 10. We have many factors of 2 to choose from, so we need only add factors of 5.

$\left[\frac{2007}{5}\right] = 401$, $\left[\frac{2007}{25}\right] = 80$, $\left[\frac{2007}{125}\right] = 16$, $\left[\frac{2007}{625}\right] = 3$ and

$401 + 80 + 16 + 3 = 500$ factors of 5.

S07A30 $\mathbf{\frac{3}{2}}$. $S = \log_3 3^{1/3} + \log_3 9^{2/9} + \log_3 27^{3/27} + \cdots \log_3 (3^n)^{n/3^n} + \ldots = \frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \cdots \frac{n^2}{3^n} + \ldots$.

$$S - \frac{S}{3} = \frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \cdots \frac{n^2}{3^n} + \ldots - \frac{1}{3}\left(\frac{1}{3} + \frac{4}{9} + \frac{9}{27} + \cdots \frac{n^2}{3^n} + \ldots\right) = \frac{1}{3} + \frac{3}{9} + \frac{5}{27} + \cdots \frac{2n-1}{3^n} + \ldots.$$

$$S - \frac{S}{3} - \frac{1}{3}\left(S - \frac{S}{3}\right) = \frac{1}{3} + \frac{3}{9} + \frac{5}{27} + \cdots \frac{2n-1}{3^n} + \ldots - \frac{1}{3}\left(\frac{1}{3} + \frac{3}{9} + \frac{5}{27} + \cdots \frac{2n-1}{3^n} + \ldots\right)$$

$$= \frac{1}{3} + \frac{2}{9} + \frac{2}{27} + \cdots + \frac{2}{3^n} + \ldots = \frac{1}{3} + \frac{2/9}{1 - 1/3} = \frac{2}{3} = \frac{4}{9}s \to s = \frac{3}{2}.$$