NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Soph-Frosh Division **CONTEST NUMBER 1**

PART I ***SPRING 2010*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S10SF1 A 25-foot ladder leans against a vertical wall. The top of the ladder is 20 feet above the ground. If the top of the ladder slips 13 feet down the wall, compute the number of feet that the bottom of the ladder moves away from the wall.

S10SF2 The area of a square inscribed in a semicircle, with one side of the square on the straight side of the semicircle, is 40. Compute the area of a square inscribed in the entire circle.

PART II ***SPRING 2010*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S10SF3 A grocer buys apples at 3 for $1, and sells them for 7 for $3. Compute the number of apples he must sell to get a profit of $40.

S10SF4 Express .312312312… as a fraction in lowest terms.

PART III ***SPRING 2010*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S10SF5 The numbers in a sequence are as follows: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5,… where each positive integer n appears n times. Compute the 500^{th} term in the sequence.

S10SF6 If x men can do a job in y days, compute, in terms of x, y, and z, the number of days it would take $x+z$ men to do the job, assuming that the men are working at the same rate.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Soph-Frosh Division **CONTEST NUMBER 2**

PART I ***SPRING 2010*** ***CONTEST 2*** ***TIME: 10 MINUTES***

S10SF7 The perimeter of a rectangle is P. If the width of the rectangle is one-half of the length, compute the area of the rectangle, in simplest form, in terms of P.

S10SF8 Compute the absolute value of the difference between the two roots of $x^2 + 3x - 5 = 0$.

PART II ***SPRING 2010*** ***CONTEST 2*** ***TIME: 10 MINUTES***

S10SF9 At a party, 276 handshakes took place. If each person shook hands with each other person exactly once, compute the number of people at the party.

S10SF10 In ΔABC, $AB = 80$, $m\angle B = 60°$, $BC + AC = 90$. Compute the length of the shortest side of the triangle.

PART III ***SPRING 2010*** ***CONTEST 2*** ***TIME: 10 MINUTES***

S10SF11 Two parallel chords of length 10 are drawn in a circle with radius 10. Compute the area of the region between the two chords and inside the circle.

S10SF12 Ten lines are drawn in a plane, no two of which are parallel and no three of which intersect at the same point. Compute the number of regions into which the lines divide the plane.

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Soph-Frosh Division **CONTEST NUMBER 3**

PART I ***SPRING 2010*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S10SF13 Compute the area of a regular hexagon inscribed in a circle with radius 12.

S10SF14 Compute the sum of the reciprocals of the roots of $x^2 - 5x - 8 = 0$.

PART II ***SPRING 2010*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S10SF15 Compute the smallest positive integer which leaves a remainder of 9 when divided by 10, a remainder of 8 when divided by 9..., down to a remainder of 1 when divided by 2.

S10SF16 Compute the number of ounces of pure acid that a chemist should add to 12 ounces of a mix which is 70% acid to make it 80% acid.

PART III ***SPRING 2010*** ***CONTEST 3*** ***TIME: 10 MINUTES***

S10SF17 Compute the sum of the digits of the first 100 positive odd integers.

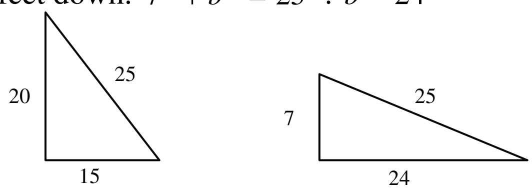
S10SF18 Compute the area of an isosceles right triangle with a perimeter of 2.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Soph-Frosh Division CONTEST NUMBER 1

Spring 2010 Solutions

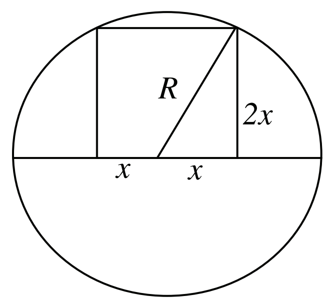
S10SF1 **9.** Draw triangles to represent the wall, the ladder and the ground.
Use the Pythagorean Theorem to find the distance from the wall after the ladder slipped 13 feet down. $7^2 + b^2 = 25^2$. $b = 24$



$24 - 15 = 9$

S10SF2 **100.** One way to consider the problem is to find the ratio of the areas of the squares. The area of the big square is $\frac{1}{2}(diagonal)^2 = \frac{1}{2}(2R)^2 = 2R^2$. Let A be the area of the square inscribed in the semicircle

$$x^2 + (2x)^2 = R^2$$
$$5x^2 = R^2$$
$$A = 4x^2 = \frac{4}{5}R^2$$
$$\frac{4}{5}R^2 : 2R^2 = \frac{2}{5}.$$



Since the area of the small square is 40, the area of the large square is 100.

S10SF3 **420.** He buys 21 apples for \$7 and sells 21 apples for \$9. Since he earns \$2 for every 21 sold, he must sell $21 \times 20 = 420$ apples to earn \$40.

S10SF4 $\mathbf{\frac{104}{333}}$**.** Let $x = .\overline{312}$. Then $1000x = 312.\overline{312}$. Subtract to get $999x = 312$.
Then $x = \frac{312}{999} = \frac{104}{333}$.

S10SF5 **32.** Use trial and error to determine the k^{th} term where the last N appears. The formula is $\frac{N(N+1)}{2}$. When $N = 31$, the last number 31 appears in the $\frac{31(31+1)}{2} = 496^{\text{th}}$ term of the sequence. Therefore, the 500^{th} number is 32.

S10SF6 $\boldsymbol{\frac{xy}{x+z}}$**.** One man can do the job in xy days. $x + z$ men can do the job in $\frac{xy}{x+z}$ days.

New York City Interscholastic Mathematics League

Soph-Frosh Division CONTEST NUMBER 2

Spring 2010 Solutions

S10SF7 $\boldsymbol{\frac{P^2}{18}}$**.** Let $x =$ width, $2x =$ length. The perimeter is $6x$, so $x = \frac{P}{6}$. Thus the area is $2x^2 = 2\left(\frac{P^2}{36}\right) = \frac{P^2}{18}$.

S10SF8 $\boldsymbol{\sqrt{29}}$**.** Use the quadratic formula to find the roots and then subtract. The requested difference is $\frac{-3+\sqrt{29}}{2} - \frac{-3-\sqrt{29}}{2} = \sqrt{29}.$

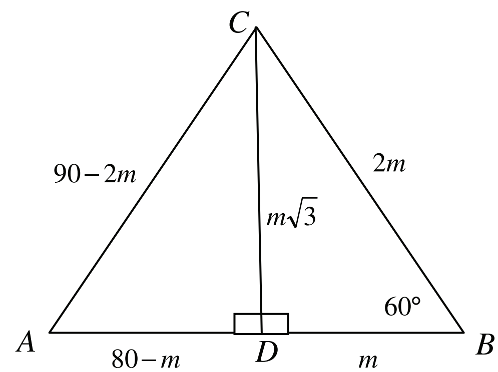
S10SF9 **24.** Let N be the number of people at the party. Then each person shook hands with N-1 people. The product $N(N$-1) counts each handshake twice, so then the number of handshakes is $\frac{N(N-1)}{2} = 276$. Solve to obtain $N = 24$ and reject-23.

S10SF10 **17.** Draw the altitude from C to $\overline{AB}$. Using ΔACD, $(90-2m)^2 = \left(m\sqrt{3}\right)^2 + (80-m)^2$.

$8100 - 360m + 4m^2 = 3m^2 + 6400 - 160m + m^2$

$200m = 1700$

$m = \frac{17}{2}$ $\qquad \overline{BC} = 2m = 17.$



Alternative Solution: Use the Law of Cosines.

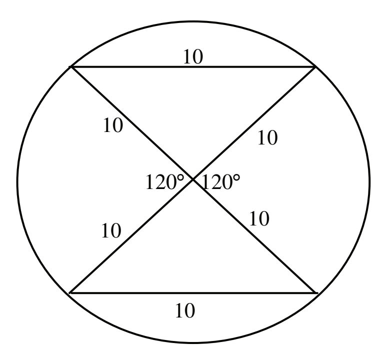
Let $\overline{BC} = x$ and $\overline{AC} = 90 - x$.

Then $(90+x)^2 = 80^2 + x^2 - 2(80x)\cos 60.$

So, $x = 17$.

S10SF11 $\boldsymbol{50\sqrt{3} + \frac{200\pi}{3}}$**.** The region consists of two equilateral triangles with side 10 and two $120°$ sectors with radius 10.

The requested area is $2\frac{100}{4}\sqrt{3} + \frac{2}{3}100\pi = 50\sqrt{3} + \frac{200\pi}{3}.$



S10SF12 **56.** Use a chart of numbers of lines and regions. When a new line is drawn, it divides each region that it enters into two regions. But the number of regions a new line enters is one more than the number of previously existing lines. Thus the second line creates 2 more regions, the third line creates 3 more regions, and, in general, for n > 1, the nth line creates n more regions. Thus the requested number of regions is $2+2+3+4+\cdots+10 = 56$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Soph-Frosh Division **CONTEST NUMBER 3**

Spring 2010 Solutions

S10SF13 **$216\sqrt{3}$.** The hexagon consists of 6 equilateral triangles with side 12. Therefore the requested area is $6\times\frac{12^2}{4}\sqrt{3}=216\sqrt{3}$.

S10SF14 $\mathbf{\frac{-5}{8}}$ **.** Let the roots be r and s. $\frac{1}{r}+\frac{1}{s}=\frac{r+s}{rs}$. Then in the equation $a^2+bx+c=0$, $-b$ is the sum of the roots and c is the product of the roots. Therefore $r+s=5$ and $rs=-8$. Thus $\frac{1}{r}+\frac{1}{s}=\frac{-5}{8}$.

S10SF15 **2,519.** The solution must be one less than a multiple of 10, one less than a multiple of 9, one less than a multiple of 8, …. Thus, the solution must be one less than the least common multiple of 2,3,4,5,6,7,8, 9, and 10 namely $2520-1=2519$.

S10SF16 **6.** Let x be the number of ounces of acid added. Then $x+.7(12)=.8(12+x)$ so $x=6$.

S10SF17 **1,000.** The first number is 1 and the last number is 199. In the units digits, there are 20 of each of the digits 1,3,5,7,9, so the sum of the units digits is $20(1+3+5+7+9)=500$. In the tens digits, there are 10 of each of the digits 1,2,3,4,5,6,7,8,9, so the sum of the tens digits is $10(45)=450$. In the hundreds digits, there are 50 ones. Thus the requested sum is $500+450+50=1000$.

S10SF18 **$3-2\sqrt{2}$.** Let x be a leg of the isosceles right triangle.

The perimeter is $2x+x\sqrt{2}=2$

$x(2+\sqrt{2})=2$

$x=\frac{2}{2+\sqrt{2}}$

Then the area is

$$A=\frac{1}{2}x^2=\frac{1}{2}\left(\frac{2}{2+\sqrt{2}}\right)^2=\frac{2}{6+4\sqrt{2}}$$

$$=\frac{1}{3+2\sqrt{2}}\frac{(3-2\sqrt{2})}{(3-2\sqrt{2})}=3-2\sqrt{2}$$