New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER ONE** | **SPRING 2006**

PART I: 10 minutes | **NYCIML Contest One** | **Spring 2006**

S06J1. Julie saves \$.01 on September 1, \$.03 on September 2, \$.05 on September 3, and continues to increase her savings by \$.02 a day for the entire month. Compute her total savings for September.

S06J2. A circle is inscribed in a regular hexagon. The area of the circle is 64π. Compute the area of the hexagon.

PART II: 10 minutes | **NYCIML Contest One** | **Spring 2006**

S06J3. There are x jellybeans in a jar. Archie eats $\frac{1}{2}$ of the jellybeans. Betty eats $\frac{2}{3}$ of the remaining jellybeans. Veronica eats $\frac{6}{7}$ of the jellybeans that remain after Betty has eaten. Compute the minimum x $(x > 0)$ for which this is possible. (Everyone eats only an integral number of jellybeans.)

S06J4. Compute x: $\sqrt{3-x} = x\sqrt{3-x}$.

PART III: 10 minutes | **NYCIML Contest One** | **Spring 2006**

S06J5. The diameter of circle O is equal to seven times the reciprocal of the circumference of the circle. Compute the area of the circle.

S06J6. Two sides of an obtuse triangle measure 12 and 15. Compute the number of possible integral lengths of the third side.

ANSWERS:

S06J1. **\$9.00**

S06J2. $\mathbf{128\sqrt{3}}$

S06J3. **42**

S06J4. **1, 3**

S06J5. $\mathbf{\frac{7}{4}}$

S06J6. **12**

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S06J1. This is an arithmetic progression, so $\frac{30}{2}(1+59) = \mathbf{\$9.00}$. Or the sum of the first n odd integers is n^2 or \$9.00.

S06J2. The hexagon consists of 6 equilateral triangles, each with altitiude 8 and side $\frac{16}{\sqrt{3}}$. $6 \cdot \frac{64}{3}\sqrt{3} = \mathbf{128\sqrt{3}}$.

S06J3. The minimum will occur if there is 1 jellybean left. Working backwards, Veronica found 7 jellybeans, Betty found 21 jellybeans, and there were **42** jellybeans originally.

S06J4. Squaring both sides, we get $x^2(3-x) = (3-x)$. So x can be 3, -1, or 1. Checking these answers, we reject -1, so $x =$ **1, 3**.

S06J5. $\frac{7}{2\pi r} = 2r \rightarrow 4\pi r^2 = 7 \rightarrow \pi r^2 = \mathbf{\frac{7}{4}}$.

S06J6. By the triangle inequality, if x is the length of the third side, $4 \le x \le 26$. For the triangle to be obtuse, the square of the length of the largest side must be greater than the sum of the squares of the lengths of the two other sides. Thus x can be 4,5,6,7,8,20,21,22,23,24,25, or 26, for a total of **12** values.

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JUNIOR DIVISION | **CONTEST NUMBER TWO** | **SPRING 2006**

PART I: 10 minutes | **NYCIML Contest Two** | **Spring 2006**

S06J7. If x represents the same digit in the base 10 number $x9x8x9x9$, and $x9x8x9x9$ is a number divisible by 9, compute x.

S06J8. Consider a list of the first 50 positive integers, with the property that each integer is divisible by 3 and is also one less than a perfect square. (i.e. 3, 15, 24, …) compute the 50th number on the list.

PART II: 10 minutes | **NYCIML Contest Two** | **Spring 2006**

S06J9. Compute all real x: $\left(x^2-x-1\right)^{x+4}=1$.

S06J10. A triangle and a trapezoid have equal areas. The altitude of the trapezoid is equal to an altitude of the triangle drawn to a side of length 20. Compute the length of the median (the segment joining the midpoints of the legs) of the trapezoid.

PART III: 10 minutes | **NYCIML Contest Two** | **Spring 2006**

S06J11. Compute the smallest positive integer that is divisible by 15 and contains only the digits 0 and 7.

S06J12. In triangle ABC, $AB=50, m\measuredangle B=60^\circ$, $BC+AC=60$. Compute the length of the shortest side of the triangle.

ANSWERS:

S06J7. **7**

S06J8. **5775**

S06J9. **-4, -1, 0, 2**

S06J10. **10**

S06J11. **7770**

S06J12. $\mathbf{\frac{110}{7}}$

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JUNIOR DIVISION CONTEST NUMBER TWO SPRING 2006

SOLUTIONS

S06J7. The sum of the digits of the number must be divisible by 9. $35+4x$ is divisible by 9 only if $x = 7$.

S06J8. All of the numbers of the form $n^2-1=(n-1)(n+1)$ are divisible by 3, except when n is divisible by 3. The 50th number on the list 2,4,5,7,8,10… is 76. Thus the 50th number on our desired list is $76^2-1=\mathbf{5775}$.

S06J9. There are three possible cases:

$x^2-x-1=1 \rightarrow x=-1,2$

$x^2-x-1=-1$ and $x+4$ is even $\rightarrow x=0$.

$x+4=0 \rightarrow x=-4$

Thus $x =$ **-4, -1, 0, 2.**

S06J10 The length of the median of the trapezoid is ½ the sum of the lengths of the bases. $mh=\frac{1}{2}20h \rightarrow m=\mathbf{10}$.

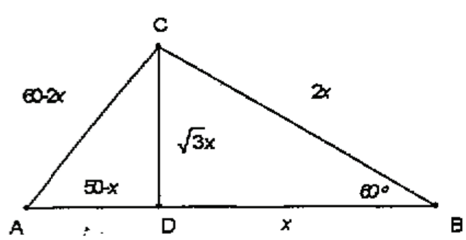
S06J11. To be divisible by 5, the number must end in a 0. To be divisible by 3, the number must have 3 sevens. Thus the answer is **7770**.

S06J12. Let $BD=x$. $CD=\sqrt{3}x$. $BC=2x$. Using triangle ADC,

$(50-x)^2+3x^2=(60-2x)^2$

$4x^2-100x+2500=3600-240x+4x^2$

$140x=1100 \rightarrow x=\frac{55}{7} \rightarrow 2x=\mathbf{\frac{110}{7}}$.



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JUNIOR DIVISION | **CONTEST NUMBER THREE** | **SPRING 2006**

PART I: 10 minutes | **NYCIML Contest Three** | **Spring 2006**

S06J13. A circle with radius 4 has diameter $\overline{AB}$. ABC is an equilateral triangle and $\overline{AC}$ intersects the circle at D. Compute BD.

S06J14. Compute a: $\left(\sqrt[3]{\sqrt[4]{x^2}}\right)\left(\sqrt[5]{\sqrt[6]{x^3}}\right)=x^a$.

PART II: 10 minutes | **NYCIML Contest Three** | **Spring 2006**

S06J15. Three fair diced are rolled and their sum is 6. Compute the probability that all 3 dice show a 2.

S06J16. If x is an integer and $300 \le x \le 600$, compute the number of possible values of x such that it consists of 3 digits in ascending order.

PART III: 10 minutes | **NYCIML Contest Three** | **Spring 2006**

S06J17. A circle is inscribed in a rhombus that has diagonals of length 16 and 30. Compute the radius of the circle.

S06J18. In a sequence of increasing positive integers, each term after the second is the sum of the two terms which immediately precede it. If the tenth term is 301, compute the third term.

ANSWERS:

S06J13. $4\sqrt{3}$

S06J14. $\frac{4}{15}$

S06J15. $\frac{1}{10}$

S06J16. **31**

S06J17. $\frac{120}{17}$

S06J18. **10**

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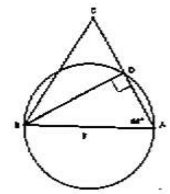
SOLUTIONS



S06J13. ABD is a 30-60-90 right triangle. $BD = \mathbf{4\sqrt{3}}$.

S06J14.

$$\left(\sqrt[3]{\sqrt[4]{x^2}}\right) = \left(\left(x^2\right)^{\frac{1}{4}}\right)^{\frac{1}{3}} = x^{\frac{1}{6}}$$

$$\left(\sqrt[5]{\sqrt[6]{x^3}}\right) = \left(\left(x^3\right)^{\frac{1}{6}}\right)^{\frac{1}{5}} = x^{\frac{1}{10}}$$

$$x^{\frac{1}{6}} \cdot x^{\frac{1}{10}} = x^{\frac{4}{15}} \rightarrow a = \mathbf{\frac{4}{15}}.$$

S06J15. There is one way for all the dice to show a 2, and 9 other permutations that add to 6: $(1,1,4)(1,4,1),(4,1,1),(1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)$. Thus the probability is $\mathbf{\frac{1}{10}}$.

S06J16. In the 300's, 2 numbers must be chosen from 4, 5, 6, 7, 8, and 9. ${}_6C_2 = 15$. In the 400's, 2 numbers must be chosen from 5, 6, 7, 8, and 9. ${}_5C_2 = 10$. In the 500's, 2 numbers must be chosen from 6, 7, 8, and 9. ${}_4C_2 = 6$. $15 + 10 + 6 = \mathbf{31}$.

S06J17. The center of the rhombus is the point of intersection of the diagonals and there are 4 right triangles formed. Thus the radius of the circle is the altitude to the hypotenuse of an 8-15-17 triangle. The area of the triangle is

$$\frac{1}{2} \cdot 8 \cdot 15 = 60 = \frac{1}{2} \cdot r \cdot 17 \rightarrow r = \mathbf{\frac{120}{17}}.$$

S06J18 The terms can be represented as:

$a_1, a_2, a_1 + a_2, a_1 + 2a_2, 2a_1 + 3a_2, 3a_1 + 5a_2, 5a_1 + 8a_2, 8a_1 + 13a_2, 13a_1 + 21a_2, 21a_1 + 34a_2$

$21a_1 + 34a_2 = 301$.

Since 21 and 301 are divisible by 7, a_2 must also be divisible by 7, since 34 is not. 14 is too large, so $a_2 = 7$ and $a_1 = 3$. $a_1 + a_2 = \mathbf{10}$.