NYCIML — Interscholastic Mathematics League

SOPH FROSH DIVISION | **CONTEST NUMBER ONE** | **SPRING 2007**

PART I: 10 minutes | **NYCIML Contest One** | **Spring 2007**

S07SF1. A $10'' \times 10'' \times 10''$ cube is painted, then cut into 1000 $1'' \times 1'' \times 1''$ cubes. Compute the number of cubes that are painted on exactly one side.

S07SF2. An isosceles trapezoid has sides of length 5, 5, 7 and 13. Compute the length of a diagonal of the trapezoid.

PART II: 10 minutes | **NYCIML Contest One** | **Spring 2007**

S07SF3. Arturo drives from home to school and averages 30 mph. If he returns on the same route, compute what his average rate must be on the return trip in order to average 40 mph for the entire round trip.

S07SF4. Point P is 6 inches from the center of a circle whose radius is 10. Compute the number of chords with integral length which pass through P.

PART III: 10 minutes | **NYCIML Contest One** | **Spring 2007**

S07SF5. In a store, John buys 5 apples, 8 pears and 11 oranges, paying \$2.63. Sam buys 3 apples, 5 pears and 7 oranges, and pays \$1.65. Cindy buys 1 apple, 1 pear and 1 orange. Compute the amount that she pays.

S07SF6. If 796, 1157 and 1594 are divided by the positive integer q, they all leave a remainder of r. Compute r.

ANSWERS:

S07SF1. **384**

S07SF2. $\mathbf{2\sqrt{29}}$

S07SF3. **60**

S07SF4. **8**

S07SF5. **.31 or 31 cents**

S07SF6. **17**

New York City Interscholastic Mathematics League

SOPH FROSH DIVISION | **CONTEST NUMBER TWO** | **SPRING 2007**

PART I: 10 minutes | **NYCIML Contest Two** | **Spring 2007**

S07SF7. Compute the sum of the digits of the first 200 positive integers.

S07SF8. If $[x]$ represents the greatest integer less than or equal to x, compute x: $x[x]=40$.

PART II: 10 minutes | **NYCIML Contest Two** | **Spring 2007**

S07SF9. If $\sqrt{5x^3+5x^3+5x^3+5x^3+5x^3}=625$, compute x.

S07SF10. P is a point within rectangle $ABCD$ such that $PA=3, PB=4, PC=5$. Compute PD.

PART III: 10 minutes | **NYCIML Contest Two** | **Spring 2007**

S07SF11. Express as a fraction in lowest terms: $.26\overline{3}$.

S07SF12. Compute all ordered pairs of positive integers (x,y) that satisfy: $xy+4x-13=x^2$.

ANSWERS:

S07SF7. **1902**

S07SF8. **20/3**

S07SF9. **25**

S07SF10. $\mathbf{3\sqrt{2}}$

S07SF11. $\mathbf{\frac{79}{300}}$

S07SF12. $\mathbf{(1,10),(13,10)}$

New York City Interscholastic Mathematics League

SOPH FROSH DIVISION **CONTEST NUMBER THREE** **SPRING 2007**

PART I: 10 minutes **NYCIML Contest Three** **Spring 2007**

S07SF13. Compute the units digit of: 2007^{2007}.

S07SF14. A man is 4 miles north and 9 miles west of his cabin. He is also 4 miles south of a river which flows east to west. He wishes to go to the river, collect water and then go to his cabin. Compute the length of the shortest route that will accomplish this.

PART II: 10 minutes **NYCIML Contest Three** **Spring 2007**

S07SF15. Compute the two roots of: $\left(x-\frac{1}{4}\right)\left(x-\frac{1}{4}\right)+\left(x-\frac{1}{4}\right)\left(x-\frac{1}{8}\right)=0$.

S07SF16. Compute the smallest positive integer x such that $(252)(x)$ is a perfect cube.

PART III: 10 minutes **NYCIML Contest Three** **Spring 2007**

S07SF17. Fifteen balls are numbered consecutively 1 to 15. If 2 different balls are picked at random, compute the probability that their sum is odd.

S07SF18. The nine-digit number 302,586,7ab is divisible by 72. Compute (a,b).

ANSWERS:

S07SF13. **3**

S07SF14. **15**

S07SF15. $\frac{1}{4},\frac{3}{16}$

S07SF16. 294

S07SF17. $\frac{8}{15}$

S07SF18. **(6, 8)**

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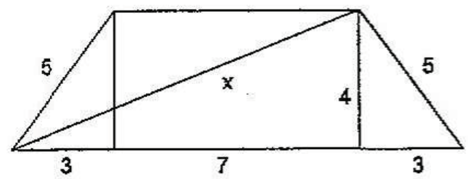
SOPH FROSH DIVISION **CONTEST NUMBER ONE** **SPRING 2007**

SOLUTIONS

S07SF1. The cubes painted on one side consist of the 8×8 squares in the middle of all 6 sides. $8\times 8\times 6=\mathbf{384}$.

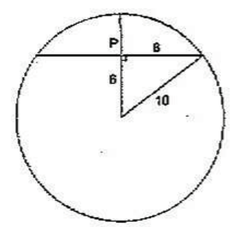
S07SF2.



$x^2=4^2+10^2$ $x=\sqrt{116}=\mathbf{2\sqrt{29}}$.

S07SF3. Let D be the distance between home and school and x be the rate on the return trip. Then $\frac{D}{30}$ is the time for the outbound trip and $\frac{D}{x}$ is the time for the return trip. The total distance divided by the total time = 40. $\frac{2D}{\frac{D}{30}+\frac{D}{x}}=40$ $\frac{60x}{x+30}=40$

$x=\mathbf{60}$.



S07SF4. The longest chord is the diameter through P which has length 20. The shortest is the chord perpendicular to the diameter, with length 16. There are two each of length 17, 18 and 19. Thus there are **8** in total.

S07SF5. $5x+8y+11z=2.63$

$3x+5y+7z=1.65$

Multiply the top equation by 2 and the bottom by 3 to get:

$10x+16y+22z=5.26$

$9x+15y+21z=4.95$

Subtracting, we get: $x+y+z=\mathbf{.31}$

S07SF6. When 2 numbers have the same remainder when divided by the same number, their difference is divisible by that number. $1157-796=361=19\times 19$ and $1594-1157=437=19\times 23$. Thus $q=19$ and dividing any of the numbers by 19, we get $r=\mathbf{17}$.

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SOPH FROSH DIVISION **CONTEST NUMBER TWO** **SPRING 2007**

SOLUTIONS

S07SF7. There will be 20 of each digit in the units column and 20 of each digit in the tens column. $2 \cdot 20(0+1+2+3+4+5+6+7+8+9)=40(45)=1800$. Adding 100 ones for the hundreds column through 199 and 2 for 200, the result is **1902.**

S07SF8. The number must be between 6 and 7. Then $[x]=6$ and $x=\frac{40}{6}=\mathbf{\frac{20}{3}}$.

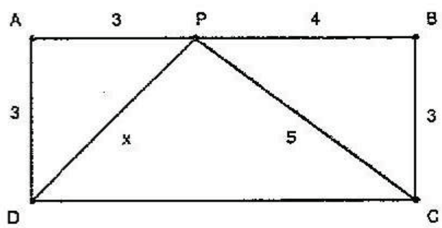
S07SF9. $\sqrt{25x^3}=5^4$

$25x^3=5^8$

$x^3=5^6$

$x=5^2=\mathbf{25}$.

S07SF10. Assume that P is on $\overline{AB}$.



$x^2=3^2+3^2$

$x=\mathbf{3\sqrt{2}}$.

S07SF11. Let $x=.26\overline{3}$. Then $10x=2.6\overline{3}$ and $9x=2.37$. $x=\frac{2.37}{9}=\frac{237}{900}=\mathbf{\frac{79}{300}}$.

S07S12. $x(y+4-x)=13$. Since 13 is prime, x must be either 1 or 13. In either case, $y=10$. Thus the solutions are $\mathbf{(1,10)}$ **and** $\mathbf{(13,10)}$.

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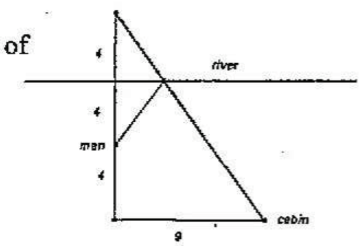
SOPH FROSH DIVISION **CONTEST NUMBER THREE** **SPRING 2007**

SOLUTIONS

S07SF13. The units digit of the powers of a number ending in 7 run in cycles of 4: 7, 9, 3, 1, 7, 9, 3, 1, etc. Since 2007 is 3 more than a multiple of 4, the units digit will be the third in the cycle, **3**.

S07SF14. Suppose that he is 4 miles north of the river, instead of south. Then the shortest route is the hypotenuse of a right triangle with legs 9 miles and 12 miles, which is **15** miles.



S07SF15.
$$\left(x-\frac{1}{4}\right)\left(x-\frac{1}{4}+x-\frac{1}{8}\right)=\left(x-\frac{1}{4}\right)\left(2x-\frac{3}{8}\right)=0$$
$$x=\mathbf{\frac{1}{4},\frac{3}{16}}$$

S07SF16. For a number to be a perfect cube, the exponent of every prime in its prime factorization must be a multiple of 3. $252=2^2\cdot 3^2\cdot 7$, so the answer is $2\cdot 3\cdot 7^2=\mathbf{294}$.

S07SF17. For the sum to be odd, one must be even and the other odd. There are $8\cdot 7=56$ ways for this to happen, and the total number of possibilities is ${}_{15}C_2=105$. The probability is $\frac{56}{105}=\mathbf{\frac{8}{15}}$.

S07SF18. The number must be divisible by 8 and 9. To be divisible by 8, 7*ab* must be divisible by 8. Then *ab* can be 04, 12, 20, 28, 36, 44, 52, 60, 68, 76, 84 or 92. To be divisible by 9, the sum of the number's digits must be divisible by 9. So $a+b=5$ or $a+b=14$. The only one which works is 68, so the answer is $\mathbf{(6,8)}$.