New York City
Interscholastic
Mathematics
League

SENIOR B DIVISION | **CONTEST NUMBER ONE** | **SPRING 2005**

PART I: 10 minutes | **NYCIML Contest One** | **Spring 2005**

S05B01. Solve for all values of x: $x+\sqrt{8-x}=6$.

S05B02. Compute the area of the region bounded by the graphs of: $x=5$, $x=0$, $y=-2$, and $y=|2x-3|$.

PART II: 10 minutes | **NYCIML Contest One** | **Spring 2005**

S05B03. A circle is inscribed in an equilateral triangle with sides of length 6. Compute the sum of the areas of the regions outside the circle and inside the triangle.

S05B04. A convex polygon has 44 diagonals. Compute the number of sides of the polygon.

PART III: 10 minutes | **NYCIML Contest One** | **Spring 2005**

S05B05. Compute the tens digit (t) and units digit (u) of: $1!+2!+3!+\cdots+50!$. Write your answer as an ordered pair (t, u).

S05B06. In an isosceles trapezoid, the length of the longer base equals the length of the diagonal. The length of the shorter base equals the length of the leg. Compute the measure in degrees of the smallest angle of the trapezoid.

ANSWERS

1. **4**
2. **24.5**
3. $\mathbf{9\sqrt{3}-3\pi}$
4. **11**
5. **(1, 3)**
6. **72**

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SENIOR B DIVISION | **CONTEST NUMBER TWO** | **SPRING 2005**

PART I: 10 minutes | **NYCIML Contest Two** | **Spring 2005**

S05B07. The probability of Gary passing a test is $\frac{2}{3}$ and the probability of Bill passing the test is $\frac{4}{5}$. Compute the probability that exactly one of them will pass the test.

S05B08. Solve for all values of x: $\left[\frac{3x}{4}\right]=5$, where $[x]$ is the greatest integer less than or equal to x.

PART II: 10 minutes | **NYCIML Contest Two** | **Spring 2005**

S05B09. A $10\times10\times10$ cube is painted, and then cut into 1000 $1\times1\times1$ cubes. Compute the number of these cubes that are painted on exactly one face.

S05B10. Compute: $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\ \cdots\ +\frac{1}{10100}$.

PART III: 10 minutes | **NYCIML Contest Two** | **Spring 2005**

S05B11. Three fair dice are thrown. Compute the probability that their sum is odd and their product is even.

S05B12. A cube is inscribed in a sphere of radius 3. Compute the volume of the cube.

ANSWERS

7.	$\frac{2}{5}$	10.	$\frac{100}{101}$
8.	$\frac{20}{3}\le x<8$	11.	$\frac{3}{8}$
9.	**384**	12.	$24\sqrt{3}$

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SENIOR B DIVISION | **CONTEST NUMBER THREE** | **SPRING 2005**

PART I: 10 minutes | **NYCIML Contest Three** | **Spring 2005**

S05B13. Fifteen billiard balls are arranged in rows to form an equilateral triangle with 5 balls on a side. Using the same type of configuration, compute the total number of balls an equilateral triangle with 100 balls on a side contains.

S05B14. If $\log_2\left(\log_4\left(\log_6 x\right)\right)=0$, compute x.

PART II: 10 minutes | **NYCIML Contest Three** | **Spring 2005**

S05B15. Consider the set of 4-digit natural numbers such that each number has no repeating digit. Compute the number of these that are in ascending order.

S05B16. A square with side 8 is inscribed in a semicircle. Compute the area of the square inscribed in a circle with the same radius as the semicircle.

PART III: 10 minutes | **NYCIML Contest Three** | **Spring 2005**

S05B17. For the number $\underline{9}\ \underline{A}\ \underline{8},\ \underline{B}\ \underline{9}\ \underline{C},\ \underline{8}\ \underline{D}\ \underline{4}$, the numbers 1, 2, 6, and 7 are picked at random to replace A, B, C, and D, not necessarily in that order. Compute the probability that the resulting number is divisible by 99.

S05B18. Three fair dice are thrown and their sum is 7. Compute the probability that at least one die shows a 1.

ANSWERS

13. **5050**
14. **1296**
15. **126**
16. **160**
17. **1**
18. $\mathbf{\frac{4}{5}}$

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SENIOR B DIVISION	**CONTEST NUMBER FOUR**	**SPRING 2005**
PART I: 10 minutes	**NYCIML Contest Four**	**Spring 2005**

S05B19. Compute the number of the first 100 positive integers that have an odd number of positive integral factors.

S05B20. Solve for x: $x[x]=40$, where $[x]$ is the greatest integer less than or equal to x.

PART II: 10 minutes	**NYCIML Contest Four**	**Spring 2005**

S05B21. If $\log_7 x = a$, express $\log_{343} x^2$ in terms of a in simplest form with no logarithms.

S05B22. If $x+\frac{1}{x}=5$, compute $x^3+\frac{1}{x^3}$.

PART III: 10 minutes	**NYCIML Contest Four**	**Spring 2005**

S05B23. Jessica has three teenage sons. If the product of their ages is 3780, compute the age of the oldest son.

S05B24. Compute the length of the largest altitude in a triangle whose sides are 5, 6, and 9.

ANSWERS

19. **10**
20. $\mathbf{6\frac{2}{3}}$
21. $\mathbf{\frac{2a}{3}}$
22. **110**
23. **18**
24. $\mathbf{4\sqrt{2}}$

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SENIOR B DIVISION | **CONTEST NUMBER FIVE** | **SPRING 2005**

PART I: 10 minutes | **NYCIML Contest Five** | **Spring 2005**

S05B25. Compute the number of distinct equilateral triangles with integral side lengths that have an area less than 50.

S05B26. In a row of 6 students, Ellen sits in front of Tonya. Compute the probability that Ellen sits in the seat *directly* in front of Tonya.

PART II: 10 minutes | **NYCIML Contest Five** | **Spring 2005**

S05B27. The 5 odd digits are used to form 120 different 5- digit numbers, with no repetition of a digit in any number. If they are arranged in ascending numerical order, compute the 83rd number on the list.

S05B28. A circle has radius 10", and a chord parallel to a diameter is 5" from the diameter. Compute the area of the region inside the circle and between the chord and the diameter.

PART III: 10 minutes | **NYCIML Contest Five** | **Spring 2005**

S05B29. Compute the number of equations of the form $x^2+bx+c=0$ that have real roots, if b and c are positive integers less than or equal to 6.

S05B30. $\sin 75^\circ - \cos 75^\circ = \sin x$. If x is acute, compute x.

ANSWERS

25. **10**
26. $\mathbf{\frac{1}{3}}$
27. **73915**
28. $\frac{50\pi}{3}+25\sqrt{3}$ or $\frac{50\pi+75\sqrt{3}}{3}$
29. **19**
30. **45°**

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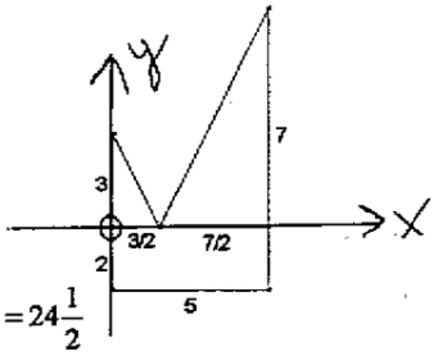
SENIOR B DIVISION **CONTEST NUMBER ONE** **Spring 2005**

SOLUTIONS

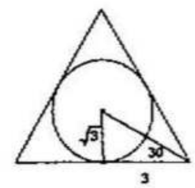
S05B1 $\sqrt{8-x}=6-x$, $8-x=36-12x+x^2$, $x^2-11x+28=0$, $x=4$ or $x=7$, 7 does not check. $\therefore x=4$

S05B2 The area consists of a 2×5 rectangle and 2 triangles.



$10+\frac{9}{4}+\frac{49}{4}=24\frac{1}{2}$

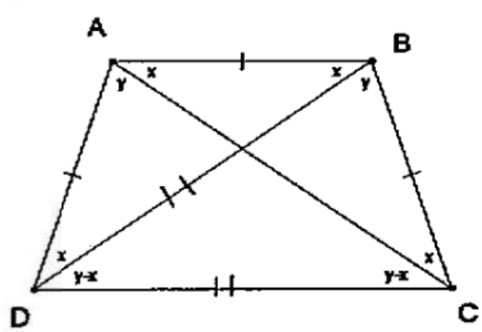
S05B3 The radius of the circle is $\sqrt{3}$. The area is $\frac{6^2\sqrt{3}}{4}-3\pi=9\sqrt{3}-3\pi$.



S05B4 A convex polygon with N sides has $\frac{N(N-3)}{2}$ diagonals. $N(N-3)=88$, $N^2-3N-88=0$, $N=11$.

S05B5 We are only concerned with $1!+\cdots+9!$ since after that, the last 2 digits are zeroes. $1!+\cdots+9!=409113$, thus the answer is $(1,3)$.

S05B6 $\Delta ABC: 3x+y=180$, $\Delta ACD: 3y-x=180$. Solving, $y=72, x=36$. Base angle=72.



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SENIOR B DIVISION **CONTEST NUMBER TWO** **Spring 2005**

SOLUTIONS

S05B7 $\frac{2}{3}\bullet\frac{1}{5}+\frac{4}{5}\bullet\frac{1}{3}=\frac{2}{15}+\frac{4}{15}=\frac{2}{5}$

S05B8 $5\le\frac{3x}{4}<6$, $20\le 3x<24$, $\frac{20}{3}\le x<8$

S05B9 The small cubes painted on one face came from an 8×8 square in the middle of each of the 6 faces of the original large cube. $64\times 6=384$ cubes..

S05B10 $\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\cdots+\frac{1}{10100}$

$=\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\frac{1}{4\times 5}+\frac{1}{5\times 6}+\cdots+\frac{1}{100\times 101}$

$\frac{1}{N(N+1)}=\frac{1}{N}-\frac{1}{N+1}$, so we get $1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots-\frac{1}{100}+\frac{1}{100}-\frac{1}{101}$ and

this telescopes to $1-\frac{1}{101}=\frac{100}{101}$

S05B11 The only way for their sum to be odd and their product even is if 2 are even and one is odd. The probability of this is: $3\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{3}{8}$.

S05B12 The diagonal of a cube with side x is $x\sqrt{3}$. $x\sqrt{3}=6$, $3x^3\sqrt{3}=216$, $x^3=\frac{216}{3\sqrt{3}}=\frac{72}{\sqrt{3}}=24\sqrt{3}$.

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SENIOR B DIVISION **CONTEST NUMBER THREE** **Spring 2005**

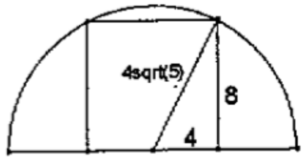
SOLUTIONS

S05B13 The number of balls is $1+2+3+\cdots 100=\frac{100(101)}{2}=5050$.

S05B14 $2^0=\log_4(\log_6 x)=1$, $4=\log_6 x$, $x=6^4=1296$.

S05B15 Zero cannot be used, since it would be the first digit. Using the other 9 digits, there are ${}_9C_4=126$ numbers.

S05B16 The radius of the semicircle is $4\sqrt{5}$. The big square has diagonal $8\sqrt{5}$ and therefore area =160.



S05B17 Since the sum of the digits is 54, the number is divisible by 9. Since the odd-placed digits have a sum of 38, and the even-placed digits have a sum of 16, the difference is 22 and the number is divisible by 11. Since the number is divisible by both 9 and 11, it must be divisible by 99, so the probability is 1.

S05B18 The possibilities are: $(1,1,5)$ - 3 ways, $(1,2,4)$ - 6 ways, $(1,3,3)$ - 3 ways, $(2,2,3)$ - 3 ways. Since 12 out of 15 have a 1, the probability is $\frac{4}{5}$.

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SENIOR B DIVISION **CONTEST NUMBER FOUR** **Spring 2005**

SOLUTIONS

S05B19 The only numbers with an odd number of integral factors are the perfect squares. 1, 4, 9, 16, 25, 36, 49, 64, 81, 100. 10 numbers.

S04B20 x must be a number between 6 and 7. Therefore, $[x]=6$, $6x=40$, $x=\frac{40}{6}=\frac{20}{3}=6\frac{2}{3}$.

S04B21 $\log_{343} x^2 = 2\log_{343} x = 2\log_{343} 7^a = 2a\log_{343} 7 = 2a\cdot\frac{1}{3}=\frac{2a}{3}$

S04B22 $\left(x+\frac{1}{x}\right)^2=25$, $x^2+\frac{1}{x^2}+2=25$, $x^2+\frac{1}{x^2}=23$,
$\left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}\right)=5\times23=115$, $x^3+\frac{1}{x^3}+x+\frac{1}{x}=x^3+\frac{1}{x^3}+5=115$, $x^3+\frac{1}{x^3}=110$.

S04B23 $3780=2^2\times3^3\times5\times7$. One son's age is a multiple of 7 so he must be 14 years old. Another's age is a multiple of 5 so he must be 15 years old. The oldest is therefore **18** years old.

S04B24 Using Heron's formula, the area of the triangle is $\sqrt{10\times4\times5\times1}=\sqrt{200}=10\sqrt{2}$. Using the shortest side, $10\sqrt{2}=\frac{1}{2}\times5\times h$, $h=4\sqrt{2}$.

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SENIOR B DIVISION **CONTEST NUMBER FIVE** **Spring 2005**

SOLUTIONS

S04B25 Using the formula $A=\frac{S^2\sqrt{3}}{4}$, if a side is 10, the area is $25\sqrt{3}$ which is less than 50. If a side is 11, the area is $\frac{121\sqrt{3}}{4}$ which is greater than 50, so there are 10 triangles.

S04B26 If Ellen sits in the first seat, there are 5 places Tonya can sit. If she sits in the second, 4, etc. $5+4+3+2+1=15$ ways. In 5 of these ways, Tonya sits in the seat directly behind Ellen. $\frac{5}{15}=\frac{1}{3}$.

S04B27 The first 24 start with 1. The next 24 start with 3 and the next 24, 5. The 73^{rd}-78^{th} start with 71. The fifth number after that, the 83^{rd} number, is 73915.

S04B28 The area consists of two $30°$ sectors, plus a triangle with base $10\sqrt{3}$ and height 5. $\frac{1}{6}\times 100\pi+25\sqrt{3}=\frac{50\pi}{3}+25\sqrt{3}$ or $\frac{50\pi+75\sqrt{3}}{3}$.

S04B29 $b^2-4c\geq 0$

If $b=1$, no c is possible

$b=2$, $c=1$

$b=3$, $c=1,2$

$b=4$, $c=1,2,3,4$

$b=5$, $c=1,2,3,4,5,6$

$b=6$, $c=1,2,3,4,5,6$

19 possible equations.

S04B30 Using the sum formulas for $\sin(45^\circ+30^\circ)$ and $\cos(45^\circ+30^\circ)$,

$$\sin 75^\circ-\cos 75^\circ=\sin 30^\circ\cos 45^\circ+\sin 45^\circ\cos 30^\circ-\left(\cos 30^\circ\cos 45^\circ-\sin 30^\circ\sin 45^\circ\right)=\frac{\sqrt{2}}{2}$$

Thus $x=45^\circ$