NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 1

PART I *FALL 2005* *CONTEST 1* *TIME: 10 MINUTES*

F05A1 On Jeopardy, each contest matches three people. In a tournament, in each match there is always one winner and the winner advances to the next round and the other two players are eliminated. The tournament continues until one person remains. If 243 players enter the tournament, compute the number of contests that must be played to determine the champion.

F05A2 Triangle GAB has sides $GA = 28$, $AB = 32$ and $GB = 40$. The altitude drawn to side GB intersects that side at H. Compute GH.

PART II *FALL 2005* *CONTEST 1* *TIME: 10 MINUTES*

F05A3 Compute the numerical value of $\dfrac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$.

F05A4 Compute all real values of x: $\dfrac{x+1}{x+2} + \dfrac{x+7}{x+8} = \dfrac{x+3}{x+4} + \dfrac{x+5}{x+6}$.

PART III *FALL 2005* *CONTEST 1* *TIME: 10 MINUTES*

F05A5 Compute all real values of x: $|x|^2 = |2x - 120|$.

F05A6 Donald Trimp is building a theater with k rows. In the first row there are 15 seats, 17 seats in the second row, 19 seats in the third row, and so on. Compute the smallest value for k such that the number of seats is a perfect square.

ANSWERS:

F05A1	**121**
F05A2	**17**
F05A3	$\sqrt{3}$
F05A4	**-5**
F05A5	**10, -12**
F05A6	**18**

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 2

PART I ***FALL 2005*** ***CONTEST 2*** ***TIME: 10 MINUTES***

F05A7 Compute the number of real roots that satisfy the equation: $5^{\sqrt{5-x}} = \left(5^{\sqrt{5-x}}\right)^x$.

F05A8 Isosceles triangles are cut off of each corner of an 8-inch square. The resulting shape is a regular octagon. The area of the octagon may be expressed as $a + b\sqrt{2}$. Compute $a + b$.

PART II ***FALL 2005*** ***CONTEST 2*** ***TIME: 10 MINUTES***

F05A9 The sum of an infinite geometric sequence is 3. The sum of the squares of the terms of the sequence is $\frac{9}{2}$. Compute the common ratio of the original geometric sequence.

F05A10 A circle passes through the three points N (-2, 2), Y (6, 6) and C (8, 2). Compute the length of the radius of the given circle.

PART III ***FALL 2005*** ***CONTEST 2*** ***TIME: 10 MINUTES***

F05A11 Compute all real values of x: $2x^2 - x - \sqrt{2x^2 - x - 6} = 12$.

F05A12 Working together, Ben and Jerry take 15 hours to build a model car. Working together, Ben and Tom can build the same model car in 20 hours. Working together, Tom and Jerry take 25 hours to build the model car. Compute the number of hours it will take Tom, Ben, and Jerry to build the model car working together. (Assume that each works at a constant rate.)

ANSWERS:		
	F05A7	**2**
	F05A8	**0**
	F05A9	$\mathbf{\frac{1}{3}}$
	F05A10	**5**
	F05A11	**-2.5, 3**
	F05A12	$\mathbf{\frac{600}{47}}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 3

PART I *FALL 2005* *CONTEST 3* *TIME: 10 MINUTES*

F05A13 Compute x: $2\log_6(x+5)-2\log_6(x+6)=\log_6 5-1$.

F05A14 Compute: $\dfrac{\cos 28^\circ+\cos 36^\circ+\cos 54^\circ+\cos 62^\circ}{\cos 9^\circ+\cos 17^\circ}$.

PART II *FALL 2005* *CONTEST 3* *TIME: 10 MINUTES*

F05A15 Two perpendicular chords intersect in circle O. One chord is divided into two parts of lengths 4 and 6. The other chord has a length of 14. Compute the length of the radius of the circle.

F05A16 Four players, playing bridge, are each dealt 13 cards from a standard deck of playing cards. Charles Goren has a hand with 9 spades and 4 clubs. Compute the probability that his partner, Grace Goren, has the remaining 4 spades in her hand.

PART III *FALL 2005* *CONTEST 3* *TIME: 10 MINUTES*

F05A17 Find the first set of three consecutive positive integers such that the smallest is a multiple of 3, the second is a multiple of 5 and the largest is a multiple of 7.

F05A18 Solve for all ordered triples (p, q, r) if

$$2pq+p+q=115$$
$$2qr+q+r=49$$
$$2pr+p+r=10$$

ANSWERS:		
	F05A13	$\sqrt{30}$
	F05A14	$\sqrt{2}$
	F05A15	$5\sqrt{2}$
	F05A16	$\frac{55}{6327}$
	F05A17	54, 55, 56
	F05A18	(3, 16, 1), (-4, -17, -2)

New York City Interscholastic Mathematics League

New York City Interscholastic Mathematics League
Senior A Division — CONTEST NUMBER 4

PART I — *FALL 2005* — *CONTEST 4* — *TIME: 10 MINUTES*

F05A19 — a,b,c,d are non-zero real numbers. Compute all possible values of: $\frac{a}{|a|}+\frac{b}{|b|}+\frac{c}{|c|}+\frac{d}{|d|}+\frac{abcd}{|abcd|}$.

F05A20 — Compute all possible values of the real number k for which the following equation has one real root: $2005^{x}+k2005^{-x}=10$.

PART II — *FALL 2005* — *CONTEST 4* — *TIME: 10 MINUTES*

F05A21 — $x^2+4hx=1$ and the sum of the squares of the roots of the equation is 10. Compute h^2.

F05A22 — ΔABC is an equilateral triangle with side 6. $\overline{BC}$ is extended through C to D so that $\overline{CD}=6$. If E is the midpoint of $\overline{AB}$, and $\overline{DE}$ intersects $\overline{AC}$ at F, compute the area of quadrilateral $BEFC$.

PART III — *FALL 2005* — *CONTEST 4* — *TIME: 10 MINUTES*

F05A23 — Compute the number of integral values of x: $\left|\frac{x+12}{x+2}\right|\geq 3$.

F05A24 — Compute the right-most non-zero digit when 100! is computed.

ANSWERS:

F05A19	**-3, 1, 5**
F05A20	$\mathbf{k\leq 0}$ **or** $\mathbf{k=25}$
F05A21	$\frac{1}{2}$
F05A22	$6\sqrt{3}$
F05A23	**7**
F05A24	**4**

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division CONTEST NUMBER 5

PART I ***FALL 2005*** ***CONTEST 5*** ***TIME: 10 MINUTES***

F05A25 A fair coin is tossed repeatedly. Compute the probability that heads will appear four times before tails appears twice.

F05A26 If x, y, z are real numbers, compute all possible values of $x+y+z$:

$x+y-z=3$

$xy+9=0$

$x^2+y^2+z^2=27.$

PART II ***FALL 2005*** ***CONTEST 5*** ***TIME: 10 MINUTES***

F05A27 A box contains 4 red, 5 blue and 6 white marbles. Compute the number of samples of four marbles, chosen randomly and without replacement, such that every color will be represented.

F05A28 In trapezoid $QUIK$, bases $\overline{QU}$ and $\overline{IK}$ have lengths of 15 and 45 respectively. The non-parallel sides $\overline{QK}$ and $\overline{UI}$ have lengths of 30 and 20 respectively. $\overline{AB}$, a line segment parallel to $\overline{QU}$ is drawn, with A on $\overline{QK}$ and B on $\overline{UI}$. $\overline{AB}$ divides the trapezoid into two trapezoids of equal perimeter. Compute $AQ: AK$.

PART III ***FALL 2005*** ***CONTEST 5*** ***TIME: 10 MINUTES***

F05A29 Compute the sum of the infinite series: $\frac{2}{8}+\frac{20}{8^2}+\frac{56}{8^3}+\frac{272}{8^4}+\ldots+\frac{4^n+(-2)^n}{8^n}+\ldots$

F05A30 A beetle walking on a Cartesian coordinate plane starts at the origin. It walks one unit up to (0, 1). The beetle now turns 90° to the right and walks 2 units and then turns 90 ° to the right and walks 3 units. The beetle continues this process until it walks 2005 units in one direction. Compute the coordinates of the beetle when it stops.

ANSWERS:

F05A25	$\frac{3}{16}$
F05A26	**3, -3**
F05A27	**720**
F05A28	**4:1 or 4**
F05A29	$\frac{4}{5}$
F05A30	**(-1002, 1003)**

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division CONTEST NUMBER 1
Fall 2005 Solutions

F05A1 In each match 1/3 of the contestants move on to the next round, so 243 contestants require 81 matches with 81 winners. The second round would require 27 matches, The third round would need 9 matches. The fourth round only 3 matches and the final is a single match. The total (81+27+9+3+1) is **121**. OR: Since 242 must lose and 2 people lose in each match, there were **121** matches.

F05A2 Let $GH = x$ and $AH = h$ then $HB = 40\text{-}x$. AHG and AHB are right triangles. Using the Pythagorean Theorem twice, we get two equations $x^2+h^2=28^2$ and $(40-x)^2+h^2=32^2$. Subtract the second equation from the first and we get $x^2-(40-x)^2=-240$. Simplify the equation we get $80x-1600=-240$ therefore $x=\mathbf{17}$.

OR: Use Heron's Formula to find the area, and then compute the area by $\frac{1}{2}bh$, etc.

F05A3 $$\frac{\cos 15^\circ+\sin 15^\circ}{\cos 15^\circ-\sin 15^\circ}=\left(\frac{\cos 15^\circ+\sin 15^\circ}{\cos 15^\circ-\sin 15^\circ}\right)\left(\frac{\cos 15^\circ+\sin 15^\circ}{\cos 15^\circ+\sin 15^\circ}\right)=\frac{\cos^2 15^\circ+\sin^2 15^\circ+2\cos 15^\circ\sin 15^\circ}{\cos^2 15^\circ-\sin^2 15^\circ}$$

$$=\frac{1+\sin 30^\circ}{\cos 30^\circ}=\frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\sqrt{3}$$

F05A4 One could multiply by the LCD and solve the resulting algebraic monstrosity. However, the realization that all the terms would be equal to one if the numerators were increased by one, leads to a simpler equation: $\frac{1}{x+6}+\frac{1}{x+4}=\frac{1}{x+2}+\frac{1}{x+8}$. Combine both fractions by finding common denominators we get: $\frac{2x+10}{(x+6)(x+4)}=\frac{2x+10}{(x+2)(x+8)}$. Since the numerators are equal, the numerator must be 0 or the denominators are equal. The denominators are never equal so, $2x+10=0 \quad \therefore \quad x=-5$.

F05A5 Using $|a|=\pm a$, the equation simplifies to $x^2-2x+120=0$ or $x^2+2x-120=0$. The first quadratic has no real roots (the discriminant is –476). The second quadratic yields: $(x+12)(x-10)=0 \;\therefore x=\mathbf{-12,10}$.

F05A6 Adding up the seats in each row gives us an arithmetic series $15+17+19+21+(15+2(k-1))=k(k+14)=k^2+14k$. To make k^2+14k a perfect square (n^2) we need to add 49 to get $(k+7)^2=n^2+49=p^2$ (another perfect square). Therefore $p^2-n^2=49$ or $(p+n)(p-n)=49$. 49 can be factored into (49)(1) or (7)(7). We set $p+n=49$ and $p-n=1$ giving $p=25$ so $k=18$. The other factor pair doesn't work out.

New York City Interscholastic Mathematics League
Senior A Division Contest Number 2
Fall 2005 Solutions

F05A7 Since the bases are the same the exponents must be equal, $\sqrt{5-x} = x\sqrt{5-x}$.
$(1-x)\sqrt{5-x} = 0 \therefore x = 1$ or 5. There are **2** roots.

F05A8 The area of the octagon = the area of the square – the area of the triangles. The area of the square is 64. Let x = the leg of the isosceles triangle, since two triangles are removed from each side of the square then each side of the octagon must be 8-2x. Use the Pythagorean Theorem, to get $x^2 + x^2 = (8-2x)^2$. Solve the quadratic equation and we get $x = 8 \pm 4\sqrt{2}$. Since $0 < x < 8$, $x = 8 - 4\sqrt{2}$. The area of the triangles is:
$4 \cdot \frac{(8-4\sqrt{2})^2}{2} = 192 - 128\sqrt{2}$. The area of the octagon is $128\sqrt{2} - 128$ $\therefore$ $a + b = \mathbf{0}$.

F05A9 If we denote the first geometric sequence as $a, ar, ar^2, ar^3 \ldots$ then the second sequence is $a^2, a^2r^2, a^2r^4, a^2r^6 \ldots$ The sum of the first sequence is $\frac{a}{1-r} = 3$ or $a = 3(1-r)$. The sum of the second sequence is $\frac{a^2}{1-r^2} = \frac{9}{2} = \frac{a^2}{(1-r)(1+r)} \therefore \frac{a}{1+r} = \frac{3}{2}$ or $a = \frac{3}{2}(1+r)$.
$3(1-r) = \frac{3}{2}(1+r) \quad \therefore r = \mathbf{\frac{1}{3}}$.

F05A10 The center is the intersection of any two perpendicular bisectors of any two chords. The perpendicular bisector of $\overline{NC}$ is x=3. The perpendicular bisector of $\overline{YC}$ is $y = .5x + .5$. So when $x = 3$, $y = 2$. The center is (3, 2). The radius is **5**.

F05A11 Let $a^2 = 2x^2 - x - 6$ then the equation becomes: $a^2 + 6 - \sqrt{a^2} = 12$ or $a^2 - |a| - 6 = 0$. So a = 3, -3, 2, or –2 therefore $2x^2 - x - 6 = 9$ or 4. Thus x = **-2.5 or 3**. (x = 2.5 and –2 are extraneous roots).

F05A12 Let B = the number of hours it takes Ben to do the job, working alone. Let T = the number of hours it takes Tom to do the job, working alone. Let J = the number of hours it takes Jerry to do the job, working alone. We get three equations:
$15\left(\frac{1}{B} + \frac{1}{J}\right) = 1,\ 20\left(\frac{1}{B} + \frac{1}{T}\right) = 1,\ 25\left(\frac{1}{T} + \frac{1}{J}\right) = 1$
$\therefore \frac{1}{B} + \frac{1}{J} = \frac{1}{15},\ \frac{1}{B} + \frac{1}{T} = \frac{1}{20},\ \frac{1}{T} + \frac{1}{J} = \frac{1}{25}$
by adding the three equations we get
$\frac{2}{T} + \frac{2}{B} + \frac{2}{J} = \frac{47}{300}$ so $\frac{1}{T} + \frac{1}{B} + \frac{1}{J} = \frac{47}{600} \therefore time = \mathbf{\frac{600}{47}}$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division CONTEST NUMBER 3

Fall 2005 Solutions

F05A13

$$2\log_6\left(\frac{x+5}{x+6}\right)=\log_6\left(\frac{5}{6}\right)$$

$$\left(\frac{x+5}{x+6}\right)^2=\frac{5}{6}$$

$$\frac{x^2+10x+25}{x^2+12x+36}=\frac{5}{6}$$

$$6x^2+60x+150=5x^2+60x+180$$

$$x^2-30=0$$

$$x=\pm\sqrt{30}\therefore x=\mathbf{\sqrt{30}}$$

F05A14

$$\cos 28^\circ=\cos\left(45^\circ-17^\circ\right)=\cos 45^\circ\cos 17^\circ+\sin 45^\circ\sin 17^\circ$$

$$\cos 62^\circ=\cos\left(45^\circ+17^\circ\right)=\cos 45^\circ\cos 17^\circ-\sin 45^\circ\sin 17^\circ$$

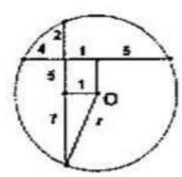
$$\cos 36^\circ=\cos\left(45^\circ-9^\circ\right)=\cos 45^\circ\cos 9^\circ+\sin 45^\circ\sin 9^\circ$$

$$\cos 54^\circ=\cos\left(45^\circ+9^\circ\right)=\cos 45^\circ\cos 9^\circ-\sin 45^\circ\sin 9^\circ$$

$$\therefore \cos 28^\circ+\cos 62^\circ+\cos 36^\circ+\cos 54^\circ=2\cos 45^\circ\cos 17^\circ+2\cos 45^\circ\cos 9^\circ$$

Divide by $\cos 9^\circ+\cos 17^\circ$ and we get $2\cos 45^\circ=\mathbf{\sqrt{2}}$

F05A15 Let x = one segment of the intersected chord.
$x(14-x)=24\rightarrow x=2$, or $x=12$. Drawing segments as in the diagram, we get:
$r^2=1^2+7^2=50\rightarrow r=\sqrt{50}=\mathbf{5\sqrt{2}}$.



F05A16 Grace must receive 9 other cards in addition to getting the remaining 4 spades. The probability that Grace has the remaining four spades

$$=\frac{{}_4C_4\cdot{}_{35}C_9}{{}_{39}C_{13}}=\frac{1\cdot 35!}{9!26!}\cdot\frac{13!26!}{39!}=\frac{13\cdot 12\cdot 11\cdot 10}{39\cdot 38\cdot 37\cdot 36}=\mathbf{\frac{55}{6327}}$$

F05A17 To find these numbers first multiply 3,5, and 7 to get 105. Add 3,5 and 7 onto 105 to get 108, 110, 112. This guarantees that the numbers are divisible by 3, 5 and 7 respectively. Now divide by 2 to get **54, 55, 56**.

F05A18 Multiply each equation by 2, add 1 to each and then factor to obtain

$(2p + 1)(2q + 1) = 231$
$(2q + 1)(2r + 1) = 99$
$(2p + 1)(2r + 1) = 21$

Multiply all three equations to get $(2p + 1)^2(2q + 1)^2(2r + 1)^2 = (21)(99)(231)$
Therefore $(2p + 1)(2q + 1)(2r + 1) = \pm 693$
And $(2p + 1, 2q + 1, 2r + 1) = (7, 33, 3)$ or $(-7, -33, -3)$
Therefore the answer for (p, q, r) is $(3, 16, 1)$ or $(-4, -17, -2)$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division   CONTEST NUMBER 4   Fall 2005 Solutions

F05A19 The value of $\frac{x}{|x|}=1$ when $x>0$ and $\frac{x}{|x|}=-1$ when $x<0$. If a, b, c, and d are positive then the sum = 5. If 3 are positive and 1 negative then the sum = 1. If exactly 2 are positive then the sum = 1. If only one is positive then the sum = -3. If all are negative then the sum = -3. thus the answers are **-3,1,5**.

F05A20 Let $A=2005^x$ then $A+\frac{k}{A}=10$. $A^2+k=10A$ or $A^2-10A+k=0$. If $k>0$ you get one root when the discriminant = 100- $4k=0$ $\therefore k=25$ (Otherwise you get 2 positive roots when $0<k<25$ and no real roots when $k>25$.)
If $k=0$, then the original equation is linear $A=10$ (with one root). If $k<0$ then one root for A is negative and one root is positive. But 2005^x cannot be negative so you get one root for x. So k can be **$k\leq 0$ or 25.**

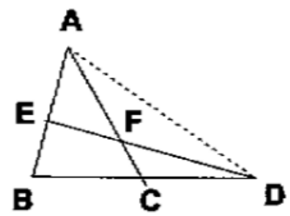
F05A21 $x^2+4hx-1=0$ the sum of the roots is $-4h$ and the product of the roots is -1
If we call the roots a and b, then $a+b=-4h$ and $ab=-1$.

$(a+b)^2=a^2+b^2+2ab \qquad 16h^2=10-2=8 \therefore h^2=\mathbf{\frac{1}{2}}$.

F05A22 Area of $\Delta ABC=\frac{6^2}{4}\sqrt{3}=9\sqrt{3}$

Area of $\Delta ACD=9\sqrt{3}$ since it has the same base and height.

$FC=\frac{1}{3}AC,$ since the medians intersect $\frac{2}{3}$ of the way to the opposite side.



Area $\Delta FCD=\frac{1}{3}\left(9\sqrt{3}\right)=3\sqrt{3},$ Area $\Delta DBE=9\sqrt{3}\rightarrow$ Area $BEFC=9\sqrt{3}-3\sqrt{3}=\mathbf{6\sqrt{3}}.$

OR: Quadrilateral $BEFC$ can be split into ΔBEF and ΔBCF. Notice that F is the centroid of ΔABD, therefore ΔBEF and ΔBCF are each $\frac{1}{6}$ of ΔABD.

F05A23 Multiply both sides by $|x+2|$

$|x+12|\geq 3|x+2|$ however $x\neq -2$

Square both sides and get $x^2+24x+144\geq 9x^2+36x+36$

Or $2x^2+3x-27\leq 0$

$(2x+9)(x-3)\leq 0 \therefore -4.5\leq x\leq 3 \wedge x\neq -2$

$x=-4$, -3, -1, 0, 1, 2, 3 or **7** values.

F05A24 To calculate the first non-zero digit, we look at the last digit of each factor as well as the multiples of 5 (in the case of multiples of 10 we will look at the first non-zero digit). In 100! each digit would appear 10 times at the end. Each digit would appear an additional time in a multiple of 10. (Also we factor out a 5 from 50 to get a 1.) The product of the digits 1, 2, 3, 4, 6, 7, 8, 9 ends in 6. Multiplying them 11 times will leave a 6 at the end. There are ten numbers ending in five. If we divide out the 5, the units digits would be 1,3,5,7,9 (twice). The 5's come from 25 and 75. 25 and 75 will factor out a second 5 and leave us with 1 and 3. The numbers ending in 5 have a last digit of 3 when multiplied and the fives are removed. Each 5 requires a factor of 2, the fives will need 13 twos. Therefore we need to divide the 8 (last digit of 6 times 3) by the last digit of 2^{13} which is a 2. The last non-zero digit is 8 divided by 2 or **4**.

New York City Interscholastic Mathematics League

Senior A Division CONTEST NUMBER 5

Fall 2005 Solutions

F05A25 For 4 heads to appear before 2 tails, there can be 4 heads in a row or 4 heads out of 5 tosses. P(4 Heads) = $\left(\frac{1}{2}\right)^4=\frac{1}{16}$, P(4 out of 5 heads)= ${}_5C_4\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1=\frac{5}{32}$

but this includes the probability of getting 4 heads then a tail = $\frac{1}{32}$

$P(\text{4 heads before 2 tails})=\frac{1}{16}+\frac{5}{32}-\frac{1}{32}=\mathbf{\frac{3}{16}}$.

F05A26

$$\begin{cases} x+y=z+3 \\ (x+y)^2=(z+3)^2 \\ x^2+y^2+2xy=z^2+6z+9 \\ 27-z^2-18=z^2+6z+9 \\ 2z^2+6z=0 \\ z=0,-3 \quad \text{and} \quad x+y+z=\mathbf{3 \text{ or } -3} \end{cases}$$

F05A27 We are choosing four marbles and each color is present, we can have 2 red, 1 blue, and 1 white marble or 1 red, 2 blue, and 1 white marble or 1 red, 1 blue, and 2 white marbles.

$${}_4C_2\times{}_5C_1\times{}_6C_1=180$$
$${}_4C_1\times{}_5C_2\times{}_6C_1=240$$
$${}_4C_1\times{}_5C_1\times{}_6C_2=300$$
$$180+240+300=\mathbf{720}$$

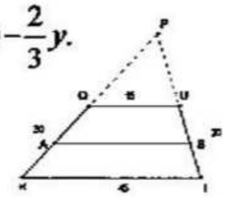
F05A28 Extend $\overline{KQ}$ and $\overline{UI}$ until they meet at a point, P. $\frac{PQ}{PK}=\frac{15}{45}$ and $\frac{PU}{PI}=\frac{15}{45}$

$\frac{PQ}{PK}=\frac{PQ}{PQ+30}=\frac{15}{45}$ and $\frac{PU}{PI}=\frac{PU}{PU+20}=\frac{15}{45}\therefore PQ=15$ and $PU=10$.

If we let $AB=y$, then $QA=y\text{-}15$, $AK=45-y$, $UB=\frac{2}{3}y-10$ and $BI=30-\frac{2}{3}y$.

Set the perimeters of the two trapezoids equal and find $y=39$

$\therefore\frac{QA}{AK}=\frac{24}{6}=\mathbf{4}$



F05A29 There are two infinite geometric series being added here:

$$\left(\frac{4}{8}\right)^n+\left(\frac{-2}{8}\right)^n=\left(\frac{1}{2}\right)^n+\left(\frac{-1}{4}\right)^n=\frac{\frac{1}{2}}{1-\frac{1}{2}}+\frac{\frac{-1}{4}}{1-\frac{-1}{4}}=\mathbf{\frac{4}{5}}$$

F05A30 Every four steps the beetle walks 2 units south and 2 units west. So after 2004 steps (501 groups of 4) the beetle is at (-1002,-1002). It then walks 2005 units north so it is at **(-1002, 1003)**.