NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 1

PART I   ***SPRING 2013***   ***CONTEST 1***   ***TIME: 10 MINUTES***

S13A1   If $a_1 = 20$, $a_2 = 1$, $a_3 = 3$ and $a_n = a_{n-1} - a_{n-2} + a_{n-3}$ for $n \geq 4$, compute a_{2013}.

S13A2   If $1! + 2! + 3! + 6! + 7! + 8! + 10! = (191A)^2$, compute the digit A.

PART II   ***SPRING 2013***   ***CONTEST 1***   ***TIME: 10 MINUTES***

S13A3   Compute all two-digit numbers x such that the number formed by the last two digits of x^2 is x.

S13A4   Circle O is inscribed in hexagon $ABCDEF$, where the lengths of sides AB, BC, CD, DE and EF form an increasing arithmetic progression, in that order. If $FA = 12$, compute the perimeter of hexagon $ABCDEF$.

PART III   ***SPRING 2013***   ***CONTEST 1***   ***TIME: 10 MINUTES***

S13A5   Compute all integers a such that $(a^2 - 7a + 11)^2 + 1$ is a prime.

S13A6   If the positive numbers r, s and t are the roots of the polynomial $ax^3 + bx^2 + cx + d$ and $r + 1, s + 1$ and $t + 1$ are the roots of the polynomial $dx^3 + cx^2 + bx + a$, compute $r + s + t$.

ANSWERS:

S13A1	$\mathbf{20}$
S13A2	$\mathbf{7}$
S13A3	$\mathbf{25, 76}$
S13A4	$\mathbf{72}$
S13A5	$\mathbf{2, 3, 4, 5}$
S13A6	$\mathbf{\frac{-3+3\sqrt{5}}{2}}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 2

PART I   ***SPRING 2013***   ***CONTEST 2***   ***TIME: 10 MINUTES***

S13A7   Compute $(\cos 2°)(\cos 4°) \cdots (\cos 176°)(\cos 178°)$

S13A8   The four-digit number $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}$ is one less than a perfect square and $2(\underline{a}\ \underline{b}) = \underline{c}\ \underline{d}$. Compute all possible four-digit numbers $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}$.

PART II   ***SPRING 2013***   ***CONTEST 2***   ***TIME: 10 MINUTES***

S13A9   Compute $\sin(2\arctan\frac{1}{3})$.

S13A10   The difference between the arithmetic mean and the geometric mean of a^3, b^3 and c^3 is 2013. If a, b and c are positive integers whose sum is between 15 and 55, compute $a + b + c$.

PART III   ***SPRING 2013***   ***CONTEST 2***   ***TIME: 10 MINUTES***

S13A11   If $16! = a!\,b!\,c!$, where $a > b > c$ are positive integers, compute the ordered triple (a, b, c).

S13A12   In regular hexagon $ABCDEF$, $\overline{BE}$ and $\overline{DF}$ intersect at G. Points P, Q, and R are on $\overline{AB}$, $\overline{CD}$ and $\overline{EF}$ respectively such that $\overline{GP} \perp \overline{AB}$, $\overline{GQ} \perp \overline{CD}$, and $\overline{GR} \perp \overline{EF}$. If $AB = 4$, compute $GP + GQ + GR$.

ANSWERS:

S13A7	$\mathbf{0}$
S13A8	$\mathbf{1224, 4488}$
S13A9	$\mathbf{\frac{3}{5}}$
S13A10	$\mathbf{33}$
S13A11	**(14,5,2)**
S13A12	$\mathbf{6\sqrt{3}}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 3

PART I ***SPRING 2013*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S13A13 For $1 \leq i \leq 2013$, $x_i + x_{i+1} = i$. Compute $x_1 - x_{2013}$.

S13A14 The last four digits of a perfect square are $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}$, where a, b, c and d are some permutation of $2, 0, 1$ and 2. Compute the ordered quadruple (a, b, c, d).

PART II ***SPRING 2013*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S13A15 If $1 - \tan^2 x + \tan^4 x - \tan^6 x + \cdots = \frac{1}{2}$, compute all possible values for $\cos x$.

S13A16 In ΔABC with $AB > AC$, point D is on BC such that AD is an angle bisector. Points E and F are the midpoints of AB and BC, respectively, and point G is the intersection of the extensions of AD and EF. Compute $m\angle AGB$.

PART III ***SPRING 2013*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S13A17 The positive integers a and b are relatively prime. Compute the integer $\frac{a^2+b^2}{ab}$.

S13A18 The "cross-number puzzle" is completed by putting the proper digit into each box. All answers to clues are three-digit numbers (they do not begin with 0). Compute the answer to 3-Down.

Across
1. A perfect cube
4. A perfect square
5. A perfect cube and a perfect square

Down
1. A multiple of 17
2. A sum of consecutive factorials
3. A prime

ANSWERS:

S13A13	$\mathbf{-1006}$
S13A14	$(\mathbf{2, 2, 0, 1})$
S13A15	$\pm\frac{\sqrt{2}}{2}$
S13A16	$\mathbf{90^\circ}$
S13A17	2
S13A18	269

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 4

PART I   ***SPRING 2013***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S13A19   Compute $\frac{10^2(10^2-1)(10^2-4)(10^2-9)\cdots(10^2-9^2)}{9^2(9^2-1)(9^2-4)(9^2-9)\cdots(9^2-8^2)}$.

S13A20   The distinct roots of the equation $x^2+bx+c=0$ are r, t, $r-t$ and $r+t$. Compute b in terms of r and t.

PART II   ***SPRING 2013***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S13A21   If $x^{\lceil x \rceil}=\frac{25}{8}$, where $\lceil\quad\rceil$ denotes the ceiling function, compute the value of x.

S13A22   If $x_1+x_2+\cdots+x_{n-1}+x_n=n^2x_n$ for $n>0$ and $x_1=2013$, compute x_{2013}.

PART III   ***SPRING 2013***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S13A23   The integer n is the product of distinct primes. If n has 8 odd divisors, compute all possible values for the total number of divisors of n.

S13A24   The perimeter of ΔABC is 36 and the area is 36. Compute $\left(\tan\frac{A}{2}\right)\left(\tan\frac{B}{2}\right)\left(\tan\frac{C}{2}\right)$.

ANSWERS:

S13A19	380
S13A20	$\boldsymbol{-r}$
S13A21	$\frac{5\sqrt{2}}{4}$
S13A22	$\mathbf{\frac{1}{1007}}$
S13A23	**8, 16**
S13A24	$\mathbf{\frac{1}{9}}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 5

PART I   ***SPRING 2013***   ***CONTEST 5***   ***TIME: 10 MINUTES***

S13A25 Three positive numbers are in a geometric progression and their squares are in an arithmetic progression. Compute the ratio of the largest number to the smallest number.

S13A26 Compute $\log_3 \sqrt[3]{3} + \log_3 \sqrt[9]{9} + \log_3 \sqrt[27]{27} + \cdots$.

PART II   ***SPRING 2013***   ***CONTEST 5***   ***TIME: 10 MINUTES***

S13A27 The median of $20, 13$ and x is equal to the mean. Compute all possible values for x.

S13A28 If $x - y = 1$ and $x^4 + y^4 = 7$, compute all possible values for $x^2 + y^2$.

PART III   ***SPRING 2013***   ***CONTEST 5***   ***TIME: 10 MINUTES***

S13A29 Compute $\sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$, where the positive number x is the answer to this problem.

S13A30 Compute:

$$\sum_{i=1^\circ}^{89^\circ} \frac{1}{1 + \tan^{2013} i}$$

ANSWERS:

S13A25	$\mathbf{1}$
S13A26	$\mathbf{\frac{3}{4}}$
S13A27	$\mathbf{6, 27}$
S13A28	$\mathbf{-5, 3}$
S13A29	$\mathbf{2}$
S13A30	$\frac{89}{2}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 1

SPRING 2013 Solutions

S13A1 **20**. Adding the equations $a_n = a_{n-1} - a_{n-2} + a_{n-3}$ and $a_{n+1} = a_n - a_{n-1} + a_{n-2}$ gives $a_{n+1} = a_{n-3}$, which means our sequence has period 4. Thus $a_{2013} = a_1 = \boxed{20}$.

S13A2 **7**. Observe that the sum $1! + 2! + 3! + 6! + 7! + 8! + 10!$ ends in a 9 and is a multiple of 3. If x^2 ends in a 9, then x must end in a 3 or a 7. Since 1913 is not a multiple of 3, A must be $\boxed{7}$.

S13A3 $\mathbf{25, 76}$. If x^2 and x share their last two digits, then their difference is a multiple of 100. Thus $100|x(x-1)$. Since $x - 1$ and x are consecutive integers, they are relatively prime. Therefore $4|(x-1)$ or $4|x$ and $25|(x-1)$ or $25|x$. The latter condition tells us that $x = 25, 26, 50, 51, 75,$ or 76. Only $\boxed{25, 76}$ satisfy the first condition.

S13A4 $\mathbf{72.}$ Tangents to a circle from a common exterior point are equal, so we can label the tangents a, b, c, d, e and f. Notice that $AB + CD + EF = CD + DE + FA$. Let $a, a + r, a + 2r, a + 3r$ and $a + 4r$ be the side lengths of AB, BC, CD, DE and EF, respectively. Then $FA = a + 2r$. The perimeter of the hexagon is $6a + 12r = 6(a + 2r) = \boxed{72}$.

S13A5 **2,3,4,5**. Notice that a and $a - 7$ are of opposite parity and therefore $a^2 - 7a$ is even. That means that $a^2 - 7a + 11$ is odd and $(a^2 - 7a + 11)^2 + 1$ is even. The only even prime is 2 so we can conclude that $a^2 - 7a + 11 = \pm 1$, whose solutions are $\boxed{2,3,4,5}$.

S13A6 $\mathbf{\frac{-3+3\sqrt{5}}{2}}$**.** For $f(x) = ax^3 + bx^2 + cx + d$, we have that $dx^3 + cx^2 + bx + a = f(\frac{1}{x})$. Thus, $\frac{1}{r}, \frac{1}{s}$ and $\frac{1}{t}$ are the roots of $dx^3 + cx^2 + bx + a$. Without loss of generality, let $r \leq s \leq t$. Then $\frac{1}{t} \leq \frac{1}{s} \leq \frac{1}{r}$ and we have $\frac{1}{t} = r + 1, \frac{1}{s} = s + 1$ and $\frac{1}{r} = t + 1$.

From the first and third equations, we can conclude that $r = t$ which tells us that $r = s = t$. The second equation tells us that $s^2 + s - 1 = 0$ or $s = \frac{-1\pm\sqrt{5}}{2}$. Since s is positive, we have that

$$r + s + t = 3\left(\frac{-1+\sqrt{5}}{2}\right) = \boxed{\frac{-3+3\sqrt{5}}{2}}.$$

New York City Interscholastic Mathematics League

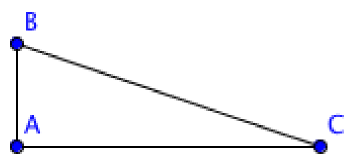
Senior A Division   CONTEST NUMBER 2

SPRING 2013 Solutions

S13A7   **0**. Since the product contains $\cos 90° = 0$, the product is $\boxed{0}$.

S13A8   **1224, 4488**. $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d} = 100(\underline{a}\,\underline{b}) + 2(\underline{a}\,\underline{b}) = 102(\underline{a}\,\underline{b}) = (x-1)(x+1)$. Since $x-1$ and $x+1$ are of the same parity and $102(\underline{a}\,\underline{b})$ is even, both $x-1$ and $x+1$ are even. One of $x-1$ and $x+1$ have to be divisible by 17, so $x-1=34$, $x+1=34$, $x-1=68$, or $x+1=68$. Only $x-1=34$ and $x+1=68$ make $(x-1)(x+1)$ a multiple of 3. Thus $\underline{a}\,\underline{b} = 12$ or 44 and $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d} = \boxed{1224}$ or $\boxed{4488}$.

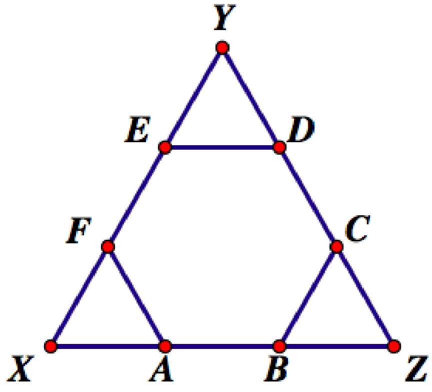
S13A9 $\mathbf{\frac{3}{5}}$. In triangle ABC, if $AB=1$ and $AC=3$, then $m\angle C = \arctan\frac{1}{3}$. Therefore $\sin\left(2\arctan\frac{1}{3}\right) = 2\sin\arctan\frac{1}{3}\cos\arctan\frac{1}{3} = 2\sin C\cos C = 2\left(\frac{1}{\sqrt{10}}\right)\left(\frac{3}{\sqrt{10}}\right) = \boxed{\frac{3}{5}}$.



S13A10   **33**. We have that $\frac{a^3+b^3+c^3}{3} - \sqrt[3]{a^3b^3c^3} = 2013$ or $a^3+b^3+c^3-3abc = 6069$. Since $a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-ac-bc)$, $a+b+c$ must be a divisor of 6069. The only divisor between 15 and 55 is $\boxed{33}$.

S13A11   **(14, 5, 2)**. If $a<13$, then 13 does not divide $a!$, $b!$ or $c!$ and therefore does not divide $16!$ - Contradiction. Thus $a \geq 13$. If $a=13$, then we have that $14*15*16 = b!\,c!$. By a similar argument, $b \geq 7$ which means that 3^2 divides $b!$. Since 3^2 does not divide $14*15*16$, a cannot be 13. If $a=14$, then $15*16 = b!\,c!$. Again, we can conclude $b \geq 5$ and in fact $b=5$ gives $c=2$. Thus our ordered triple is $(14,5,2)$.

S13A12 $6\sqrt{3}$. Let us extend sides AB, CD, and EF until they intersect at points X, Y and Z as below. Notice that triangle XYZ is equilateral. For any point P inside the hexagon, the perpendicular distances from P to AB, CD and EF are the same as the perpendicular distances from P to XY, YZ and ZX. The sum of these distances is the same as the height of the equilateral triangle – to see this, notice that the area of XYZ is equal to the sum of the areas of PXZ, PYZ and PXY; dividing both sides by half of the length of a side of the equilateral triangle produces the result. Thus the sum is $\boxed{6\sqrt{3}}$.



New York City Interscholastic Mathematics League

Senior A Division CONTEST NUMBER 3

SPRING 2013 Solutions

S13A13 **-1006**. $x_1 - x_{2013} = (x_1 + x_2) - (x_2 + x_3) + (x_3 + x_4) - \cdots - (x_{2012} + x_{2013})$
$= (1-2) + (3-4) + \cdots + (2011 - 2012) = (-1) * 1006 = \boxed{-1006}$.

S13A14 $(\mathbf{2}, \mathbf{2}, \mathbf{0}, \mathbf{1})$. The units digit of a perfect square cannot be 2 and if a perfect square ends in 0, it must end in 00; therefore $d = 1$. Since the perfect square is odd and all odd perfect squares are $1\ (mod\ 8)$, $\underline{b}\ \underline{c}\ \underline{d}$ must be $1\ (mod\ 8)$, which only happens of $b = 2$ and $c = 0$. Thus, the ordered quadruple is (2,2,0,1). Note: $149^2 = 22201$.

S13A15 $\pm\frac{\sqrt{2}}{2}$. We have a geometric sequence with common ratio $\tan^2 x$ whose sum is $\frac{1}{1-(-\tan^2 x)}$. Therefore $\frac{1}{1+\tan^2 x} = \frac{1}{\sec^2 x} = cos^2 x = \frac{1}{2}$, or $\cos x = \boxed{\pm\frac{\sqrt{2}}{2}}$.

S13A16 **90°**. Let $m\angle BAG = x$ and $m\angle ABG = y$. Notice that $EF||AC$ and therefore $EG||AC$. Thus, AG is a transversal and $m\angle EGA = m\angle GAC = m\angle BAG = x$. We can conclude that ΔAEG is isosceles and $AE = EG$. Since $AE = EB$, ΔBEG is also isosceles and $m\angle ABG = m\angle EGB = y$. We can now see that ΔAGB has angle x, y and $x + y$ which means $x + y + (x + y) = 180°$. Therefore $m\angle AGB = x + y = \boxed{90°}$.

S13A17 **2**. If $\frac{a^2+b^2}{ab}$ is an integer, then $ab|a^2 + b^2$. This tells us that $a|a^2 + b^2$ or $a|b^2$. Thus, any prime factor, p, of a divides b^2 and must divide b. However, a and b are relatively prime which means a cannot have any prime factors. The same argument can be made for b and so a and b are positive integers with no prime factors, meaning $a = b = 1$ and $\frac{a^2+b^2}{ab} = \boxed{2}$.

S13A18 **269**. For 5-Across, a number that is a square and a cube must be of the form x^6. The only such number that is three-digits is $27^2 = 9^3 = 729$. For 2-Down, the first few factorials are 1, 2, 6, 24, 120, and 720. The sum of some consecutive number of these must end in a 2. This happens for $2 = 2$, $2 + 6 + 24 = 32$, $2 + 6 + 24 + 120 = 152$ and $2 + 6 + 24 + 120 + 720 = 872$. Therefore, 2-Down is 152 or 872. Since no three-digit perfect cube (125, 216, 343, 512, 729) has 8 for a tens digit, 2-Down must be 152 and 1-Across is either 216 or 512. Since the tens digit of 4-Across, which is a perfect square, is odd the units digit must be 6. If 1-Across was 216, then 5-Down would be 669, which is not a prime. Thus 1-Across is 512 and 5-Down is $\boxed{269}$.

New York City Interscholastic Mathematics League

Senior A Division CONTEST NUMBER 4

SPRING 2013 Solutions

S13A19 **380**. $n^2(n^2-1)\cdots(n^2-(n-1)^2) = n^2(n-1)(n+1)\cdots(1)(2n-1) = n(2n-1)!$ Therefore $\frac{10^2(10^2-1)(10^2-4)(10^2-9)\cdots(10^2-9^2)}{9^2(9^2-1)(9^2-4)(9^2-9)\cdots(9^2-8^2)} = \frac{10(19!)}{9(17!)} = \frac{(10)(18)(19)}{9} = \boxed{380}$.

S13A20 $-\boldsymbol{r}$. If $r = t$, then the roots are $r, r, 0$, and $2r$. If $r \neq 0$, then the $0, r$ and $2r$ are distinct and the quadratic has 3 distinct roots, an impossibility. If $r = 0$, then the quadratic has one distinct root. Thus $r \neq t$ and r and t are the roots of the quadratic. If $r + t = t$, then $r = 0$ and the roots are $0, t, -t$ and t. By the same argument above, we get one distinct root or three, which means $r + t \neq t$. Therefore $r + t = r$ and $t = 0$, making the roots $r, 0, r$ and r, which are distinct if $r \neq 0$. Finally, $b = -(r+t) = \boxed{-r}$.

S13A21 $\boldsymbol{\frac{5\sqrt{2}}{4}}$. If $x \leq 1$, then $x^{\lceil x \rceil} \leq 1^1 < \frac{25}{8}$. If $x > 2$, then $x^{\lceil x \rceil} > 2^2 > \frac{25}{8}$. Therefore $1 < x \leq 2$ and $\lceil x \rceil = 2$. Thus $x^2 = \frac{25}{8}$ or $x = \boxed{\frac{5\sqrt{2}}{4}}$.

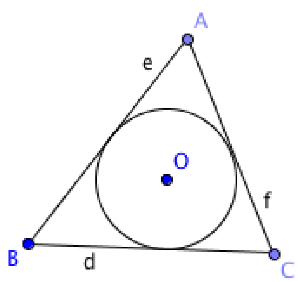
S13A22 $\boldsymbol{\frac{1}{1007}}$. From $x_1 + x_2 + \cdots + x_n = n^2 x_n$ and $x_1 + x_2 + \cdots + x_{n-1} = (n-1)^2 x_{n-1}$, we can conclude that $x_n = n^2 x_n - (n-1)^2 x_{n-1}$ or $(n-1)^2 x_{n-1} = (n^2-1)x_n$. This means that $\frac{x_n}{x_{n-1}} = \frac{n-1}{n+1}$. We can now produce a telescoping product:

$\frac{x_{2013}}{x_1} = \frac{x_2}{x_1} * \frac{x_3}{x_2} * \ldots * \frac{x_{2013}}{x_{2012}} = \frac{1}{3} * \frac{2}{4} * \frac{3}{5} * \ldots * \frac{2012}{2014} = \frac{1*2}{2013*2014}$, and so $x_{2013} = (2013)\left(\frac{1*2}{2013*2014}\right) = \boxed{\frac{1}{1007}}$.

S13A23 **8, 16**. If n is odd, then all of n's divisors are odd which means n has $\boxed{8}$ divisors. If n is even, then $\frac{n}{2}$ is odd and the divisors of $\frac{n}{2}$ are the odd divisors of n and $\frac{n}{2}$ has 8 divisors. Since the number of divisors of a number that is the product of a primes is 2^a (why?), $\frac{n}{2}$ is the product of three distinct odd primes. Then $n = 2pqr$, for p, q, r distinct primes, and has $\boxed{16}$ divisors.

S13A24 $\boldsymbol{\frac{1}{9}}$. Let O be the incircle of ΔABC, with radius r, and label the tangent segments as below. Notice that $d + e + f = s$, where s is the semiperimeter and $d + f = a$, $e + f = b$ and $d + e = c$. Thus $d = s - b$, $e = s - a$ and $f = s - c$. Since O is the intersection of the angle bisectors of the triangle, we have that $\tan\frac{A}{2} = \frac{r}{e}$, $\tan\frac{B}{2} = \frac{r}{d}$ and $\tan\frac{C}{2} = \frac{r}{f}$.

$\tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2} = \left(\frac{r}{s-a}\right)\left(\frac{r}{s-b}\right)\left(\frac{r}{s-c}\right) = \frac{r^3 s^3}{s^3(s-a)(s-b)(s-c)} = \frac{K^3}{s^2K^2} = \frac{K}{s^2} = \frac{36}{18^2} = \boxed{\frac{1}{9}}$, where K is the area of ΔABC.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 5

SPRING 2013 Solutions

S13A25 **1**. Let a, b and c be the three numbers. Then $\frac{c}{b} = \frac{b}{a}$ and $c^2 - b^2 = b^2 - a^2$. The first equation tells us that $ac = b^2$ and a substitution into the second questions gives $a^2 + c^2 = 2ac$, which means $a = c$. Thus, the ratio of the largest number to the smallest number is $\boxed{1}$.

S13A26 $\mathbf{\frac{3}{4}}$. $\log_3 \sqrt[3^n]{3^n} = \log_3 3^{\frac{n}{3^n}} = \frac{n}{3^n}$. The sum can now be written as $\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \cdots$. Let $x = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \cdots$. Then $\frac{x}{3} = \frac{1}{9} + \frac{2}{27} + \frac{3}{81} + \cdots$. Taking the difference gives $\frac{2x}{3} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots = \frac{1}{2}$, and $x = \frac{3}{4}$.

S13A27 $\mathbf{6, 27.}$ If $x \le 13$, then $\frac{x+13+20}{3} = 13$, or $x = 6$. If $13 < x < 20$, then $\frac{x+13+20}{3} = x$, or $x = \frac{33}{2}$, which is not an integer. If $20 \le x$, then $\frac{x+13+20}{3} = 20$, or $x = 27$. Thus $x = \boxed{6}$ or $\boxed{27}$.

S13A28 $\mathbf{-5, 3.}$ Let $a = x^2 + y^2$. $(x - y)^2 = a - 2xy = 1$ and $x^4 + y^4 = a^2 - 2x^2y^2 = 7$. Then the first equation gives $(2xy)^2 = 4x^2y^2 = a^2 - 2a + 1$ and the second gives $4x^2y^2 = 2a^2 - 14$. Thus, $a^2 - 2a + 1 = 2a^2 - 14 \Rightarrow a^2 + 2a - 15 = (a+5)(a-3) = 0$, $a = \boxed{-5,3}$.

S13A29 **2**. We have $x = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}} = \sqrt{x + x} = \sqrt{2x}$, or $x^2 = 2x$, and $x = \boxed{2}$.

S13A30 $\mathbf{\frac{89}{2}.}$ $\frac{1}{1+\tan^{2013} i} + \frac{1}{1+\tan^{2013}(90-i)} = \frac{\cos^{2013} i}{\sin^{2013} i+\cos^{2013} i} + \frac{\cos^{2013}(90-i)}{\sin^{2013}(90-i) +\cos^{2013}(90-i)}$ $= \frac{\cos^{2013} i}{\sin^{2013} i+\cos^{2013} i} + \frac{\sin^{2013} i}{\sin^{2013} i+\cos^{2013} i} = 1$, which means pairing complements in our sum produces 1. We have 44 pairs of complements and $\frac{1}{1+\tan^{2013} 45} = \frac{1}{2}$ giving us a sum of $44 + \frac{1}{2} = \boxed{\frac{89}{2}}$.