NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior B Division CONTEST NUMBER 1

PART I ***SPRING 2010*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S10B01 If $\frac{1}{a} - \frac{1}{b} = 20$ and $a - b = -8$, compute the value of the product ab.

S10B02 The sides of a triangle are of length 4, $2\sqrt{5}$ and x. Compute all possible integer values of x such that the largest angle of the triangle is less than $90°$.

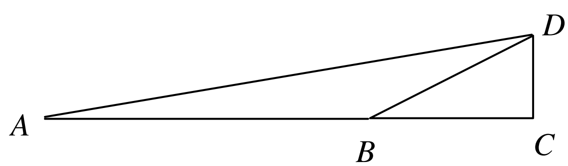
PART II ***SPRING 2010*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S10B03 Let $f(x)$ be a function such that $f(1) = 1$, $f(2) = 3$, and $f(n) = 2f(n-1) - f(n-2)$. Compute the value of $f(2010)$.

S10B04 In the game *Combat*, a red army is worth 10 points, a blue army is worth 3 points, and a white army is worth 1 point. While I am playing I notice that the total value of my red armies is 40 points more than the total value of my blue and white armies put together. Also, the number of red armies is one-third the total number of blue and white armies, and the total value of all my armies is 240 points. Compute the total number of armies I have.

PART III ***SPRING 2010*** ***CONTEST 1*** ***TIME: 10 MINUTES***

S10B05 In the diagram below, which is not drawn to scale, $m\angle C = 90°$, $m\angle CBD = 30°$ and the ratio of BC to AB is 1 to 3. Compute the value of $\sin A$.



S10B06 Compute the number of perfect squares that are factors of $(5! + 6! + 7!)^3$, where $n!$ represents n factorial.

New York City Interscholastic Mathematics League

Senior B Division   Contest Number 2

Part I   ***Spring 2010***   ***Contest 2***   ***Time: 10 Minutes***

S10B07   Compute the sum of all of the positive factors of 999.

S10B08   Given $X(2, 3)$ and $Y(5, 2)$, line segment $\overline{XY}$ is reflected across the line $x + y = 3$. If the reflection of X is X' and the reflection of Y is Y', compute the area of quadrilateral $XYY'X'$.

Part II   ***Spring 2010***   ***Contest 2***   ***Time: 10 Minutes***

S10B09   If $233_4 + 4K5_6 = 1401_5$, where the subscripts indicate bases, compute the value of the digit K.

S10B10   A pyramid whose base is a regular hexagon has lateral edges of four times the length of each edge of the base. If the volume of the pyramid is $60\,\text{cm}^3$, compute the length of one edge of the base in centimeters.

Part III   ***Spring 2010***   ***Contest 2***   ***Time: 10 Minutes***

S10B11   If $\sin 2\alpha = \dfrac{\sqrt{55}}{8}$ and $\cos\alpha = \dfrac{-\sqrt{5}}{4}$, compute $\tan\alpha$.

S10B12   Kim picked a whole number x greater than 10, added one, multiplied this result by two, added one again, multiplied this result by three, added one for the last time, and multiplied this result by four, giving her a final result of y. Compute the smallest two values of x such that y is a perfect cube.

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Senior B Division   Contest Number 3

Part I   ***Spring 2010***   ***Contest 3***   ***Time: 10 Minutes***

S10B13   In rectangle $ABCD$, $AB = 28$ and $AD = 22$. If E and F are the midpoints of $\overline{AB}$ and $\overline{AD}$ respectively, compute the area of quadrilateral $AECF$.

S10B14   Compute the greatest whole number x satisfying $\sqrt{3x} - \sqrt{x} < 2$.

Part II   ***Spring 2010***   ***Contest 3***   ***Time: 10 Minutes***

S10B15   Kate has four children. Cara is 6 years old, and her siblings are ages 2, 7, and 15, respectively. Each child's weekly allowance is proportional to his or her age, and the total of their allowances is \$60. Cara spends 40% of her weekly allowance on lunches, \$5.00 on karate lessons, and she saves the rest. How much, in dollars and cents, does Cara save each week?

S10B16   Compute all real values of x such that $\log_2(x-1) + \log_2(x+2) - \log_2(3x-1) < 1$.

Part III   ***Spring 2010***   ***Contest 3***   ***Time: 10 Minutes***

S10B17   Given the set of natural numbers, $\{1, 2, 3, 4, \ldots\}$, compute the 2010$^{\text{th}}$ number in this set that is not divisible by 2, 3 or 5.

S10B18   If $\tan x = \sqrt{5}$, compute the value of $\tan 3x$.

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Senior B Division   Contest Number 4

Part I   ***Spring 2010***   ***Contest 4***   ***Time: 10 Minutes***

S10B19   Let S = {37, 82, 48, A, 94, 30, 2A, 120, 3A}. If the arithmetic mean of S is 67, compute the median of S.

S10B20   Compute the smallest positive integer value of k such that the trinomial $x^2 + kx + 2010$ is factorable over the integers.

Part II   ***Spring 2010***   ***Contest 4***   ***Time: 10 Minutes***

S10B21   Let PQRSTUVW be a cube with edge length 12. If X is the center of face TUVW, compute the length of PX.

S10B22   A box contains 6 red, 5 blue and 4 white marbles. Four marbles are chosen at random without replacement. Compute the probability that there is at least one marble of each color among the four chosen.

Part III   ***Spring 2010***   ***Contest 4***   ***Time: 10 Minutes***

S10B23   Compute the number of possible distinct ordered pairs of digits (A, B) such that the six-digit base-10 integer $346AB2$ is divisible by 36.

S10B24   If a certain number is added in turn to 1, 7, and 4, in that order, the resulting sums form a geometric sequence. Compute the sum of the first six terms of this geometric sequence.

New York City Interscholastic Mathematics League

Senior B Division Contest Number 5

Part I ***Spring 2010*** ***Contest 5*** ***Time: 10 Minutes***

S10B25 Compute the number of possible integer values of a such that the quadratic equation $2x^2 + ax + 3 = 0$ has no real solutions.

S10B26 Compute all of the real valued solutions to the equation $\sqrt{5x-4} - \sqrt{x+8} = 2$

Part II ***Spring 2010*** ***Contest 5*** ***Time: 10 Minutes***

S10B27 Compute the absolute value of the difference between the largest and smallest two-digit natural numbers (where the first digit is not zero) with the property that four times the sum of the number's digits is three more than the number.

S10B28 Compute the sum of all of the real solutions of the equation $|x-3| \cdot |x+4| = 8$.

Part III ***Spring 2010*** ***Contest 5*** ***Time: 10 Minutes***

S10B29 In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are the bases with lengths $AB = 4$ and $CD = 12$. The midpoints of sides $\overline{AD}$ and $\overline{BC}$ are E and F respectively, and point X lies on $\overline{EF}$. Compute the ratio of the area of triangle XCD to the area of the trapezoid.

S10B30 Let $z = 1 + ci$ be a complex number where c is a positive real number. If z^3 is a real number, compute the sum $z + z^2 + z^3 + z^4 + z^5$. Express your answer in $a + bi$ form.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior B Division CONTEST NUMBER 1

Spring 2010 Solutions

S10B01. $\mathbf{\frac{2}{5}}$ or **0.4**. First, subtracting the fractions gives $\frac{1}{a}-\frac{1}{b}=\frac{b-a}{ab}=20$. Next, $a-b=-8$ implies that $b-a=8$, and substituting this into the first equation yields $\frac{8}{ab}=20 \Rightarrow ab=\frac{8}{20}=\frac{2}{5}$.

S10B02. **3, 4, 5.** If the lengths of the sides of an acute triangle are a, b and c (with c being the longest side), then $a^2+b^2>c^2$. For this problem we need to consider two cases: i) $2\sqrt{5}$ is the longest side; and ii) x is the longest side. In case i) we have $x^2+4^2>(2\sqrt{5})^2 \Rightarrow x^2>4 \Rightarrow x>2$ (because x cannot be negative). In case ii) we have $4^2+(2\sqrt{5})^2>x^2 \Rightarrow 36>x^2 \Rightarrow 6>x$. Case i) gives us a range of $2<x<2\sqrt{5}$, and case ii) gives us a range of $2\sqrt{5}<x<6$. Therefore $2<x<6$ and x can be 3, 4 or 5.

S10B03. **4019**. Try to identify a pattern. If $f(1)=1$ and $f(2)=3$, then $f(3)=2\cdot f(2)-f(1)$ $=6-1=5$. Next, $f(4)=2\cdot f(3)-f(2)=10-3=7$ and $f(5)=2\cdot f(4)-f(3)$ $=14-5=9$. It seems that $f(n)=2n-1$. We can prove this by induction. We already have the base case $n=3$; for the inductive step, we can assume that $f(n-1)=2(n-1)-1=2n-3$ and $f(n-2)=2(n-2)-1=2n-5$. Therefore $f(n)=2(2n-3)-(2n-5)=4n-6-2n+5=2n-1$, which proves our formula is correct, and as a result $f(2010)=2\cdot 2010-1=4019$. Alternatively, we could note that $f(n-1)=(f(n)+f(n-2))/2$. This indicates that each term is the average of the terms immediately before and after it, which means that the outputs of the function form an arithmetic sequence. We know that the common difference is 2, so all we have to do is compute $f(1)+2009\cdot 2$, which comes to 4019.

S10B04. **56**. Let R be the number of red armies, B be the number of blue armies, and W be the number of white armies. The first piece of given information gives us $10R=3B+W+40 \Rightarrow 10R-3B-W=40$. The second piece of information gives us $R=\frac{1}{3}(B+W)$, and the last piece of information gives $10R+3B+W=240$. Adding the first equation to the last equation yields $20R=280 \Rightarrow R=14$. Substituting this value into the second equation gives us $B+W=42$. Therefore, $R+B+W=14+42=56$.

S10B05. $\mathbf{\frac{1}{7}}$. Let $BC=x$. Then, $AB=3x$ and $CD=\frac{x}{\sqrt{3}}$ (by using the 30-60-90 triangle). So, using right triangle ACD, $AD^2=(4x)^2+\left(\frac{x}{\sqrt{3}}\right)^2=16x^2+\frac{x^2}{3}=\frac{49x^2}{3}$ and $AD=\frac{7x}{\sqrt{3}}$. Therefore,

$$\sin A=\frac{CD}{AD}=\frac{\frac{x}{\sqrt{3}}}{\frac{7x}{\sqrt{3}}}=\frac{1}{7}.$$

S10B06. **80**. First, find the prime factorization of $(5!+6!+7!)^3$ by factoring out 5! from the terms inside the parentheses: $(5!+6!+7!)^3 = (5!(1+6+6\cdot 7))^3 = (2^3\cdot 3\cdot 5\cdot 7^2)^3 = 2^9\cdot 3^3\cdot 5^3\cdot 7^6$. The perfect square factors of this number will only contain even powers of the prime factors 2, 3, 5, and 7. There are five choices for 2 ($2^0, 2^2, 2^4, 2^6, 2^8$), two choices for 3 ($3^0, 3^2$), two choices for 5 ($5^0, 5^2$), and four choices for 7 ($7^0, 7^2, 7^4, 7^6$). Therefore, by the Fundamental Counting Principle, there are $5\cdot 2\cdot 2\cdot 4 = 80$ perfect square factors.

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Senior B Division   Contest Number 2

Spring 2010 Solutions

S10B07. **1520**. The prime factorization of 999 is $3^3 \cdot 37$ and the factors are 1, 3, 9, 27, 37, 111, 333, and 999. Therefore, the sum of the factors is $1+3+9+27+37+111+333+999=1520$. Challenge: can you find a general formula that can find the sum of the factors of any number? (You'll probably need to examine the prime factorization.)

S10B08. **12**. The line $x+y=3$ has slope -1, so XX' and YY' will both have slope 1, since the line connecting a point to its reflection must be perpendicular to the axis of reflection. By checking the lattice points these segments pass through, we can identify (1, 2) and (3, 0) as the points where XX' and YY' respectively intersect $x+y=3$. Knowing that these are the respective midpoints of XX' and YY', we can identify $X'(0,1)$ and $Y'(1,-2)$, showing that quadrilateral $XYY'X'$ is a trapezoid. Since we know the coordinates of its vertices, we can calculate that base XX' has length $2\sqrt{2}$ and base YY' has length $4\sqrt{2}$. Since $x+y=3$ is perpendicular to the bases, the height of the trapezoid is the distance between the points (3, 0) and (1, 2) $\Rightarrow 2\sqrt{2}$. Therefore, the area of the trapezoid is

$$\frac{1}{2}\cdot 2\sqrt{2}\left(2\sqrt{2} \quad 4\sqrt{2}\right)+\sqrt{2}\left(6\sqrt{2}\right)=12.$$

S10B09. **5**. Convert the equation into base ten, then solve for K: $233_4 + 4K5_6 = 1401_5$
$\Rightarrow (2\cdot 16+3\cdot 4+3)+(4\cdot 36+6K+5)=125+4\cdot 25+1 \Rightarrow 196+6K=226 \Rightarrow 6K=30 \Rightarrow K=5$.

S10B10. $\mathbf{2\sqrt[6]{5}}$. Let x be the length of an edge of the base and let h be the height of the pyramid. Since the length of the side of a regular hexagon is equal to the radius of the hexagon (the distance between the hexagon's center and one of its vertices), the radius of the base is x. There is a right triangle formed by the radius of the base, the lateral edge (whose length is $4x$) and the height of the pyramid. Using the Pythagorean Theorem we have $x^2+h^2 \quad (4x)^2 \quad h \quad x\sqrt{15}$. The volume of a pyramid is $V=\frac{1}{3}Bh$, where B is the area of the pyramid's base. The area of the base is equal to the area of the six equilateral triangles that make up the hexagon, or $6\cdot\frac{x^2\sqrt{3}}{4} = \frac{3x^2\sqrt{3}}{2}$.

Since the volume of the pyramid is 60 we have $\frac{1}{3}\cdot\frac{3x^2\sqrt{3}}{2}\cdot x\sqrt{15}=60$. Solving this last equation yields

$$\frac{3x^3\sqrt{5}}{2}=60 \Rightarrow x^3=\frac{40}{\sqrt{5}}=8\sqrt{5} \Rightarrow x=2\sqrt[6]{5}.$$

S10B11. $\mathbf{\frac{\sqrt{55}}{5}}$. Using the double-angle formula for sine, $\sin 2\alpha = 2\sin\alpha\cos\alpha = \frac{\sqrt{55}}{8}$

$$\Rightarrow 2\sin\alpha\cdot\frac{-\sqrt{5}}{4}=\frac{\sqrt{55}}{8} \Rightarrow \sin\alpha=\frac{\sqrt{55}}{-4\sqrt{5}}=\frac{-\sqrt{11}}{4}. \text{ Now, } \tan\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{-\sqrt{11}}{4}}{\frac{-\sqrt{5}}{4}}=\frac{\sqrt{11}}{\sqrt{5}}=\frac{\sqrt{55}}{5}.$$

S10B12. **40 and 169**. From the given information, Kim's result is $4(3(2(x+1)+1)+1)$ $= 4(3(2x+3)+1) = 4(6x+10) = 24x+40 = 8(3x+5)$. Since $8 = 2^3$ we need $3x+5$ to be a perfect cube. We must check a series of equations: $3x+5 = 4^3 = 64 \Rightarrow$ x is not an integer, $3x+5 = 5^3 = 125 \Rightarrow x = 40$, $3x+5 = 6^3 = 216 \Rightarrow$ x is not an integer, $3x+5 = 7^3 = 343 \Rightarrow$ x is not an integer, $3x+5 = 8^3 = 512 \Rightarrow x =$ 169. Therefore, the two smallest values that work are 40 and 169. Alternatively, note that $3x + 5$ is one less than a multiple of 3. It can shown that for any integer n, n^3 is one less than a multiple of 3 if and only if n is one less than a multiple of 3. (As an extra challenge, see if you can prove this.) Therefore, we only need to check $3x + 5 = 2^3$ ($x = 1$; too small), $3x + 5 = 5^3$ ($x = 40$), and $3x + 5 = 8^3$ ($x = 169$).

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Senior B Division Contest Number 3

Spring 2010 Solutions

S10B13. **308**. The areas of triangles *EBC* and *CDF* are each $\frac{1}{4}$ of the area of rectangle *ABCD*. Therefore, the area of quadrilateral *AECF* is $\frac{1}{2}\cdot 22\cdot 28 = 308$.

S10B14. **7**. We can factor out $\sqrt{x}$ on the left side, yielding $(\sqrt{3}-1)\sqrt{x} \quad 2 < \sqrt{x} \Rightarrow \frac{2}{\sqrt{3}-1}$. From there we can multiply the numerator and denominator by $\sqrt{3}+1$ to get $\sqrt{x} < \frac{2(\sqrt{3}+1)}{2} \quad \sqrt{x} \Rightarrow \sqrt{3} \quad 1$ $\Rightarrow x \quad 4 < 2\sqrt{3}+$. Since $4+2\sqrt{3}\approx 7.4$, the largest possible whole number value for *x* is 7.

S10B15. **$2.20**. The sum of the ages of the children is 30. Therefore, each child gets $\frac{60}{30} = 2$ times their age as an allowance. Cara is 6 and gets $12 per week of which she spends $\$12(0.4) = \$4.80$ on lunch. As a result she spends $\$4.80 + \$5.00 = \$9.80$ per week and saves a total of $\$12 - \$9.80 = \$2.20$ per week.

S10B16. $\mathbf{1 < x < 5}$. Given the logarithms on the left-hand side, first notice that *x* must be greater than 1. Now use the product/quotient rules for logs, raise both sides with base 2 and solve the resulting inequality: $\log_2(x-1)+\log_2(x+2)-\log_2(3x-1) < 1 \Rightarrow \frac{(x-1)(x\quad 2)+}{3x-1} \quad 2 \Rightarrow \frac{x^2+x\quad 2-\quad 6x-2}{3x-1} \quad 0$ $\Rightarrow \frac{x(x-5)}{3x-1} < 0$. This last inequality is true when $1 < x < 5$. Another way to solve the inequality $\frac{(x-1)(x\quad 2)+}{3x-1} < 2$ is to multiply both sides by the denominator, which we know must be positive since $x > 1$. That gives us $(x-1)(x\quad 2)+\ 6x\quad 2\quad x^2 - x\quad 2 \Rightarrow 6x\quad 2\ + x^2\ -5x\quad 0$. This inequality is true when $0 < x < 5$, but we know that $x > 1$, so the answer is the range $1 < x < 5$.

S10B17. **7537**. Consider the set of numbers from 1 to 30. There are 8 elements of this set not divisible by 2, 3, or 5: 1, 7, 11, 13, 17, 19, 23, and 29. Since 30 is the least common multiple of 2, 3 and 5, for every set of 30 consecutive whole numbers from $30n + 1$ to $30(n + 1)$, eight will not be divisible by 2, 3, or 5. Next, $2010 \div 8 = 251$ with a remainder of 2. Since the second number in the set of numbers from 1 to 30 not divisible by 2, 3, or 5 is 7, the 2010th number of this type is $251\times 30 \quad 7 + 7537$.

S10B18 $\frac{\sqrt{5}}{7}$. The angle addition formula for tangent is $\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}$ and, as a result, the double angle formula for tangent is $\tan 2x = \frac{2\tan x}{1-\tan^2 x}$. Therefore, upon substitution and simplifying, the triple angle formula for tangent is $\tan 3x = \frac{\tan 2x + \tan x}{1-\tan 2x\tan x} = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$. If

$\tan x = \sqrt{5}$, then $\tan 3x = \dfrac{3 \cdot \sqrt{5} - 5\sqrt{5}}{1 - 3 \cdot 5} = \dfrac{-2\sqrt{5}}{-14} = \dfrac{\sqrt{5}}{7}$.

New York City Interscholastic Mathematics League

Senior B Division Contest Number 4

Spring 2010 Solutions

S10B19. **64**. The sum of the elements of S is $411+6A$. If the mean of S is 67, then $\frac{411+6A}{9}=67 \Rightarrow A=32$. So, the elements of S (in order) are: 30, 32, 37, 48, 64, 82, 94, 96, and 120. As a result, the median of S is 64.

S10B20. **97**. In order to find the smallest possible value of *k*, we must find two numbers that multiply to 2010 whose sum is a minimum. The pairs of factors that multiply to 2010 are (1, 2010), (2, 1005), (3, 670), (5, 402), (6, 335), (10, 201), (15, 134), and (30, 67). The pair with the smallest sum is (30, 67), with a sum of 97.

S10B21 $\mathbf{6\sqrt{6}}$. Find a suitable right triangle and use the Pythagorean Theorem. In particular, *PX* will be the hypotenuse of a right triangle whose legs are an edge of the cube and the radius of face *TUVW*. Since the radius of face *TUVW* is $6\sqrt{2}$, we have $12^2+(6\sqrt{2})^2=PX^2 \Rightarrow PX=\sqrt{216}=6\sqrt{6}$.

S10B22. $\mathbf{\frac{48}{91}}$. The sample space has ${}_{15}C_4=\frac{15!}{4!\,11!}=1365$ elements. The successful events are two reds, one blue, and one white; two blues, one red, and one white; or two whites, one red, and one blue. Therefore, the probability of having at least one of each color is

$$\frac{{}_6C_2\cdot{}_5C_1\cdot{}_4C_1+{}_5C_2\cdot{}_6C_1\cdot{}_4C_1+{}_4C_2\cdot{}_6C_1\cdot{}_5C_1}{{}_{15}C_4}=\frac{15\cdot5\cdot4+10\cdot6\cdot4+6\cdot6\cdot5}{1365}=\frac{720}{1365}=\frac{48}{91}.$$

S10B23. **6**. A number is divisible by 4 if and only if the number formed by its last two digits is divisible by 4. Since 346*AB*2 is divisible by 4, *B*2 must be divisible by 4, so *B* = 1, 3, 5, 7, or 9. A number is divisible by 9 if and only if the sum of its digits is divisible by 9. Since 346*AB*2 is divisible by 9, $15+A+B$ is divisible by 9. If *B* = 1, then *A* = 2. If *B* = 3, then *A* = 0 or 9. If B = 5, then *A* = 7. If *B* = 7, then *A* = 5. Finally, if *B* = 9, then *A* = 3. Therefore, there are 6 ordered pairs.

S10B24. $\mathbf{-\frac{21}{8}}$ or $\mathbf{-2\frac{5}{8}}$ or **–2.625**. Let the number added be *x*. Then *x* + 1, *x* + 7, *x* + 4 is a geometric sequence and the middle term is the geometric mean of the other terms, so that $(x+7)^2=(x+1)(x+4)\Rightarrow x^2+14x+49=x^2+5x+4\Rightarrow 9x=-45\Rightarrow x=-5$. Hence, the sequence is $-4,2,-1$. Since the common ratio is $-\frac{1}{2}$, the first six terms are $-4,2,-1,\frac{1}{2},-\frac{1}{4},\frac{1}{8}$. The sum of these six numbers is $-\frac{21}{8}$.

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Senior B Division   Contest Number 5

Spring 2010 Solutions

S10B25. **9**. We want the discriminant of the quadratic equation to be less than zero. The discriminant is $a^2 - 4 \cdot 2 \cdot 3 = a^2 - 24$. Thus $a^2 < 24 \Rightarrow a = 0, \pm 1, \pm 2, \pm 3 \pm 4$. Therefore, there are 9 possibilities for *a*.

S10B26. **8**. Add $\sqrt{x+8}$ to both sides and square both sides. Then rearrange the equation and square both sides again to get a quadratic equation. Finally, solve the quadratic equation and be on the look out for extraneous roots. $\sqrt{5x-4} - \sqrt{x+8} = 2 \Rightarrow \sqrt{5x-4} = \sqrt{x+8} + 2$
$\Rightarrow 5x - 4 = x + 8 + 4\sqrt{x+8} + 4 \Rightarrow x - 4 = \sqrt{x+8} \Rightarrow x^2 - 8x + 16 = x + 8 \Rightarrow x^2 - 9x + 8 = 0$
$\Rightarrow (x-1)(x-8) = 0 \Rightarrow x = 8$ (notice that $x = 1$ is extraneous).

S10B27. **36**. Call the two-digit number *AB*. Then from the given and with some simplification we have: $4(A+B) = 10A + B + 3 \Rightarrow B - 2A = 1$. Now, if $A = 1$, $B = 3$. If $A = 2$, $B = 5$. If $A = 3$, $B = 7$. Finally, if $A = 4$, $B = 9$. Therefore there are four numbers with the given property: 13, 25, 37, and 49. The difference between the largest and smallest of these numbers is $49 - 13 = 36$.

S10B28. **-2**. The given equation can be re-written: $|x-3| \cdot |x+4| = 8 \Rightarrow |x^2 + x - 12| = 8$
$\Rightarrow x^2 + x - 12 = \pm 8$. One equation is $x^2 + x - 20 = 0$; the other is $x^2 + x - 4 = 0$. The sum of the roots of both of these equations is the opposite of the coefficient on the linear term or –1. Hence, the sum of the solutions of the original equation is $-1 + (-1) = -2$. (Note: Since the problem asks for all *real* roots, you should first check the discriminant to make sure there are no complex roots involved, but in this case both discriminants are positive.)

S10B29. $\mathbf{\frac{3}{8}}$. Let the altitude of trapezoid *ABCD* be *h*. Then the area of trapezoid *ABCD* is $\frac{1}{2}h(4+12) = 8h$. Also, the height of triangle *XCD* is $\frac{1}{2}h$, so its area is $\frac{1}{2} \cdot 12 \cdot \frac{1}{2}h = 3h$. Therefore, the ratio of the area of triangle *XCD* to the area of the trapezoid is $\frac{3h}{8h} = \frac{3}{8}$.

S10B30. $\mathbf{-1-21i\sqrt{3}}$. First, $z^3 = (1+ci)^3 = 1 + 3ci - 3c^2 - c^3 i$ and since z^3 is a real number, we have $3c - c^3 = 0$. Therefore, because $c > 0$, $c = \sqrt{3}$. Hence, $z^3 = 1 - 3c^2 = 1 - 9 = -8$. Now, using the formula for the sum of a geometric series whose first term is *z* and whose common ratio is *z*,
$z + z^2 + z^3 + z^4 + z^5 = \frac{z - z^6}{1 - z} = \frac{z - (z^3)^2}{1 - z} = \frac{1 + i\sqrt{3} - (-8)^2}{1 - (1 + i\sqrt{3})} = \frac{i\sqrt{3} - 63}{-i\sqrt{3}}$. This fraction can be simplified by multiplying the numerator and denominator by $i\sqrt{3}$. This gives us

$$\frac{i\sqrt{3} - 63}{-i\sqrt{3}} \cdot \frac{i\sqrt{3}}{i\sqrt{3}} = \frac{-3 - 63i\sqrt{3}}{3} = -1 - 21i\sqrt{3}.$$