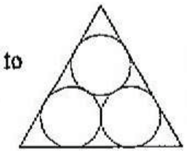
New York City
Interscholastic
Mathematics
League

SENIOR B DIVISION **CONTEST NUMBER ONE** **FALL 2007**

PART I: 10 minutes

F07B01 The points $(2,7)$ and $(6,3)$ are equidistant from the line $y = kx$. Compute the positive value of k.

F07B02 In the diagram, the three circles are mutually tangent and are tangent to the sides of the equilateral triangle. If the radius of each circle is 1, compute the area of the triangle.



PART II: 10 minutes **NYCIML Contest One** **Fall 2007**

F07B03 If the graphs of the parabolas defined by $y = x^2 + 2x + 6$ and $y = -x^2 + 10x + m$ are tangent to one another, compute m.

F07B04 If $i = \sqrt{-1}$, and if $1 + (1+i) + (1+i)^2 + (1+i)^3 + (1+i)^4 + (1+i)^5$ can be expressed as $a + bi$ where a and b are real numbers, compute $(a,\ b)$.

PART III: 10 minutes **NYCIML Contest One** **Fall 2007**

F07B05 A bag contains only purple, yellow and red marbles. Archie selects a marble at random. The probability of **not** selecting a red is $\frac{2}{3}$. The probability of **not** selecting a yellow is $\frac{3}{4}$. Compute the probability that Archie selects a purple.

F07B06 Rectangle $ABCD$ is inscribed in rhombus $PQRS$, such that A is the midpoint of $\overline{PQ}$, diagonal $\overline{PR} \perp \overline{AD}$, $AD = 6$ and $AB = 8$.
Compute the area of the rhombus.

ANSWERS

1. **5/4**
2. $\mathbf{4\sqrt{3}+6}$
3. **-2**
4. **(-8,1)**
5. $\frac{5}{12}$
6. **96**

New York City Interscholastic Mathematics League

SENIOR B DIVISION **CONTEST NUMBER TWO** **FALL 2007**

PART I: 10 minutes

F07B07 Freddie can do a job in 3 hours. Michael can do the same job in 4 hours. Jason can do the same job in 5 hours. If Freddie and Michael work together for 12 **minutes**, then Freddie and Jason work together for 15 **minutes**, compute the number of **minutes** that it will take Michael and Jason working together to finish the job.

F07B08 The nth term of a Clio sequence $C_n = 1-2+3-4+\cdots+(-1)^{n-1}n$ for all $n = 1, 2, 3, \ldots$ Compute the value of: $C_1+C_2+C_3+C_4+\cdots+C_{1024}$.

PART II: 10 minutes **NYCIML Contest Two** **Fall 2007**

F07B09 If a and b are real numbers and $0 < a < b$, what is the median of the set: $\left\{a, b, \frac{a+b}{2}, \sqrt{a^2+b^2}, \frac{1}{2}\sqrt{a^2+b^2}\right\}$?

F07B10 A drawer contains 4 black socks and 4 red socks. Helen and Jim take turns removing one sock at random without replacement. If Helen picks first, compute the probability that Helen will be the first to match a pair of socks.

PART III: 10 minutes **NYCIML Contest Two** **Fall 2007**

F07B11 $x^2+3x+25$ has roots j and c. Compute the value of j^2+c^2.

F07B12 Compute the last two digits of $2007^{\left(31^{10}\right)}$?

ANSWERS

7. **100**
8. **0**
9. $\frac{a+b}{2}$
10. **27/35**
11. **-41**
12. **07 (or just 7)**

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SENIOR B DIVISION **CONTEST NUMBER THREE** **FALL 2007**

PART I: 10 minutes

F07B13 In rectangle $ABCD$, point E is the midpoint of $\overline{AB}$ and point F is the midpoint of $\overline{BC}$. If $DE = \sqrt{17}$ and $AF = 3\sqrt{2}$, compute AC.

F07B14 The greatest common factor of two positive integers is 60. The least common multiple of the same two integers is 12,600. Compute the number of positive integral factors that the product of the two positive integers have?

PART II: 10 minutes **NYCIML Contest Three** **Fall 2007**

F07B15 2007 is a multiple of 9 which differs by 1 from 2008, a multiple of 8. Compute the next largest multiple of 9 which differs by 1 from a multiple of 8.

F07B16 Siggy rolls 4 fair six-sided dice. Compute the probability that Siggy finds the sum of the largest <u>three</u> numbers to be 17?

PART III: 10 minutes **NYCIML Contest Three** **Fall 2007**

F07B17 The sum of the first $2n$ positive integers exceeds the sum of the first n positive integers by 610. Compute n.

F07B18 In a plane, one vertex of a square lies on a line. Another vertex lies 3 units away from the line, and a third vertex lies 5 units away from the line. If no other points of the square intersect the line, compute the largest possible area for this square.

ANSWERS

13. $2\sqrt{7}$
14. **192**
15. **2025**
16. **1/24**
17. **20**
18. **34**

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SENIOR B DIVISION **CONTEST NUMBER FOUR** **FALL 2007**

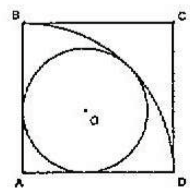
PART I: 10 minutes

F07B19 If the value of $|x|+|y|$ exceeds the value of $|x+y|$ by 5, and $x>10$, compute the value of y.

F07B20 If 14! is evaluated, it is $871c8291200$ where c is a digit in the number. Compute the value of c.

PART II: 10 minutes **NYCIML Contest Four** **Fall 2007**

F07B21 In the diagram below, $\overparen{BD}$ is the arc of a circle centered at vertex A of square $ABCD$. The circle O is tangent to $\overline{AB}$, $\overline{AD}$, and $\overparen{BD}$. If the area of the square is 16, compute the radius of circle O.



F07B22 Given that $\log\left(\frac{x^3}{y^4}\right)=-1$ and $\log\left(\sqrt[18]{x^9y^2}\right)=1$, compute the value of $\log(xy)$.

PART III: 10 minutes **NYCIML Contest Four** **Fall 2007**

F07B23 Compute the value of $\sin 34^\circ\left(\tan 17^\circ+\cot 17^\circ\right)$.

F07B24 *ABCDEFGH* are the vertices of a cube such that *A* and *E* are the endpoints of an internal diagonal of the cube. A ladybug is at vertex *A*. The ladybug will crawl only along edges of the cube. Compute the number of paths the ladybug can take to go from vertex *A* to vertex *E* such that she never visits any vertex more than once. (Note: an acceptable path need not visit every vertex of the cube.)

<u>ANSWERS</u>

19. **-5/2**
20. **7**
21. $\mathbf{4\sqrt{2}-4}$
22. **19/6**
23. **2**
24. **18**

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SENIOR B DIVISION **CONTEST NUMBER FIVE** **FALL 2007**

PART I: 10 minutes

F07B25 Compute the number of positive integers less than 1000 that have at least one "3" in their base 10 representation.

F07B26 In isosceles ΔABC, base $BC = 5$, and vertex angle $A = 36°$. If point D lies on $\overline{AC}$ such that $\overline{BD}$ bisects angle B, compute BD.

PART II: 10 minutes **NYCIML Contest Five** **Fall 2007**

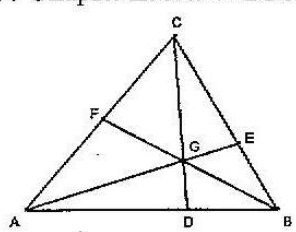
F07B27 Function f is defined as follows: $f(x+y) = 2f(x)f(y)$.
If $f(1) = 5$, compute the value of $f(3)$.

F07B28 In ΔABC with right angle C, $AC = 4$ and $BC = 3$. Point D lies on $\overline{AB}$ such that $m\angle ACD = 30°$. Compute CD.

PART III: 10 minutes **NYCIML Contest Five** **Fall 2007**

F07B29 1027 is the product of two primes. Compute these primes.

F07B30 In the diagram below, the area of $\Delta AGD = 20$, the area of $\Delta BGD = 30$, and $BE : EC = 5 : 4$. Compute the area of ΔCGF.



ANSWERS

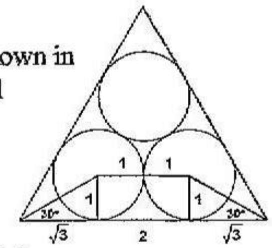
25. **271**
26. **5**
27. **500**
28. $\dfrac{72\sqrt{3}-96}{11}$
29. **13 and 79**
30. **240/11**

SENIOR B DIVISION **CONTEST NUMBER ONE** **FALL 2007**

SOLUTIONS

1. If the two points are equidistant from the line, then, by congruent triangles, the line bisects the segment joining the points. In other words, (4, 5), the midpoint of (2, 7) and (6, 3), lies on $y = kx$. So, $5 = 4k$. $\boldsymbol{k = 5/4}$.

2. If the radius is 1, then the side of the triangle is $2\sqrt{3}+2$ as shown in the diagram. . Using the formula for the area of an equilateral triangle, $A = \frac{\sqrt{3}}{4}s^2 = \frac{\sqrt{3}}{4}\left(2\sqrt{3}+2\right)^2 = \mathbf{4\sqrt{3}+6}$.



3. If we set the two equations equal to each other, we obtain: $x^2 + 2x + 6 = -x^2 + 10x + m$, or $2x^2 - 8x + (6 - m) = 0$. This equation has only one solution, so the discriminant must equal 0. $(-8)^2 - 4(2)(6-m) = 0$ yields $\boldsymbol{m = -2}$.

4. One method is to expand each expression and combine like terms. Another is to explore Pascal's triangle. The first six lines of Pascal's triangle represent the coefficients for each of the expansions. Moreover, the leftmost diagonal represents coefficients of positive real terms, the next diagonal represents imaginary terms with positive coefficients, and the next represents negative real terms, then imaginary terms with positive coefficients, and then the cycle repeats. Finally, using the "Hockey Stick Theorem" we find the sum of the elements of a diagonal is the number in the next row shifted one to the right. In this problem, these are 6, 15, 20, 15 and 6. Therefore, the solution is $6 + 15i - 20 - 15i + 6 + 1i = -8 + i$. The answer is **(-8 , 1)**.

1
1 *1*
1 *2* <u>1</u>
1 *3* <u>3</u> **1**
1 *4* <u>6</u> **4** *1*
1 *5* <u>10</u> **10** *5* **1**
1 6 *15* <u>20</u> **15** *6* 1

5. If P(not red) = 2/3, then P(red) = 1/3. P(not yellow) = P(purple or red) = P(purple) + P(red) = 3/4. Therefore, P(purple) = 3/4 – 1/3 = **5/12**.

6. $\overline{PR} \perp \overline{AD}$ and $\overline{AD} \perp \overline{AB}$, so $\overline{PR} \parallel \overline{AB}$ and ΔQAB is similar to ΔQPR. Since A is the midpoint of $\overline{PQ}$, the scale factor is 2. Thus, $PR = 2AB = 16$. Similarly, $SQ = 2AD = 12$. The area of the rhombus is $\frac{1}{2} PR \cdot SQ = \mathbf{96}$.

SENIOR B DIVISION **CONTEST NUMBER TWO SOLUTIONS** **FALL 2007**

7. Freddie's rate is 1/3, Michael's is 1/4 and Jason's is 1/5 job per hour. Working together, Freddie and Michael's combined rate is 1/3 + 1/4 = 7/12. If they work for 12 minutes, or 1/5 an hour, then they get 7/60 of the job done. Similarly, Freddie and Jason's combined rate is 1/3 + 1/5 = 8/15. If they work for 15 minutes, or 1/4 of an hour, then they get 8/60 of the job done. This leaves 45/60 or 3/4 of the job undone. Michael and Jason's combined rate is 1/4 + 1/5 = 9/20. Therefore, it will take them 5/3 of an hour, or **100 minutes**, to complete the job.

8. The first few terms of the Clio sequence are 1, -1, 2, -2, 3, -3, etc. Ultimately, $C_n = -n/2$ if n is even and $(n + 1)/2$ if n is odd. Therefore, the sum of the first 1024 terms of the Clio sequence is **0**.

9. $\mathbf{\frac{a+b}{2}}$. Because $0 < a < b$, we have immediately that $a < \frac{a+b}{2} < b < \sqrt{a^2+b^2}$.

Note also that $\frac{1}{2}\sqrt{a^2+b^2} < b$ because everything is positive and $a^2+b^2 < 4b^2$.

So we only have to compare $\frac{a+b}{2}$ and $\frac{1}{2}\sqrt{a^2+b^2}$. However, $ab > 0$ so $a^2+2ab+b^2 > a^2+b^2$, and since everything is positive we can take square roots to get $\frac{1}{2}\sqrt{a^2+b^2} < \frac{a+b}{2}$, giving us our answer. Alternatively, we can reach this last inequality by noting that $\sqrt{a^2+b^2}$ is the distance from the point (a, b) to $(0, 0)$, while $a + b$ is the length of the path from (a, b) to $(0, 0)$ which travels parallel to the axes. Then by the triangle inequality, $\sqrt{a^2+b^2} < a+b$.
Or: let $a = 3$ and $b = 4$, and we get 3, 4, 3.5, 5, 2.5. Thus the median is $3.5 = (a + b)/2$

10. Let "X" represent the color of Helen's first pick. Let "Y" represent the other color. Helen wins if she gets two of the same letter before Jim does. Thus, Helen wins on the following sequence of picks: XXX, XYX, XXYYX, XXYYY, XYYXX, and XYYXY. Note: XYX represents Helen picking X first, Jim picking Y second, and Helen picking X third. P(XXX) = (1)(3/7)(2/6) = 1/7. P(XYX) = (1)(4/7)(3/6) = 2/7. P(XXYYX) = (1)(3/7)(4/6)(3/5)(2/4) = 3/35. P(XXYYY) = (1)(3/7)(4/6)(3/5)(2/4) = 3/35. P(XYYXX) = (1)(4/7)(3/6)(3/5)(2/4) = 3/35. P(XYYXY) = (1)(4/7)(3/6)(3/5)(2/4) = 3/35. Therefore, P(Helen wins) = **27/35**. [It seems amazing that Helen's likelihood of winning is so high!]

11. From the equation, the sum of the roots is -3 and the product of the roots is 25. Therefore, $(j + c)^2 = j^2 + 2jc + c^2 = (-3)^2$, or $j^2 + 50 + c^2 = 9$. Thus, $j^2 + c^2 =$ **- 41**.

12. The first thing to note is that the last two digits of 2007^n is the same as the last two digits of 7^n. We can now examine powers of 7: 7^1 yields 07, 7^2 yields 49, 7^4 yields the last two digits of $49^2 = (50-1)(50-1) = 2500-100+1 = 2401$ or 01. Thus, $7^{\text{mulitple of 4}}$ ends in "01". We now need to see what remainder 31^{10} leaves when divided by 4. Note that 32 is a multiple of 4 and that 32-1 = 31. Were we to expand $(32-1)^{10}$, the first 10 terms of the expansion would have at least 1 multiple of 32, and thus each term would be a multiple of 4. Only the last term does not contain a 32, namely $(-1)^{10} = 1$. Thus, 31^{10} leaves a remainder of 1 when divided by 4. [This is essentially what mod rules tell us.] Therefore, the last two digits of the expression are **"07" or just 7.**

Happy Halloween !!!

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SENIOR B DIVISION **CONTEST NUMBER THREE** **FALL 2007**

SOLUTIONS

13. If $AE = x$ and $BF = y$, then $AF^2 = 4x^2 + y^2 = 18$ and $DE^2 = x^2 + 4y^2 = 17$. Summing these expressions yields $5x^2 + 5y^2 = 35$. Therefore, $x^2 + y^2 = 7$. $AC^2 = 4x^2 + 4y^2 = 28$. Thus, $AC = 2\sqrt{7}$.

14. The product of the least common multiple and the greatest common factor of two numbers is in fact the product of the two numbers. Therefore, the product of the two numbers is $2^2 3^1 5^1 \cdot 2^3 3^2 5^2 7^1 = 2^5 3^3 5^3 7^1$. Thus, there are $(5+1)(3+1)(3+1)(1+1) =$ **192** factors.

15. **2025.** 2016 is divisible by both 8 and 9, so 2025 is one more than 2024, a multiple of 8. (Each subsequent example can be found by adding multiples of 72 to these values.)

16. In order to get a "17", Siggy needs 6-6-5-1, 6-6-5-2, 6-6-5-3, 6-6-5-4, or 6-6-5-5. However, each of these results can be in any order, in other words, in order to find the probability of 6-6-5-1, we need to consider all arrangements of 6, 6, 5, 1. $P(6\text{-}6\text{-}5\text{-}1) = (4!/2!)\cdot(1/6)^4 = 12/(6^4) = 2/(6^3)$. P(6-6-5-2), P(6-6-5-3) and P(6-6-5-4) are the same as P(6-6-5-1). $P(6\text{-}6\text{-}5\text{-}5) = (4!/2!2!)\cdot(1/6)^4 = 6/(6^4) = 1/(6^3)$. Therefore, $P(17) = 9/(6^3) =$ **1/24.**

17. The sum of the first n positive integers is $\frac{n(n+1)}{2}$. Thus, the sum of the first $2n$ positive integers is $\frac{2n(2n+1)}{2}$. We have $\frac{2n(2n+1)}{2} - \frac{n(n+1)}{2} = 610$. Multiplying by 2 and combining like terms yields $3n^2 + n = 1220$. We could solve this quadratic, but knowing n is a positive integer makes testing values go rather quickly, particularly if we write the left side as $n(3n + 1) = 1220$. $\boldsymbol{n = 20}$.

18. Let A, B, C, and D be the vertices of the square such that B lies on the line. Let points E and F lie on the line such that $\overline{AE}$ and $\overline{CF}$ are perpendicular to the line. Let $AE = 5$ and $CE = 3$. Note that ΔAEB and ΔBFC are congruent. Therefore, $EB = 3$, $BF = 5$ and the side of the square equals $\sqrt{34}$. Thus, the area of the square is **34**. The only other possible setup is to have $\overline{AE}$ and $\overline{DF}$ perpendicular to the line with $AE = 3$ and $DF = 5$. This, however, would result in a smaller square.

SENIOR B DIVISION CONTEST NUMBER FOUR SOLUTIONS FALL 2007

19. If $|x|+|y| \neq |x+y|$, then either x or y is negative while the other is positive. Here, we know x is positive, so y must be negative. Thus, the equation $|x|+|y|=|x+y|+5$ can be rewritten as $x-y=x+y+5$ or as $x-y=-x-y+5$, depending upon whether $x+y$ is positive or negative. The first equation yields $y = -5/2$ while the second yields $x = 5/2$. As the second is impossible, $y = \mathbf{-5/2}$. As an alternative, let $x=17$. Then we have $17+|y|=|17+y|+5$, so $12+|y|=|17+y|$. y cannot be positive, as that would yield $12+y=17+y$. If $-17<y<0$, we have $12-y=17+y$, which yields $y=-\frac{5}{2}$. y cannot be less than -17, as that would yield $12-y=-17-y$.

20. We could multiply out 14!; however, 14! must be a multiple of 11. Therefore, $8-7+1-c+8-2+9-1+2-0+0$ must be a multiple of 11. This simplifies to $18-c$. Therefore, $c = \mathbf{7}$.

21. The side of the square is 4. Let E lie on $\overline{AD}$ such that $OE=r$. Therefore, $AO=\sqrt{2}r$. If F is the point of tangency of circle O and the arc, then AF must equal 4. So, $4=r+\sqrt{2}r$. Thus, $r=\frac{4}{\sqrt{2}+1}=\mathbf{4\sqrt{2}-4}$.

22. Using the log rules, we can rewrite the two equations as $3\log(x)-4\log(y)=-1$ and $\frac{\log(x)}{2}+\frac{\log(y)}{9}=1$. Solving this system of linear equations yields $\log(x)=\frac{5}{3}$ and $\log(y)=\frac{3}{2}$. $\log(xy)=\log(x)+\log(y)=\mathbf{\frac{19}{6}}$.

23. Let $x=17°$. $\sin(2x)(\tan x+\cot x)=(2\sin x\cos x)\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)=$
$(2\sin x\cos)\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}=\mathbf{2}$.

24. One thing to do is to just count up the paths. Here is a counting way to get the solution. The ladybug can move in six different directions as she moves from vertex to vertex. In addition, three of these are essentially the inverses of the other three. We can call these pairs of directions "up" and "down", "in" and "out", and "right" and "left".

At the starting position, the ladybug can only go in three different directions. We can assume that these are "up", "in", and "right". As such, one acceptable path from A to E is up-in-right. In fact, any ordering of these three directions will also work, so there are $3 \cdot 2 \cdot 1$ or 6 paths of length three.

The next shortest possible path has five moves—for instance up-right-down-in-up. Notice that the third movement is the inverse of the first, and the last movement is the same as the first. Thus, there are 3 different choices for the first move, 2 choices for the second move, and only 1 choice possible for the third, fourth and fifth moves. Therefore, there are $3 \cdot 2 \cdot 1 \cdot 1 \cdot 1$ or 6 paths of length five.

Finally, the ladybug can have a path of seven moves—for instance up-right-down-in-left-up-right. Notice that the third move is the inverse of the first, and the sixth move equals the first. Moreover, fifth move is the inverse of the second, and the last move equals the second. To generalize, once the first and second moves are made, the remaining moves are determined so as to avoid revisiting a vertex. Therefore, there are $3 \cdot 2 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1$ or 6 paths of length seven. Overall, there are **18** paths.

SOLUTIONS

25. There are 999 positive integers less than 1000. We can consider all of these integers as 3-digit numbers, where the numbers 10 to 99 are written as "010" to "099", and the numbers 1 to 9 are written as "001" to "009". By the counting principle, the number of positive integers that have no "3"s is just (9)(9)(9) - 1 = 729 – 1 = 728. We subtract 1 because "000" is not a positive integer. Therefore, the number of integers with at least one "3" is 999 – 728 = **271**.

26. Angles ABC and ACB are both $72°$. Therefore, $m\measuredangle DBC = 36°$, and $m\measuredangle BDC = 72°$. ΔDBC is isosceles with $BC = BD =$ **5**.

27. We can find $f(2) = f(1+1) = 2f(1)f(1) = 2(5)(5) = 50$. Thus, $f(3) = f(2+1) = 2f(2)f(1) =$ **500**.

28. ΔABC is a 3-4-5 right triangle. Let $m\measuredangle CBA = \varphi$ and let $m\measuredangle CAB = \theta$. $m\measuredangle CDA = 60° + \varphi$. Using the law of sines on ΔACD, we get $\dfrac{4}{\sin(60°+\varphi)} = \dfrac{CD}{\sin\theta}$,

or $CD = \dfrac{4\sin\theta}{\sin 60°\cos\varphi + \cos 60°\sin\varphi}$. From the large triangle, we get $\sin\theta = \dfrac{3}{5}$,

$\sin\varphi = \dfrac{4}{5}$ and $\cos\varphi = \dfrac{3}{5}$. So, $CD = \dfrac{\frac{12}{5}}{\frac{3\sqrt{3}}{10} + \frac{4}{10}} = \dfrac{24}{3\sqrt{3}+4} = \mathbf{\dfrac{72\sqrt{3}-96}{11}}$.

29. We can express 1027 as $10^3 + 3^3 = (10+3)(10^2 - 10\cdot 3 + 3^2) = 13\cdot 79$. Thus, the numbers are **13** and **79**.

30. There is quite a bit going on in this problem. Essentially, this problem is the proof of Ceva's theorem, which states that $\dfrac{AD}{DB}\cdot\dfrac{BE}{EC}\cdot\dfrac{CF}{FA} = 1$. First, ΔAGD and ΔBGD both have the same altitude to side $\overline{AB}$. Therefore, $AD : DB = 2 : 3$. In other words, the ratio of the base lengths to the ratio of the areas are equal if both triangles have the same altitude. The ratio of the areas of ΔACD and ΔBCD is also 2 to 3 because they have bases $\overline{AD}$ and $\overline{DB}$ respectively and share the same altitude to $\overline{AB}$. By the law of proportions, we obtain:

2: 3 = area ΔADC : area ΔBDC = area ΔADG : area ΔBDG =
area ΔADC - area ΔADG: area ΔBDC- area ΔBDG = area ΔAGC : area ΔBGC .

We obtain a similar result using $BE : EC = 5 : 4$ = area ΔBGA : area ΔCGA. Moreover, we can state that $CF : FA$ = area ΔBGC : area ΔBGA. We can now take the product:
$(AD / DB)\cdot(BE / EC)\cdot(CF / FA) = (2/3)\cdot(5/4)\cdot(CF / FA) =$
(area ΔAGC / area ΔBGC)·(area ΔBGA / area ΔAGC)·(area ΔBGC / area ΔBGA) = 1.

Therefore, $CF : FA = 6 : 5$ = area ΔCGF : area ΔFGA. However, we can use $BE : EC = 5 : 4$ = area ΔBGA : area ΔCGA = 50 : (area ΔCGF + area ΔFGA) to obtain area ΔCGF + area ΔFGA = 40. Solving this yields area ΔCGF = **240/11**.