NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 1

PART I | ***FALL 2011*** | ***CONTEST 1*** | ***TIME: 10 MINUTES***

F11A1 Larry selects a 13-digit number while David selects a 10-digit number. Let n be the number of digits in the product of those two numbers. Compute all possible values of n.

F11A2 The roots of the polynomial $x^6 - 12x^5 + Ax^4 + Bx^3 + Cx^2 + Dx + E$ form a geometric progression. If the sum of the reciprocals of the roots is 6, compute E.

PART II | ***FALL 2011*** | ***CONTEST 1*** | ***TIME: 10 MINUTES***

F11A3 Compute the least positive integer s for which $s^3 + 3s^2 + 3s + 18$ is divisible by 17.

F11A4 Jan selected an integer n, where $16 < n < 26$. He then computed n^{2010} and deleted the unit's digit. If the last 5 digits of the new number are 16537, compute n.

PART III | ***FALL 2011*** | ***CONTEST 1*** | ***TIME: 10 MINUTES***

F11A5 Compute the number of positive integer divisors of $50p$, for a prime $p > 2011$.

F11A6 In ΔABC, $AB = 6$, $BC = 8$, and $AC = 10$. Points M and N are on BC and AC, respectively, and MN intersects the angle bisector of $\angle C$ at P. If $MP = 2$ and $PN = 5$, compute the area of ΔMNC.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 2

PART I   ***FALL 2011***   ***CONTEST 2***   ***TIME: 10 MINUTES***

F11A7   If 20! = 2432A02008176640000, compute the digit A.

F11A8   In ΔABC, $AC=BC=1$ and $m\angle C=90°$. Compute the distance between the incenter and the circumcenter of ΔABC.

PART II   ***FALL 2011***   ***CONTEST 2***   ***TIME: 10 MINUTES***

F11A9   Alpha, Beta, and Carl each pick a different prime number and then multiply it by 154. They notice that the new numbers formed all have the same number of divisors. Find the minimum possible value for the sum of the three primes.

F11A10   In ΔRST, $RS=7$, $ST=8$, and $RT=9$. The angle bisector of $\angle R$ intersects the angle bisector of $\angle T$ at M and it intersects ST at N. Compute MN.

PART III   ***FALL 2011***   ***CONTEST 2***   ***TIME: 10 MINUTES***

F11A11   If $Ax^4 + Bx + 28$ is divisible by $x^2 - 3x + 2$, compute the ordered pair (A,B).

F11A12   Let $A(n)$ represent the number of terminating zeroes in $n!$. Compute n, where:

$$n + A(n) + A\big(A(n)\big) + A\Big(A\big(A(n)\big)\Big) + \cdots = 2011$$

New York City Interscholastic Mathematics League

Senior A Division   Contest Number 3

Part I   ***Fall 2011***   ***Contest 3***   ***Time: 10 Minutes***

F11A13 $P(n)$ is equal to the product of the digits of $n!$. Compute:

$$P(0) + P(1) + \cdots + P(2011)$$

F11A14 When expanded, $(x-1)(x-2)\cdots(x-2011) = a_{2011}x^{2011} + a_{2010}x^{2010} + \ldots + a_0$. We can write $a_1 + a_3 + a_5 + \ldots + a_{2011}$ as $(a)(b!)$, where a and b are integers and $a < b$. Compute the ordered pair (a, b).

Part II   ***Fall 2011***   ***Contest 3***   ***Time: 10 Minutes***

F11A15 Compute the units digit of $[(2009)(2011)(2013)]^n$, where n is the answer to this problem.

F11A16 If $Ax^{2012} - Bx^{2011} - 17$ is divisible by $x^2 + x + 1$, compute the ordered pair (A,B).

Part III   ***Fall 2011***   ***Contest 3***   ***Time: 10 Minutes***

F11A17 If $x + y + z + 1 = x + 2y + 4z + 2 = 5x + 4y + 2z + 3$, compute $x + y + z$.

F11A18 ΔABC is an equilateral triangle of side length 1 where J is the midpoint of AB, K is on BC, and L is the trisection point of AC closer to C. Compute the minimum value of $JK + KL$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 4

PART I ***FALL 2011*** ***CONTEST 4*** ***TIME: 10 MINUTES***

F11A19 If $3^{\tan x} = 81^{\sin x}$, compute all possible values of $\cos x$.

F11A20 If $(x+2z):(2y+z):(2x+y) = 1:3:5$ and $x+y+z=18$, compute z.

PART II ***FALL 2011*** ***CONTEST 4*** ***TIME: 10 MINUTES***

F11A21 The number 2011 is a 4-digit number where one of the digits is equal to the sum of the other three (2=0+1+1). Compute the number of 4-digit numbers greater than 9000 that also have this property.

F11A22 Compute the number formed by the last two digits of $[2007 * 2009 * 2011]^n$, where n is the answer to this problem.

PART III ***FALL 2011*** ***CONTEST 4*** ***TIME: 10 MINUTES***

F11A23 In ΔMAT, AH is a median and $AH=HT$. Compute m$\angle$A in degrees.

F11A24 If $\log_{4n} 96 = \log_{5n} 75\sqrt{5}$, compute n^5.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 5

PART I   ***FALL 2011***   ***CONTEST 5***   ***TIME: 10 MINUTES***

F11A25   Compute the average of 2011 consecutive integers, where the smallest integer is 2011.

F11A26   When expanded, $(x-1)(x-2)\cdots(x-2011) = a_{2011}x^{2011} + a_{2010}x^{2010} + \ldots + a_0$. We can write $a_1 + a_3 + a_5 + \ldots + a_{2011}$ as $a * b!$, where a and b are integers and $a < b$. Compute the ordered pair (a, b).

PART II   ***FALL 2011***   ***CONTEST 5***   ***TIME: 10 MINUTES***

F11A27   Equilateral ΔABC, with $AB = 19$, is inscribed in a circle. Point P is chosen on the circumference of the circle such that $AP = 16$ and $BP = 5$. Compute CP.

F11A28   Compute the 2011th digit of the number 12345678910111213...20102011.

PART III   ***FALL 2011***   ***CONTEST 5***   ***TIME: 10 MINUTES***

F11A29   If $a = \log_{10} 2$, express $\log_{125} 20$ in terms of a.

F11A30 $P(x)$ is a polynomial with integer coefficients such that $P(1) = P(2) = P(5) = 2$, $P(3) = -6$ and $P(10) = n$, where $0 < n < 2000$. Compute n.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 1

Fall 2011 Solutions

F11A1 **22,23**. If we let a and b represent Larry and David's numbers, respectively, then we have: $10^{12} \leq a < 10^{13}$ and $10^9 \leq b < 10^{10}$. Therefore $10^{21} \leq ab < 10^{23}$. 10^{21} has 22 digits so n must have at least 22 digits. 10^{23} has 24 digits and is in fact the smallest 24-digit number, which means n has at most 23-digits. If Larry had picked 10^{12} and David had picked 10^9, the product would have 22 digits, while if Larry had picked $10^{13}-1$ and David had picked $10^{10}-1$, the product would have 23 digits, so both values are possible for n. (Challenge: Why does $(10^{13}-1)(10^{10}-1)$ have 23 digits?)

F11A2 **8.** Let a, ar, ar^2, ar^3, ar^4 and ar^5 represent the roots of the polynomial. Since the sum of the roots is 12, we have $a+ar+ar^2+ar^3+ar^4+ar^5$=12. We also have $\frac{1}{a}+\frac{1}{ar}+\frac{1}{ar^2}+\frac{1}{ar^3}+\frac{1}{ar^4}+\frac{1}{ar^5}=\frac{a+ar+ar^2+ar^3+ar^4+ar^5}{a^2r^5}=\frac{12}{a^2r^5}=6$, or $a^2r^5=2$. The product of the roots is $a^6r^{15}=2^3=$ 8.

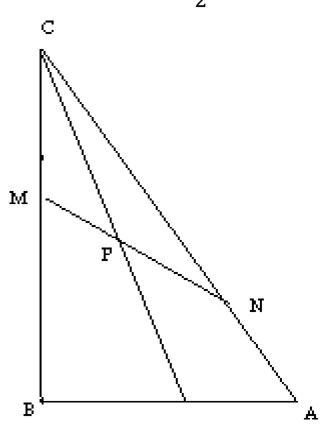
F11A3 **16**. $s^3+3s^2+3s+18=(s+1)^3+17$, which means that if $s^3+3s^2+3s+18$ is divisible by 17, so is $(s+1)^3$. Since s is prime, this also means that 17 divides $(s+1)$. The smallest positive integer that is one less than a multiple of 17 is 16.

F11A4 **24**. Let us observe that the tens digit of n^{2010} (which is a perfect square) is 7. If the tens digit of a perfect square is odd, then the unit's digit of that square must be 6 (and vice-versa). Therefore the unit's digit of n^{2010} is 6. What can n then be? Clearly n must be even and not a multiple of 10. If the units digit of n is 2, then we use the fact that $2^5 = 2$ (mod 10) to see that 2^{2010} (mod 10) = 2^{402} (mod 10) = 4 2^{80} (mod 10) = 4 2^{16} (mod 10) = 8 2^3 (mod 10) = 4 (mod 10). Since 8 = -2 (mod 10), then 8^{2010} (mod 10) = $(-1)^{2010}(2)^{2010}$ = 4 (mod 10). The only remaining even number in the range that is not a multiple of 10 is 24, so this must be the answer.

Challenge: Why is the tens digit of a perfect square odd iff the units digit is 6?

F11A5 **12**. $50p = 2^1 5^2 p^1$, which means the number of divisors is (1+1)(2+1)(1+1)=12.

F11A6 $\mathbf{\frac{147}{13}}$. By the Angle Bisector Theorem, we know $\frac{MC}{CN}=\frac{2}{5}$. If we let $MC = 2x$ and $CN = 5x$, we can apply the law of cosines on triangle MCN: $49 = 29x^2 - 20x^2\left(\frac{4}{5}\right) = 13x^2$, or $\frac{49}{13}=x^2$. The area of triangle MCN is $\frac{1}{2}(MC)(CN)\sin(\angle MCN) = \frac{1}{2}(2x)(5x)\frac{3}{5} = 3x^2 = \frac{147}{13}$.



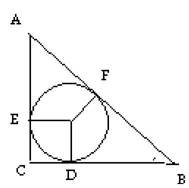
NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 2

Fall 2011 Solutions

F11A7 **9**. Since 20! is divisible by 11, we can apply the divisibility test for 11. This means that $(2+3+A+2+0+1+6+4+0+0)-(4+2+0+0+8+7+6+0+0)=A-9$ is divisible by 11, which means $A=9$.

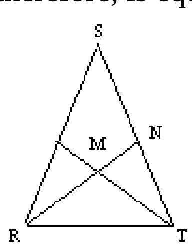
F11A8 $\mathbf{\frac{2-\sqrt{2}}{2}}$. Let *D*, *E*, and *F* be the points of tangency of the incircle with the sides of the triangle, as shown below, $EC=CD$ since they are tangents to a circle from a common point. Since, $AC=BC, AE = BD \Rightarrow AF=FB$. This means that F is the midpoint of AB. The circumcenter of a right triangle is the midpoint of the hypotenuse since the hypotenuse is a diameter of the circumcircle. Thus, F is the circumcenter and the distance between the incenter and the circumcenter is just the inradius of the incircle. The area of the triangle is $\frac{1}{2}(1)(1)=\frac{1}{2}$. The semi-perimeter is $\frac{1}{2}\left(1+1+\sqrt{2}\right)=\frac{2+\sqrt{2}}{2}$. Since $K=rs$, where K is the area of the triangle, $r=\frac{K}{s}=\frac{\frac{1}{2}}{\frac{2+\sqrt{2}}{2}}=\frac{2-\sqrt{2}}{2}$.



Alternate solution. Let D, E, and F be the points of tangency of the incircle (with center *O*) with the sides of the triangle. The circumcenter of right triangle *ABC* is the midpoint of the hypotenuse, since the hypotenuse is a diameter of the circumcircle. Due to symmetry, *F* is this midpoint. Since *OF* is perpendicular to *AB*, the desired distance is the radius of the incircle, which we denote *r*. By the property of tangents to a circle, *EC* = *CD* = *r*, *AE* = *AF* = *1-r*, and *BD* = *BF* = *1-r*. Then, $\sqrt{2}$ = *AB* = *AF* + *BF* = *2 – 2r* so *r* = $\frac{2-\sqrt{2}}{2}$.

F11A9 **20.** $154p=(2)(7)(11)(p)$. The number of divisors of $154p$ is 16 = (1+1)(1+1)(1+1)(1+1) for all primes not equal to 2,7, or 11, and 12 = (2+1)(1+1)(1+1) when p is 2, 7 or 11. This means the Alpha, Beta, and Carl will have the same number of divisors for their new numbers if their three primes were 2,7 and 11, or 3 primes none of which are 2, 7 or 11. If the three primes are 2, 7 and 11, their sum is $2+7+11=20$. If none of the three primes are 2,7 or 11, the minimum possible sum is $3+5+13=21$. Therefore the minimum possible sum is 20.

F11A10 $\mathbf{\frac{\sqrt{21}}{2}}$. From the Angle-Bisector Theorem, we get that $SN=\frac{7}{2}$ and $NT=\frac{9}{2}$. $RN^2=7*9-\frac{(7)(9)}{(2)(2)}\rightarrow RN=\frac{3\sqrt{21}}{2}$. Since the angle bisector through *T* is also an angle bisector of *ΔRTN*, we can apply the Angle-Bisector Theorem on *ΔRTN*. This tells us that $\frac{RM}{MN}=\frac{RT}{TN}=\frac{9}{9/2}=2$, or $RM\ =2MN$. $R\ \ N$, therefore, is equal to $3MN$, which means that $MN=\frac{1}{3}RN=\mathbf{\frac{\sqrt{21}}{2}}$.



F11A11 **(2,-30)**. Since 1 and 2 are roots of $x^2 - 3x + 2$, they must be roots of $Ax^4 + Bx + 28$, which means $A + B + 28 = 0$ and $16A + 2B + 28 = 0$. Solving, we get $(A, B) = (2, -30)$.

F11A12 **1516.** $A(n) = \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{5^2} \right\rfloor + \left\lfloor \frac{n}{5^3} \right\rfloor + \cdots < \frac{n}{5} + \frac{n}{5^2} + \frac{n}{5^3} + \cdots = \frac{n}{4}$. This means that: $2011 = n + A(n) + A\big(A(n)\big) + A\Big(A\big(A(n)\big)\Big) + \cdots < n + \frac{n}{4} + \frac{n}{4^2} + \frac{n}{4^3} + \cdots = \frac{4n}{3}$, or $n > 1508.25$. For $n = 1510$, we have that $n + A(n) + A\big(A(n)\big) + A\Big(A\big(A(n)\big)\Big) + \cdots = 1510 + (302 + 60 + 12 + 2) + A(376) + \cdots = 1510 + 376 + (75 + 15 + 3) + A(93) + \cdots = 1510 + 376 + 93 + 21 + A(21) = 1510 + 376 + 93 + 21 + 4 + A(4) = 2004$. For $n = 1515$, expanding similarly, the desired sum $n + A(n) + A\big(A(n)\big) + A\Big(A\big(A(n)\big)\Big) + \cdots$ is 2010. Therefore, for $n = 1515$, the desired sum $n + A(n) + A\big(A(n)\big) + A\Big(A\big(A(n)\big)\Big) + \cdots$ is 2011. Since $n + A(n) + A\big(A(n)\big) + A\Big(A\big(A(n)\big)\Big) + \cdots$ is strictly increasing, the answer will be unique, so we don't need to check higher n.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 3

Fall 2011 Solutions

F11A13 **18**. $P(0) = 1, P(1) = 1, P(2) = 2, P(3) = 6,$ and $P(4) = (2)(4) = 8$. For $n \geq 5$, $n!$ ends in a 0 which means $P(n) = 0$. Thus, $P(0) + P(1) + \cdots + P(2011) = 1 + 1 + 2 + 6 + 8 + 0 + 0 + \cdots + 0 = 18$.

F11A14 **(1006,2011)**. Plugging in 1 for *x*, we get $a_0 + a_1 + \cdots + a_{2010} + a_{2011} = 0$. Plugging in -1 for x, we get $a_0 - a_1 + \cdots + a_{2010} - a_{2011} = (-2)(-3)\cdots(-2012) = -2012!$. Subtracting the latter equation from the former gives us $2(a_1 + a_3 + \cdots + a_{2009} + a_{2011}) = 2012!$. Dividing by 2 gives us $a_1 + a_3 + \cdots + a_{2009} + a_{2011} = (1006)(2011!)$, which means (a, b) is $(1006{,}2011)$.

F11A15 **3**. This problem reduces down to finding the digit n such that$[(2009)(2011)(2013)]^n \equiv n \pmod{10}$. We have that $[(2009)(2011)(2013)]^n \equiv [7]^n \pmod{10}$, which cycles as follows: 7, 9, 3, 1, 7, 9, 3, 1, 7, 9,.... The only digit that satisfies the congruence is *n* = 3.

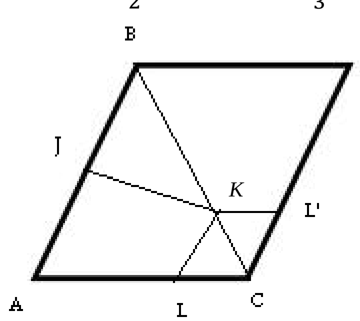
F11A16 **(-17,17)**. Let r and s be the roots of $x^2 + x + 1$. Then r and s are also roots of $(x - 1)(x^2 + x + 1) = x^3 - 1$. This means that $r^3 = s^3 = 1$. If $x^2 + x + 1$ divides $Ax^{2012} - Bx^{2011} - 17$, then r and s are roots of $Ax^{2012} - Bx^{2011} - 17$, which means $Ar^{2012} - Br^{2011} - 17 = 0$ and $As^{2012} - Bs^{2011} - 17 = 0$. Since $r^3 = s^3 = 1$, $Ar^{2012} - Br^{2011} - 17 = Ar^2 - Br - 17 = 0$ and $As^{2012} - Bs^{2011} - 17 = As^2 - Bs - 17 = 0$. This means that r and s are roots of $Ax^2 - Bx - 17$, or that $Ax^2 - Bx - 17$ is divisible by $x^2 + x + 1$. Since both polynomials are the same order, one must be a multiple of each other, which only happens when *A* = − 17 and *B* = 17.

F11A17 $-\frac{3}{4}$. The system of equations has a solution, as we can verify that no term is a linear combination of the other two. Then, we have that

$$(x + y + z + 1) + (x + y + z + 1) = (x + 2y + 4z + 2) + (5x + 4y + 2z + 3)$$
$$2(x + y + z) + 2 = 6(x + y + z) + 5$$
$$4(x + y + z) = -3, \text{or } x + y + z = -\frac{3}{4}$$

F11A18 $\frac{\sqrt{31}}{6}$. Let's reflect triangle *ABC* over *BC*, and let *L'* be the image of *L* over this reflection. Then *JK* + *KL* = *JK* + *KL'*. The minimum possible value for this sum would be achieved when *J*, *K* and *L'* are collinear. Let us consider triangle *JCL'*. *JC* is an angle bisector of angle *C*, which means the measure of angle *JCK* is 30°. The measure of angle *KCL'* is 60°, which means the measure of angle *JCL'* is 90°.

Since $JC = \frac{\sqrt{3}}{2}$ and $CL' = \frac{1}{3}$, $JL' = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{3}\right)^2} = \frac{\sqrt{31}}{6}$.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 4

Fall 2011 Solutions

F11A19 **$\pm 1,\ 1/4$**. Modifying the right hand side, we get $3^{\tan x} = 81^{\sin x} = 3^{(4\sin x)}$. The equation $4\sin x = \tan x$ yields $\sin x = 0$ or $\cos x = \frac{1}{4}$. Therefore all possible values for $\cos x$ are $\pm 1,\ 1/4$.

F11A20 **-2.** Let $x + 2z = r, 2y + z = 3r$, and $2x + y = 5r$. Adding those three equations, we get $3(x + y + z) = 9r$. Since $x + y + z = 18, r = 6$. We now have $2y + z = 18$ and $x + y + z = 18$, which can be subtracted to give us $x = y$. Solving, we get $x = y = 10$ and $z = -2$.

F11A21 **55.** Since the four-digit numbers are all greater than 9000, the largest digit of the 4 will be 9. All numbers will be of the form 9<u>ABC</u>, where A+B+C=9. This problem is equivalent to counting the number of ways we can divide 9 objects into 3 groups by placing separators between the objects. That means that the number of solutions is $\binom{11}{2} = 55$.

F11A22 **93**. This problem reduces down to finding the two digit number n such that $[(2007)(2009)(2011)]^n \equiv n \pmod{100}$. $[(2007)(2009)(2011)]^n \equiv [-7]^n \pmod{100}$, which cycles as follows: $-7, 49, -43, 1, -7, 49, \ldots \pmod{100}$, which can be rewritten as $93, 49, 57, 1, \ldots \pmod{100}$. Since all of these potential values for n are $1 \pmod 4$, and since $[-7]^{4k+1} \equiv [-7]^1 \equiv 93 \pmod{100}$, n must be 93.

F11A23 **90°**. Since MH = AH = HT, triangles MHA and AHT are both isosceles. If we let α and β represent angles M and T, respectively, we can see that m∠MAH + m∠HAT = m∠A = α + β. m∠A+ m∠M+ m∠T=2(α + β)=180 and α + β=90.

F11A24 **9.** Let $\log_{4n} 96 = \log_{5n} 75\sqrt{5} = a$. Then $96 = (4n)^a$ *and* $75\sqrt{5} = (5n)^a$. Dividing, we get: $\frac{(5n)^a}{(4n)^a} = (\frac{5}{4})^a = \frac{75\sqrt{5}}{96} = \frac{25\sqrt{5}}{32} = \left(\frac{5}{4}\right)^{5/2}$, or $a = 5/2$. Since$96 = (4n)^{5/2}, 96^2 = (4n)^5, or\ n^5 = 9$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

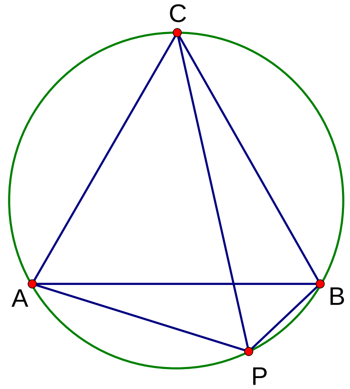
Senior A Division   CONTEST NUMBER 5

Fall 2011 Solutions

F11A25 **3016**. Let's represent the 2011 consecutive integers as $x-1005, x-1004, \ldots, x-1, x, x+1, \ldots, x+1004, and\ x+1005$. Then the sum of the integers is 2011x, and their average is x. Since the smallest integer is 2011, $x-1005=2011$, or $x=3016$.

F11A26 **(1006,2011)**. Plugging in 1 for x, we get $a_0+a_1+\cdots+a_{2010}+a_{2011}=0$. Plugging in -1 for x, we get $a_0-a_1+\cdots+a_{2010}-a_{2011}=(-2)(-3)\cdots(-2012)=-2012!$. Subtracting the latter equation from the former gives us $2(a_1+a_3+\cdots+a_{2009}+a_{2011})=2012!$. Dividing by 2 gives us $a_1+a_3+\cdots+a_{2009}+a_{2011}=1006*2011!$, which means (a,b) is $(1006,2011)$.

F11A27 **21.** Since A, B, C, and P all lie on the same circle, quadrilateral ABCP is cyclic. Applying Ptolemy's Theorem, we get $(AC)(BP)+(BC)(AP)=(AB)(CP)$. Since $AC=BC=AB=19$, we can divide the equation by 19 to get $BP+AP=CP$. Therefore $CP=5+16=\boxed{21}$.



F11A28 **7**. There are 9 one-digit numbers, 9*10=90 two-digit numbers, and 9*10*10=900 three-digit numbers. The 9 one-digit numbers give 9 digits to our number, the two-digit numbers give 180 digits, and the three digit numbers give 2700 digits. After writing out all the one digit numbers and two-digit numbers, we have 2011-180-9= 1822 digits to write out until we reach the 2011h digit. Since 1822=3*607 +1, we are looking for the first digit of the 608th three-digit number. We can find that the 608th three-digit number is 707, and therefore the 2011th digit is 7.

F11A29 $\mathbf{\frac{1+a}{3-3a}}$. $\log_{125} 20=\frac{\log 20}{\log 125}=\frac{\log 10+\log 2}{\log 1000-\log 8}=\frac{1+a}{3-3a}$.

F11A30 **722.** If P(x) is a polynomial with integer coefficients, then for integers a and b, $(a-b)|P(a)-P(b)$. This means that $(10-1)|(P(10)-P(1))$, or that 9$|(n-2)$. This also means that $(10-2)|(P(10)-P(2))$, or that 8$|(n-2)$ and that $(10-5)|(P(10)-P(5))$, or that 5$|(n-2)$. This tells us that $5*8*9=360|n-2$, or that n is 2 more than a multiple of 360. Since $0<n<2500$, this narrows n down to $362, 722, 1082, 1442,$ and 1802. Since $P(3)=-6$, we know that $(10-3)|(P(10)-P(3))$ or that $7|(n+6)$. Only $n=722$ satisfies $7|(n+6)$ and therefore $n=722$.