NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 1

PART I ***SPRING 2012*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S12A1 If $f(x) = ax^{20} + bx^{12} + 20x + 12$ and $f(-2) = 2012$, compute $f(2)$.

S12A2 If $\sin^6 x + \cos^6 x = \frac{1}{2}$, compute all possible values of $\sin 2x$.

PART II ***SPRING 2012*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S12A3 Chun randomly selects a positive two-digit number. Compute the probability that Chun's number contains the digit 3.

S12A4 In ΔABC, $AB = 11$, $AC = 10$, and $BC = 9$. Points D and E are selected on AC and AB respectively such that ΔADE and quadrilateral $BCDE$ have the same perimeter and area. Compute all possible values for AD.

PART III ***SPRING 2012*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S12A5 The sum of the squares of the digits of 2012 is a perfect square($2^2 + 0^2 + 1^2 + 2^2 = 3^2$). Compute the two smallest integers greater than 2012 with the same property.

S12A6 In ΔABC, with side lengths a, b, and c, $2a^2 + 2b^2 + 8c^2 = 2ab + 4ac + 4bc$. If the perimeter of ΔABC is 20, compute its area.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division CONTEST NUMBER 2

PART I ***SPRING 2012*** ***CONTEST 2*** ***TIME: 10 MINUTES***

S12A7 Compute the smallest integer n such that 2012 divides $n!$.

S12A8 In isosceles ΔABC, where B is the vertex angle, $m\angle B = 30°$ and the length of the circumradius is 5. Compute the area of triangle ΔABC.

PART II ***SPRING 2012*** ***CONTEST 2*** ***TIME: 10 MINUTES***

S12A9 Compute the minimum value of $|x - 20| + |x - 12|$, where x is a real number.

S12A10 Compute the sum of the real roots of the equation $9^x - 17 \cdot 3^{x+2} + 27 = 0$.

PART III ***SPRING 2012*** ***CONTEST 2*** ***TIME: 10 MINUTES***

S12A11 Compute the number of terms in the expansion $(2x + 3y + 5z)^{17}$ after like terms are combined.

S12A12 In ΔABC, $m\angle A = 3(m\angle B)$. If $AC = 4$ and $BC = 7$, compute the length of AB.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 3

PART I ***SPRING 2012*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S12A13 In isosceles ΔABC with angles B and C equal, points D and E are chosen on side AB and points F and G are chosen on side AC such that $CB = BG = GD = DF = FE = EA$. Compute the measure of angle A.

S12A14 Compute the remainder when $x^{10} + 2012x + 1$ is divided by $(x-1)^2$.

PART II ***SPRING 2012*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S12A15 Compute the number of integers between 1 and 100, inclusive, which are not divisible either by 2 or by 7.

S12A16 Compute the positive integer n for which $\frac{(\log_2 2012)(\log_3 2012)\cdots(\log_n 2012)}{n!}$ is maximal.

PART III ***FALL 2008*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S12A17 Compute the remainder when $2012^{(20^{12})}$ is divided by 5.

S12A18 If $\frac{\cos 5x}{\cos 2x} = 2\cos 3x + 1$, where $0° \le x° \le 180°$, compute all possible values of x.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 4

PART I   ***SPRING 2012***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S12A19 Compute the number of positive integer divisors in the number:
$\frac{2012+2012^2+2012^3+2012^4+2012^5}{2012^{-2}+2012^{-3}+2012^{-4}+2012^{-5}+2012^{-6}}$.

S12A20 In ΔABC, medians AD and BE are perpendicular to each other. If $AC = 6$ and $BC = 8$, compute the length of side AB.

PART II   ***SPRING 2012***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S12A21 If $(2^{2^0}+1)(2^{2^1}+1)(2^{2^2}+1)(2^{2^3}+1)(2^{2^4}+1)(2^{2^5}+1) = a^a - 1$, compute a.

S12A22 Mr. A and Mr. Y are playing a game with a bag that contains 2010 papers numbered 1 through 2010. At each turn one participant draws a paper at random with replacement and adds the number on it to the sum on the board, which is initially 0. The winner is the first participant who draws a number that makes the sum a multiple of 2011. If Mr. A goes first, what is the probability that Mr. Y wins?

PART III   ***SPRING 2012***   ***CONTEST 4***   ***TIME: 10 MINUTES***

S12A23 If $x + \frac{1}{x} = 1$, compute $\frac{1}{x^{32}} + \frac{1}{x^{16}} + \frac{1}{x^8} + \frac{1}{x^4} + \frac{1}{x^2} + \frac{1}{x} + x + x^2 + x^4 + x^8 + x^{16} + x^{32}$.

S12A24 Larry found four consecutive positive integers such that the first was a multiple of 3, the second a multiple of 7, the third a multiple of 11 and the fourth a multiple of 15. Compute the minimum possible value of the smallest of the four consecutive integers.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 5

PART I   ***SPRING 2012***   ***CONTEST 5***   ***TIME: 10 MINUTES***

S12A25   Given two concentric circles, a chord of the larger circle is tangent to the smaller circle. If the length of the chord is 17, compute the difference between the areas of the circles.

S12A26   Compute all values a such that the equation $9^x + a = 2 \cdot 3^{x+1}$ has two distinct real solutions.

PART II   ***SPRING 2012***   ***CONTEST 5***   ***TIME: 10 MINUTES***

S12A27   If $(\log_{10} x)^2 + 9(\log_{100} 17)^2 = 6(\log_{10} x)(\log_{100} 17)$, compute x.

S12A28   Compute the remainder when $2012^{(20^{12})}$is divided by 7.

PART III   ***SPRING 2012***   ***CONTEST 5***   ***TIME: 10 MINUTES***

S12A29   Compute the minimum possible value of $\log_y x + \log_{xy} y$, where $y > x \geq 1$.

S12A30   Compute the product $(\sin 10\,°)\,(\sin 50°)(\sin 70°)$.

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NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 1

Spring 2012 Solutions

S12A1 **2092**. $f(2) = a(2)^{20} + b(2)^{12} + 20(2) + 12 = a(-2)^{20} + b(-2)^{12} + 20(2) + 12$
$= a(-2)^{20} + b(-2)^{12} + 20(-2) + 12 + 20(4) = f(-2) + 80 = \mathbf{2092}$.

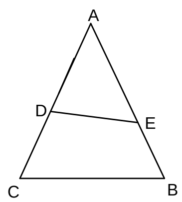
S12A2 $\frac{\pm\sqrt{6}}{3}$**.** Observe that $\sin^6 x + \cos^6 x = (\sin^2 x)^3 + (\cos^2 x)^3$, which is the sum of two cubes. Therefore we can factor the expression:

$$((\sin^2 x)^3 + (\cos^2 x)^3 = (\sin^2 x + \cos^2 x)((\sin^2 x)^2 - (\sin^2 x)(\cos^2 x) + (\cos^2 x)^2)$$
$$= (\sin^2 x + \cos^2 x)((\sin^2 x + \cos^2 x)^2 - 3(\sin^2 x)(\cos^2 x)) = 1 - 3(\sin^2 x)(\cos^2 x) = \frac{1}{2}.$$

This tells us that $\frac{\pm\sqrt{6}}{6} = (\sin x)(\cos x) = \frac{\sin 2x}{2}$, or that $\sin 2x = \frac{\pm\sqrt{6}}{3}$.

S12A3 $\frac{1}{5}$. Let us instead consider the probability that the two-digit number does not contain the digit 3. If *ab* is the two-digit number chosen, then there are 8·9 two-digit numbers that do not contain the digit 3. Since there are 9·10 two-digit numbers, the probability that a randomly selected two-digit number does not contain the digit 3 is $\frac{4}{5}$, which means the probability that it will contain the digit 3 is $1 - \frac{4}{5} = \frac{1}{5}$.

S12A4 $\frac{15\pm\sqrt{5}}{2}$**.** Let $AD = x$.Since triangle *ADE* and quadrilateral *BCDE* have the same perimeter, *AD* + *AE* = *DC* + *CB* + *BE*. Since *AD* + *AE* + *DC* + *CB* + *BE* = 30, *AD* + *AE* = 15 ⇒ $AE = 15 - x$. The area if triangle *ADE* is half the area of triangle *ABC*⇒ $\frac{1}{2}AD \cdot AE \cdot \sin A = \frac{1}{2} \cdot \frac{1}{2} AC \cdot AB \cdot \sin A$
⇒ $x(15 - x) = 55$. Solving this quadratic gives $AD = x = \frac{15\pm\sqrt{5}}{2}$**.** We can check that this is valid, that is that *AD* < *AC* and *AE* < *AB* for both values.



Alternate Solution. Barycentric coordinates! We denote *AE* = *x* and *AD* = *y*. Then, the perimeter of *ADE* is *x* + *y* + *DE* while *BCDE* has perimeter *10 – x + 11 – y + DE*. Thus, we see that *x* + *y* = 15. The barycentric coordinates of *D* are $(1 - \frac{y}{10}, 0, \frac{y}{10})$ and those of *E* are (1 - $\frac{x}{11}$, $\frac{x}{11}$, 0). Thus, the area of *ADE* as a fraction of the total area is $\begin{vmatrix} 1 & 0 & 0 \\ 1 - \frac{x}{11} & \frac{x}{11} & 0 \\ 1 - \frac{y}{10} & 0 & \frac{y}{10} \end{vmatrix} = \frac{xy}{110}$. For this to equal 0.5, we need *xy* = 55. Then the solution follows as above.

S12A5 **2021, 2036**. Let b^2 be the sum of the squares of the digits. We are considering numbers greater than 2012, so let us begin by considering numbers of the form 201<u>a</u>. This gives us $a^2 + 5 = b^2$, whose only solution is $a = 2, b = 3$, which gives us 2012. We next consider numbers of the form 202*<u>a</u>*, which gives us $a^2 + 8 = b^2$, whose only solution is $a = 1, b = 3$, giving us 2021 as a solution. Finally we consider numbers of the form 203<u>a</u>, which gives us $a^2 + 13 = b^2$, whose only solution is $a = 6$, $b = 7$, giving 2036 as a solution.

S12A6 **$4\sqrt{15}$.** Rearranging the terms gives us $(a - b)^2 + (a - 2c)^2 + (b - 2c)^2 = 0$, which can only have equality if $a = b = 2c$. Since the perimeter is 20, we get that $a = b = 8$ and $c = 4$. Applying Heron's formula, we get that $K = \sqrt{(10)(10 - 8)(10 - 8)(10 - 4)} = 4\sqrt{15}$.

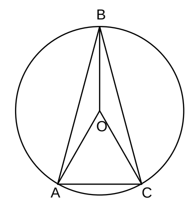
New York City Interscholastic Mathematics League

Senior A Division   CONTEST NUMBER 2

Spring 2012 Solutions

S12A7 **503**. Note that $2012 = 2^2 * 503$. Since 503 is a prime, 503 divides $n!$ iff $n \geq 503$. Since 2^2 divides 503!, 503 is the smallest possible value for *n*.

S12A8 $\mathbf{\frac{25(2+\sqrt{3})}{4}}$. Let *O* be the center of the circle and *AB* = *BC* = x. Notice that angle *AOC* is 60 degrees, making ΔAOC equilateral. Therefore *AC* = 5 and we can now use the law of cosines on ΔABC: $25 = x^2 + x^2 - 2x^2 * cos\,30 \Rightarrow x^2 = \frac{25}{2-\sqrt{3}} = 25(2+\sqrt{3})$. Therefore the area of ΔABC = $\frac{1}{2}x^2 \cdot sin30$ $= \frac{x^2}{4} = \mathbf{\frac{25(2+\sqrt{3})}{4}}$.



Or: Area(*ABC*) = $2R^2\sin A\sin B\sin C = 50 \sin 30° \sin^2 75° = 25\frac{\mathbf{1-\cos 150°}}{\mathbf{2}} = \mathbf{\frac{25(2+\sqrt{3})}{4}}$

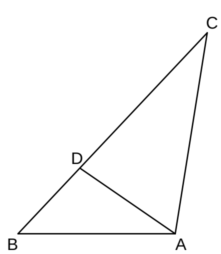
S12A9 **8.** For $x > 20$, $|x-12| > 8 \Rightarrow |x-20| + |x-12| > 8$. Similarly, for $x < 12$, $|x-20| > 8 \Rightarrow |x-20| + |x-12| > 8$. For $12 \leq x \leq 20$, $|x-20| + |x-12| = 20 - x + x - 12 = 8$. Thus the minimum possible value is 8.

Alternate solution: $|x-20| + |x-12| \geq |(x-20)-(x-12)| = 8$, and the equality holds at x = 20.

S12A10 **3**. Let r_1 and r_2 be the roots of the equation and let $y = 3^x$. Then we can write our equation as $y^2 - 153y + 27 = 0$. Notice that the roots of this new equation are 3^{r_1} and 3^{r_2} and the product of the roots is $3^{r_1+r_2} = 27 \Rightarrow r_1 + r_2 = 3$, which is the sum of the roots of the original equation. We also note that, by testing, both of these roots do indeed solve the initial equation.

S12A11 **171**. All of the terms are of the form $x^a y^b z^c$, where $a + b + c = 17$, and since every triple occurs the problem is reduced to finding the number of triples of nonnegative integers, (a, b, c), satisfying the equation. To find the number of solutions to the equation $a + b + c = 17$, we can think of 17 identical items lying across a table and we must use two separators ("stars and bars") to divide the 17 objects into three piles. The number of ways of arranging 17 items and two separators is $\binom{19}{2} = 171$.

S12A12 $\frac{3\sqrt{11}}{2}$**.** Let us draw the angle trisector of angle *A* that is closer to *AB* rather than *AC* and let *D* be the point of intersection of the trisector with segment *BC*:



If $m\angle B = \theta$, then $m\angle DAB = m\angle B = \theta$ and $m\angle CDA = m\angle CAD = 2\theta$, which means triangles BDA and *CDA* are both isosceles. Therefore *AC* = *CD* = 4 and *BD* = *AD* = 3.
Applying the law of cosines on ΔACD, we have: $4^2 = 3^2 + 4^2 - 2 \cdot 3 \cdot 4(\cos 2\theta)$, or $\cos 2\theta = \frac{3}{8}$. Again applying the law of cosines, this time on ΔBDA, we get:

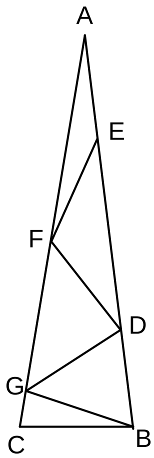
$AB^2 = 3^2 + 3^2 - 2 \cdot 3 \cdot 3(\cos(180 - 2\theta)) = 3^2 + 3^2 + 2 \cdot 3 \cdot 3 \cdot \frac{3}{8} = \frac{99}{4}$, and so $AB = \boxed{\frac{3\sqrt{11}}{2}}$.

New York City Interscholastic Mathematics League

Senior A Division   CONTEST NUMBER 3

Spring 2012 Solutions

S12A13 $\mathbf{\frac{180}{11}}$. Let $x = m\angle A$. Then $m\angle AFE = x \Rightarrow m\angle FED = 2x = m\angle FDE \Rightarrow m\angle GFD = 3x = m\angle FGD \Rightarrow m\angle GDB = 4x = m\angle GBD \Rightarrow m\angle CGB = 5x = m\angle C = m\angle B$. Thus $x + 5x + 5x = 180 \Rightarrow x = \mathbf{\frac{180}{11}}$.



S12A14 $\mathbf{2022x - 8}$. Let $y = x - 1$. Then we are asked to find the remainder when $(y+1)^{10} + 2012(y+1) + 1$ is divided by y^2. $(y+1)^{10} + 2012(y+1) + 1 = y^2 \cdot Q(y) + 2022y + 2014$, where $Q(y)$ is a polynomial in y, which means the remainder is $2022y + 2014 = 2022x - 8$.

S12A15 **43**. Using the Principle of Inclusion-Exclusion, we find that the number of integers between 1 and 100, inclusive, that are divisible by 2 or 7 is $\left\lfloor\frac{100}{2}\right\rfloor + \left\lfloor\frac{100}{7}\right\rfloor - \left\lfloor\frac{100}{2\cdot 7}\right\rfloor = 57$. Thus, the number of integers between 1 and 100, inclusive, that are not divisible by 2 or 7 is $100 - 57 = 43$.

S12A16 **4**. Let $P(n) = \frac{(\log_2 2012)(\log_3 2012)\cdots(\log_n 2012)}{n!} = \left(\frac{\log_2 2012}{2}\right)\left(\frac{\log_3 2012}{3}\right)\cdots\left(\frac{\log_n 2012}{n}\right)$. $\left(\frac{\log_n 2012}{n}\right) < 1 \leftrightarrow \log_n 2012 < n \leftrightarrow 2012 < n^n \leftrightarrow n > 4$, for integer n. Similarly, $\left(\frac{\log_n 2012}{n}\right) > 1$ for n=2,3,4. Thus, for $n \geq 4$, $P(n) > P(n+1)$. Thus $P(n)$ increases until $n = 4$ and decreases after, meaning it reaches its maximal value at $n = 4$.

S12A17 **1**. $2012^{20^{12}} (mod\ 5) \equiv 2^{20^{12}} \equiv (2^4)^{5*20^{11}} \equiv (1)^{5*20^{11}} \equiv 1 (mod\ 5)$.

S12A18 **60°, 180°**. $\frac{\cos 5x}{\cos 2x} = 2\cos 3x + 1 \Rightarrow \cos 5x = 2\cos 2x \cos 3x + \cos 2x$

$\cos 5x = \cos 5x + \cos x + \cos 2x$ by the product-to-sum identity

$$\Rightarrow 0 = \cos x + \cos 2x = 2(\cos x)^2 + \cos x - 1$$

$$= (2\cos x - 1)(\cos x + 1) \Rightarrow \cos x = \tfrac{1}{2} \text{ or } \cos x = -1.$$

For $0° \leq x \leq 180°$, the only solutions are $x = \mathbf{60°}$ **or** $\mathbf{180°}$.

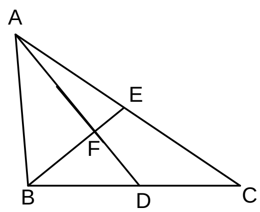
New York City Interscholastic Mathematics League

Senior A Division Contest Number 4

Spring 2012 Solutions

S12A19 **120**. $\frac{2012+2012^2+2012^3+2012^4+2012^5}{2012^{-2}+2012^{-3}+2012^{-4}+2012^{-5}+2012^{-6}} = \frac{2012^7(2012^{-2}+2012^{-3}+2012^{-4}+2012^{-5}+2012^{-6})}{2012^{-2}+2012^{-3}+2012^{-4}+2012^{-5}+2012^{-6}}$ $= 2012^7 = 2^{14}\cdot 503^7$, which has $(14+1)(7+1) = 120$ divisors.

S12A20 $\mathbf{2\sqrt{5}}$. Let F be the point of intersection of the medians and let $DF = x$ and $EF = y$. Since *AD* and *BE* are medians, we know that $AF = 2x$ and $BF = 2y$. Applying the Pythagorean Theorem on triangles *AEF* and *BFD* gives us $4x^2 + y^2 = 9$ and $x^2 + 4y^2 = 16$, respectively. Adding the two equations gives us $5x^2 + 5y^2 = 25$, or $x^2 + y^2 = 5$. Applying the Pythagorean Theorem on triangle *AFB* gives $(AB)^2 = 4x^2 + 4y^2 = 20 \Rightarrow AB = 2\sqrt{5}$.



S12A21 **16.** Multiplying the left side by $1 = (2^{2^0} - 1)$ telescopes the product and the equation becomes:

$$(2^{2^0}-1)(2^{2^0}+1)(2^{2^1}+1)(2^{2^2}+1)(2^{2^3}+1)(2^{2^4}+1)(2^{2^5}+1)$$
$$= (2^{2^1}-1)(2^{2^1}+1)(2^{2^2}+1)(2^{2^3}+1)(2^{2^4}+1)(2^{2^5}+1)$$
$$= (2^{2^2}-1)(2^{2^2}+1)(2^{2^3}+1)(2^{2^4}+1)(2^{2^5}+1) = (2^{2^3}-1)(2^{2^3}+1)(2^{2^4}+1)(2^{2^5}+1)$$
$$= (2^{2^4}-1)(2^{2^4}+1)(2^{2^5}+1) = (2^{2^5}-1)(2^{2^5}+1)$$
$$= 2^{2^6} - 1 = a^a - 1 \Rightarrow a^a = 2^{64} = 16^{16} \Rightarrow \boxed{a = 16}$$

S12A22 $\mathbf{\frac{2010}{4019}}$**.** If the two players are still playing the game at the *n*th turn, where *n* > 1, the sum on the board before the number is drawn is not a multiple of 2011. This means that the sum is $1, 2, 3, \ldots, 2009, \text{ or } 2010 \pmod{2011}$ at the beginning of the turn and there is a unique number that can be drawn to make the sum a multiple of 2011. So, at each turn after the first, the player drawing has a $\frac{1}{2010}$ chance of winning the game at that turn. Therefore the probability that Mr. Y wins is:

$$\frac{1}{2010} + \left(\frac{2009}{2010}\right)^2 \frac{1}{2010} + \left(\frac{2009}{2010}\right)^4 \frac{1}{2010} + \cdots = \frac{\frac{1}{2010}}{1 - \left(\frac{2009}{2010}\right)^2} = \boxed{\frac{2010}{4019}}$$

S12A23 **-4**. Squaring the equation $x + \frac{1}{x} = 1$ gives $x^2 + 2 + \frac{1}{x^2} = 1$, or $x^2 + \frac{1}{x^2} = -1$. By continuing to square the equation and isolating the variables, we see inductively that $x^{2^n} + \frac{1}{x^{2^n}} = -1$ for $n \geq 1$. Therefore $\frac{1}{x^{32}} + \frac{1}{x^{16}} + \frac{1}{x^8} + \frac{1}{x^4} + \frac{1}{x^2} + \frac{1}{x} + x + x^2 + x^4 + x^8 + x^{16} + x^{32} = 5(-1) + 1 = -4$

S12A24 **867.** Let $a, a+1, a+2,$ and $a+3$ be the four consecutive numbers. Note that $4a, 4a+4, 4a+8,$ and $4a+12$ are divisible by 3, 7, 11, and 15. Since $3|4a$, $3|(4a-3)$. Since $7|4a+4$, $7|(4a+4) - 7 \Rightarrow 7|4a-3$. Similarly, 11 and 15 both divide $4a-3$. Thus lcm(3,7,11,15) = 1155 divides $4a-3$. The smallest multiple of 1155 that is of the form $4a-3$ is 1155*3=3465. $3465 = 4a - 3 \Rightarrow a = 867$.

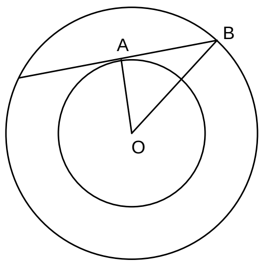
New York City Interscholastic Mathematics League

Senior A Division Contest Number 5

Spring 2012 Solutions

We are recruiting rising college freshmen to join the NYCIML Executive Board! Please announce to your students that, if they are interested, they should contact us at exec@nyciml.org.

S12A25 $\frac{289\pi}{4}$. Let O be the center, A the point of tangency and B one of the endpoints of the chord. Notice that triangle AOB is a right triangle and thus we can apply the Pythagorean Theorem. If r and R represent the radii of the smaller and bigger circles respectively, then we have $AB^2 = R^2 - r^2$. Thus the difference between the areas of the circles is $\pi(R^2 - r^2) = \pi \cdot AB^2 = \pi \cdot \left(\frac{17}{2}\right)^2 = \frac{289\pi}{4}$.



S12A26 $\mathbf{0 < a < 9}$. We can write the equation as $(3^x)^2 - 6(3^x) + a = 0$ and apply the quadratic formula: $3^x = \frac{6 \pm \sqrt{36-4a}}{2}$. The discriminant must be greater than 0, which means $9 > a$. Also, $3^x > 0$ for all x which means $6 - \sqrt{36 - 4a} > 0 \Rightarrow a > 0$. Thus $0 < a < 9$. Alternative answer: (0, 9)

S12A27 $\mathbf{17\sqrt{17}}$. Rearranging terms and factoring gives $(\log_{10} x - 3 * \log_{100} 17)^2 = 0$. $\log_{10} x = 3 * \log_{100} 17 \Rightarrow \log_{100} x^2 = \log_{100} 17^3 \Rightarrow x = \mathbf{17\sqrt{17}}$.

S12A28 **4**. $2012^{20^{12}} (mod\ 7) \equiv 3^{20^{12}} (mod\ 7)$. Observe that $3^6 \equiv 1 (mod\ 7)$ and so we want to reduce 20^{12} *modulo* 6. $20^{12}\ (mod\ 6) \equiv 2^{12} \equiv 4 (mod\ 6)$. Therefore we can write 20^{12} as $6k + 4$, for some integer k. $3^{6k+4} (mod\ 7) \equiv 3^{6k} * 3^4 \equiv (3^6)^k * 3^4 \equiv 1^k * 3^4 \equiv 4\ (mod\ 7)$.

S12A29 **1**. $1 + \log_y x + \log_{xy} y = \log_y xy + \log_{xy} y \geq 2\sqrt{(\log_y xy)(\log_{xy} y)}$, by the AM-GM Inequality. Therefore $\log_y x + \log_{xy} y \geq 2\sqrt{(\log_y xy)(\log_{xy} y)} - 1 = 1$, where the minimum is achieved when $\log_y xy = \log_{xy} y = 1$ [This equation is satisfied for $x = 1$ and any $y > 1$].

S12A30 $\frac{1}{8}$.

$$(\sin 10)(\sin 50)(\sin 70) = \frac{(\cos 10)(\sin 10)(\sin 50)(\sin 70)}{(\cos 10)} = \frac{(\sin 20)(\sin 50)(\sin 70)}{2(\cos 10)}$$

$$= \frac{(\cos 70)(\sin 50)(\sin 70)}{2(\cos 10)} = \frac{(\sin 50)(\sin 140)}{4(\cos 10)} = \frac{(\sin 50)(\cos 50)}{4(\cos 10)} = \frac{(\sin 100)}{8(\cos 10)} = \frac{(\cos 10)}{8(\cos 10)} = \frac{1}{8}.$$