

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division

CONTEST NUMBER 1

PART I *FALL 2005* *CONTEST 1* *TIME: 10 MINUTES*

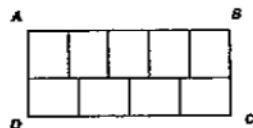
- F05SF1 Compute the number of subsets of the letters in the word "shark" that contain a "k".
- F05SF2 Compute the number of zeros at the end of $126!$.
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PART II *FALL 2005* *CONTEST 1* *TIME: 10 MINUTES*

- F05SF3 $524a7$ is divisible by 3. Compute the number of possible values for a .
- F05SF4 A farmer sells chickens and ducks. Chickens are \$4 each and ducks are \$7 each. At the end of the day, he has collected \$50. If c represents the number of chickens sold, and d the number of ducks sold, compute all possible ordered pairs (c, d) .
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PART III *FALL 2005* *CONTEST 1* *TIME: 10 MINUTES*

- F05SF5 Jack can paint a room in 8 hours. Jill can paint the same room in 6 hours. If Jill works alone for two hours and then Jack starts to help her, compute the number of minutes, to the nearest minute that Jack will work to complete painting of the room.
- F05SF6 Rectangle $ABCD$ in the diagram consists of nine congruent rectangles. If the area of $ABCD$ is 8820, compute the perimeter of $ABCD$.



ANSWERS:

F05SF1	16
F05SF2	31
F05SF3	4
F05SF4	$(2, 6), (9, 2)$
F05SF5	137
F05SF6	406



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CONTEST NUMBER 2

PART I *FALL 2005* *CONTEST 2* *TIME: 10 MINUTES*

F05SF7 Compute the number of arrangements of all of the letters in the word "sharks" that begin with a "k".

F05SF8 The three digit number $3a6$ is added to 219 and the result is divisible by 9. Compute all possible values for a .

PART II *FALL 2005* *CONTEST 2* *TIME: 10 MINUTES*

F05SF9 In regular hexagon $ABCDEF$, compute the number of diagonals that do not contain either point A or point C .

F05SF10 Compute the largest n for which 2^n is a divisor of $80!$.

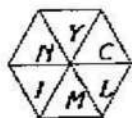
PART III *FALL 2005* *CONTEST 2* *TIME: 10 MINUTES*

F05SF11 $765ab4$ is divisible by 36. Compute all ordered pairs (a, b) for which this is possible.

F05SF12 In the town of Hogsmeade, all streets are numbered consecutively from 1 to n , and all avenues, which are perpendicular to the streets, are named consecutively from A to Z. If Fred is at 17th Street and Avenue Q and wishes to go to Zonko's Joke Shop, which is on 12th Street and Avenue K, compute the number of different paths he may take, if at no point can he be going in the opposite direction from a direction he was traveling earlier.

ANSWERS:

F05SF7	60
F05SF8	6
F05SF9	4
F05SF10	78
F05SF11	$(3, 2), (1, 4), (8, 6), (6, 8)$
F05SF12	462



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CONTEST NUMBER 3

PART I *FALL 2005* *CONTEST 3* *TIME: 10 MINUTES*

F05SF13 A triangle with sides of integral length has two sides that measure 6 and 13. Compute the number of possible lengths for the third side.

F05SF14 A rectangular solid has faces with surface areas of 9, 12, and 27. Compute the volume of the solid.

PART II *FALL 2005* *CONTEST 3* *TIME: 10 MINUTES*

F05SF15 The arithmetic mean (average) of five consecutive integers is 10. Compute the sum of the largest and the smallest of these integers.

F05SF16 Farmer Brown raises rabbits. At first, he has only pregnant mothers in his rabbit house. Five of the mother rabbits had twins and eight of the mother rabbits had triplets, while all other mothers had one child. If the total number of rabbits in the rabbit house is now 67, compute the number of mothers.

PART III *FALL 2005* *CONTEST 3* *TIME: 10 MINUTES*

F05SF17 For natural numbers a , b and c , when a is divided by 7 the remainder is 5, when b is divided by 7 the remainder is 6 and when c is divided by 7, the remainder is 1. Compute the remainder when $a + b + c$ is divided by 7.

F05SF18 The New York City Chess Team has 89 members, which includes the captain. The captain of the team is ill and has to call off the practice session. By arrangement, she calls four members and they each call four members, and so on, until all members have been notified. Compute the greatest number of members who will not have to make a call.

ANSWERS: F05SF13 11
 F05SF14 54
 F05SF15 20
 F05SF16 23
 F05SF17 5
 F05SF18 67



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division

CONTEST NUMBER 1

Fall 2005 Solutions

F05SF1 The number of subsets of "shark" that contain "k" is simply the number of subsets of the letters "shar". There are $2^4 = 16$ of these.

F05SF2 The number of zeros is equal to the number of factors of 5 in the numbers from 1 to 126.

This is equal to $\left\lfloor \frac{126}{5} \right\rfloor + \left\lfloor \frac{126}{5^2} \right\rfloor + \left\lfloor \frac{126}{5^3} \right\rfloor = 31$. ($\lfloor x \rfloor$ is the greatest integer less than or equal to x)

F05SF3 If and only if a number is divisible by 3, the sum of its digits is divisible by 3. The sum of the known digits is 18. This is already divisible by 3, so a must be divisible by 3; therefore, it can be 0, 3, 6, or 9, a total of 4 values.

F05SF4 The ordered pair with the smallest c is $(2, 6)$. From there, you can increase c by 7 and decrease d by 4. As d is 6, this can only be done once, yielding $(2, 6)$ and $(9, 2)$.

F05SF5 Every hour, Jill paints $\frac{1}{6}$ of the room; in two hours, she paints $\frac{1}{3}$ of the room. Every hour

that Jill and Jack work together, they paint $\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$ of the room. Thus, the amount of time that Jack

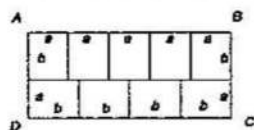
works is equal to $\frac{\frac{2}{3}}{\frac{7}{24}} = \frac{16}{7}$ hours, or 137 minutes.

F05SF6 If a and b are the dimensions of each of the nine rectangles, then $9ab = 8820 \rightarrow ab = 980$.

$5a = 4b$ and $a = \frac{4}{5}b$. $ab = 980 \rightarrow a = \frac{980}{b}$ and

$\frac{4}{5}b = \frac{980}{b} \rightarrow b^2 = 1225 \rightarrow b = 35$. Thus $a = \frac{4}{5} \cdot 35 = 28$. The perimeter of

$ABCD$ is $7a + 6b = 28 \cdot 7 + 6 \cdot 35 = 406$.





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CONTEST NUMBER 2

Fall 2005 Solutions

F05SF7 The number of arrangements of the letters in the word "sharks" that begin with a "k" is equal to the number of arrangements of the letters in "shars". There are $\frac{5!}{2!} = 60$ arrangements.

F05SF8 $3a6 + 219 = 10a + 525$. Since 525 is 3 more than a multiple of 9, $10a$ must be 3 less than a multiple of 9. The value for which this is true is 6.

F05SF9 Drawing a diagram, we can see that the only diagonals which satisfy this requirement are \overline{BD} , \overline{BE} , \overline{BF} , \overline{DF} , for a total of 4 diagonals. Note that this is not the same as considering the diagonals of a quadrilateral (since we are using only 4 vertices) which would be $\frac{4(4-3)}{2} = 2$.

F05SF10 The value of n is equal to the number of factors of 2 in the numbers from 1 to 80. This is:
 $\left\lfloor \frac{80}{2} \right\rfloor + \left\lfloor \frac{80}{2^2} \right\rfloor + \left\lfloor \frac{80}{2^3} \right\rfloor + \dots + \left\lfloor \frac{80}{2^6} \right\rfloor = 40 + 20 + 10 + 5 + 2 + 1 = 78$. ($\lfloor x \rfloor$ is the greatest integer less than or equal to x)

F05SF11 If and only if a number is divisible by 36, it is divisible by both 4 and 9. If and only if a number is divisible by 4, its last two digits are divisible by 4. By testing, we find that b can equal 2, 4, 6, or 8. If and only if a number is divisible by 9, the sum of its digits is divisible by 9. The sum of the known digits is 22, so the sum of a and b must be 5 or 14. So, (a, b) can be: $(3, 2), (1, 4), (8, 6), (6, 8)$.

F05SF12 Fred must go 11 blocks consisting of 6 avenues and 5 streets to get to Zonko's Joke Shop. Therefore there are $\frac{11!}{5!6!} = 462$ paths.



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CONTEST NUMBER 3

Fall 2005 Solutions

F05SF13 The third side can be any integer between $(13-6)$ and $(13+6)$, so it must be between 7 and 19. There are 11 possible lengths.

F05SF14 $lw = 9$, $lh = 12$, $hw = 27$. Multiplying gives us $l^2w^2h^2 = 2916$. Since the volume is lwh , it is the square root of the product of the surface areas and is 54.

F05SF15 The average of these five numbers is simply the middle number, or 10. Thus, the largest and smallest are 8 and 12, which add to 20.

F05SF16 Let m be the number of mother rabbits. There are ten children who are twins, 24 children who are triplets and $m - 13$ who are single birth. The total number of rabbits is:
 $m + (m - 13) + 10 + 24 = 67$. Solving gives us 23 mothers.

F05SF17 The remainder is $5 + 6 + 1 = 12$. But this is more than 7, so we subtract 7 to get 5.

F05SF18 There are 88 people that need to be called (the captain knows that she is ill.) Thus 22 people have to make calls and 67 do not make a call. Alternately, the captain calls 4 people, and each of them calls 4 other people, each of whom calls four other people, so that 85 people $(1 + 4 + 16 + 64)$ now know, and 4 people have yet to be notified. Thus 1 more person must make a call, so $89 - 1 - 4 - 16 - 1 = 67$ do not have to make calls.