New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER ONE** | **SPRING 2005**

PART I: 10 minutes | **NYCIML Contest One** | **Spring 2005**

S05J1. Compute the value of: $\dfrac{(1+15)\cdot(1+\frac{15}{2})\cdot(1+\frac{15}{3})\cdot\cdot\cdot(1+\frac{15}{20})}{(1+20)\cdot(1+\frac{20}{2})\cdot(1+\frac{20}{3})\cdot\cdot\cdot(1+\frac{20}{15})}$

S05J2. Compute the area of the circle inscribed in a right triangle with legs of length 5 and 12.

PART II: 10 minutes | **NYCIML Contest One** | **Spring 2005**

S05J3. A number *abcde* consists of all of the digits from 1 to 5 inclusive. *abc* is divisible by 4, *bcd* is divisible by 5, and *cde* is divisible by 3. Compute the number *abcde*.

S05J4. Kevin is twice as old as Jack was when he was as old as Jack is now. If Kevin is now 24 years old, compute Jack's age.

PART III: 10 minutes | **NYCIML Contest One** | **Spring 2005**

S05J5. The center of a sphere with radius 4 is 2 units away from a plane that intersects the sphere. Compute the area of the circle of intersection of the plane and the sphere.

S05J6. In trapezoid ABCD, M and N are midpoints of legs AD and BC respectively. Line MN intersects diagonal AC at P and diagonal BD at Q. If AB = 7 and CD = 13, Compute the length of PQ.

ANSWERS:

S05J1. **1**

S05J2. **4π**

S05J3. **12453**

S05J4. **18**

S05J5. **12π**

S05J6. **3**

New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER TWO** | **SPRING 2005**

PART I: 10 minutes | **NYCIML Contest Two** | **Spring 2005**

S05J7. Danny and Julie are driving from Shanghai to Shanglow. Julie takes two hours to drive the first 85 kilometers, and then Danny drives the remaining 100 kilometers. If they averaged 50 kilometers per hour for the entire trip, compute the number of **minutes** that Danny drove.

S05J8. A palindrome is a number that reads the same backwards and forwards, such as 23432. $0 < n < 100000$ and n is an integer. If n is randomly chosen and p/q is the probability in simplest form that n is a palindrome, compute p/q.

PART II: 10 minutes | **NYCIML Contest Two** | **Spring 2005**

S05J9. Compute x: $2^x = \frac{1}{4} \cdot \frac{2}{6} \cdot \frac{3}{8} \cdot \frac{4}{10} \cdot \ldots \cdot \frac{126}{254} \cdot \frac{127}{256}$.

S05J10. Poker chips come in three colors, red, white, and blue. Compute the number of different combinations there are of ten poker chips.

PART III: 10 minutes | **NYCIML Contest Two** | **Spring 2005**

S05J11. Three cards are placed in a hat and mixed up. One card is blue on both sides, one is yellow on both sides, and the third is blue on one side and yellow on the other side. One card is chosen without looking and then placed on a table without looking at the bottom side. If the side showing is yellow, compute the probability the other face is also yellow.

S05J12. Circles O and P have radii 16 and 9 respectively, and are tangent externally at point A. Point B is chosen on circle O such that AB = 8, and BC is drawn tangent to circle P at point C. Compute the length of BC.

ANSWERS:

S05J7. **102**

S05J8. $\mathbf{\frac{122}{11111}}$

S05J9. **-134**

S05J10. **66**

S05J11. $\mathbf{\frac{2}{3}}$

S05J12. **10**

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER THREE** **SPRING 2005**

PART I: 10 minutes **NYCIML Contest Three** **Spring 2005**

S05J13. The number x97y is divisible by 45. (x, 9, 7, and y are the digits of the number.) Find all possible ordered pairs (x, y) of digits.

S05J14. If $x = 1.69$, compute: $\dfrac{\sqrt{(x+2)^2-8x}}{\sqrt{x}-\dfrac{2}{\sqrt{x}}}$.

PART II: 10 minutes **NYCIML Contest Three** **Spring 2005**

S05J15. Compute the value of: $\dfrac{1}{2005}+\dfrac{2006\cdot 2004}{2005}-2006$.

S05J16. Circles O and P are tangent externally at point T. Chords $\overline{TA}$ and $\overline{TB}$ of circle O are extended through T to meet circle P again at points D and C respectively. If $TA = 3$, $TB = 5$, $TD = 7$, compute TC.

PART III: 10 minutes **NYCIML Contest Three** **Spring 2005**

S05J17. There are 100 students in Buck's County High School. The Physics class has 42 students, the Chemistry class has 35 students, and the Biology class has 30 students. 20 of the students in the school are in none of these classes, while 9 are in the Physics and Biology classes, 10 are in the Chemistry and Biology classes, and 11 are in the Physics and Chemistry classes. Compute the number of students who take all three of the science classes.

S05J18. In a rectangular coordinate system, there are two circles passing through the point (3, 2) and tangent to both coordinate axes. Find the sum of the radii of the two circles.

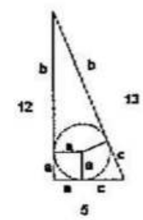
ANSWERS:

S05J13. **(2, 0), (6, 5) both required**
S05J14. **-1.3**
S05J15. **-1**
S05J16. **35/3**
S05J17. **3**
S05J18. **10**

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER ONE SOLUTIONS** **SPRING 2005**

S05J1. $=\dfrac{16\cdot\frac{17}{2}\cdot\frac{18}{3}\cdot\frac{19}{4}\cdots\frac{35}{20}}{21\cdot\frac{22}{2}\cdot\frac{23}{3}\cdot\frac{24}{4}\cdots\frac{35}{15}}=\dfrac{16\cdot17\cdot\ldots\cdot35}{20!}\cdot\dfrac{15!}{21\cdot22\cdot\ldots35}=\dfrac{35!}{35!}=\mathbf{1}$

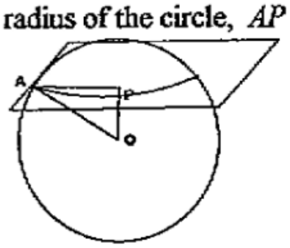


S05J2. Let the radius of the circle $= a$ and let b and c be the line segments shown in the diagram.
$a+b=12,\ a+c=5,\ b+c=13 \to a=2$. Thus the area is $\mathbf{4\pi}$.

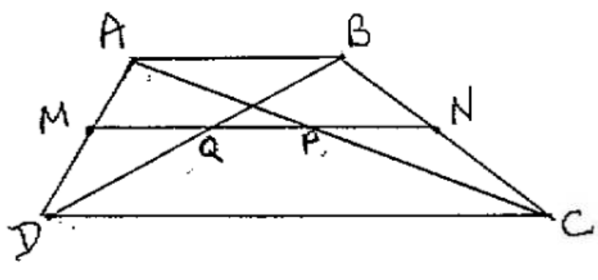
S05J3. Since *bcd* is divisible by 5, d must be 5. Since *abc* is divisible by 4, *bc* must be 12, 24, or 32. Therefore *bcd* must be 125, 245, or 325. Since *cde* is divisible by 3, *cde* must be 453. Therefore the number is **12453**.

S05J4. Jack's age now is x. Kevin is $24-x$ years older than Jack. Kevin's former age is also x. Thus we have $x=12+24-x \to x=\mathbf{18}$.

S05J5. In the diagram, OA = 4 and OP = 2. Use the Pythagorean theorem to get the radius of the circle, $AP=\sqrt{12}$. Thus the area of the circle is $\mathbf{12\pi}$.



S05J6. Since parallel lines cut off proportional segments on any transversal, Q and P are midpoints of diagonals BD and AC respectively. We now use the theorem that a line connecting the midpoints of two sides of a triangle is half the third side. In triangle ACD, MP = 13/2, while in triangle ABD, MQ = 7/2. Then PQ = MP – MQ = 13/2 – 7/2 =**3**.



SOLUTIONS

S05J7. In order to average 50 kph, they must drive the 185 kilometers in $\frac{185}{50}=3.7$ hours. Since Julie drove for two hours, Danny drove for 1.7 hours or **102** minutes.

S05J8. Organize the data into a chart:

Number of digits	**Number of Palindromic Numbers**
1	9 (any number 1 to 9)
2	9 (11,22,...99)
3	$9\cdot10 = 90$ (Form "*xyx*" 9 possibilities for *x*, 10 for y.)
4	$9\cdot10 = 90$ (Form "*xyyx*" 9 possibilities for *x*, 10 for y.)
5	$9\cdot10\cdot10 = 900$ (Form "*xyzyx*" 9 possibilities for *x*, 10 for y and 10 for *z*.)

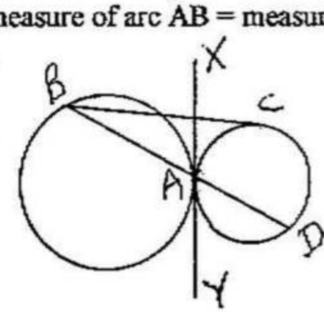
Adding the possibilities gives 1098. There are 99999 integers in the given domain. Thus the probability of a palindromic number is $\frac{1098}{99999}=\mathbf{\frac{122}{11111}}$.

S05J9. $2^x=\frac{127!}{2^{127}\cdot128!}=\frac{1}{2^{127}\cdot128}=\frac{1}{2^{127}\cdot2^7}=2^{-134}\to x=\mathbf{-134}$.

S05J10. An important distinction here is that all chips of a particular color are considered to be the same, and hence the solution is not simply 3^{10}. Instead think of the problem as trying to divide the integer 10 into the sum of 3 different non negative integers. This problem can be solved by looking at 10 as 1 + 1 + 1 + ... + 1, and thinking where to insert "dividers". To separate out 10 1's into 3 piles, we have to insert two dividers. Therefore the solution is $\binom{12}{2}$ or **66**.

S05J11. The card chosen is NOT blue on both sides. It is either yellow–yellow or yellow–blue. The bottom can be yellow from the yellow–yellow card, the "other" yellow from the yellow–yellow card, or yellow from the yellow–blue card. Thus the probability that the other face is also yellow is $\mathbf{\frac{2}{3}}$.

S05J12. Let AB intersect circle P at D, and let XY be the common tangent to the two circles (see diagram). Then $\angle XAB = \angle YAD$, so the measure of arc AB = measure of arc AD. Hence AD:AB = 9:16, and AD = 8(9/16) = 9/2. Then BC^2 = (BD)(AB) = 8(25/2) = 100, so BC = **10**.



New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER THREE SOLUTIONS** **SPRING 2005**

S05J13. Since x97y is a multiple of 5, y can only be 0 or 5. Since the number is a multiple of 9, the sum of its digits must be a multiple of 9. If y = 0, x must be 2, and if y = 5, x must be 6. So the answer is **(2, 0), (6, 5) both required**

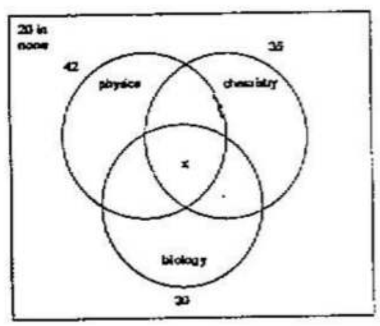
S05J14. The numerator of the given expression is equal to $\sqrt{x^2-4x+4}$, or $\sqrt{(x-2)^2}$, which is $|x-2|$. Multiplying numerator and denominator by $\sqrt{x}$, we see that the given fraction can be written as $\frac{|x-2|\sqrt{x}}{x-2}$ or $a\sqrt{x}$, where a is 1 if $x > 2$ and -1 if $x < 2$. Hence the required value is $-\sqrt{1.69} = \mathbf{-1.3}$.

S05J15.
$$\frac{1}{2005}+\frac{2006\cdot 2004}{2005}-2006=\frac{1+2006\cdot 2004-2006\cdot 2005}{2005}$$
$$=\frac{1+2006\cdot 2004-2006\cdot 2004-2006\cdot 1}{2005}=\frac{1-2006}{2005}=\mathbf{-1}$$

S05J16. If XY is the common tangent, then $\angle ATX = \angle YTD$, since $\widehat{AT} = 2\angle ATX$, $\widehat{TD} = 2\angle YTD$, $\widehat{AT} = \widehat{TD}$ and $\angle B = (1/2)\widehat{AT} = (1/2)\widehat{TD} = \angle C$. Also, $\angle ATB = \angle CTD$, so triangles ATB, DTC are similar. Thus $AT{:}TB = DT{:}TC$, so $3{:}5 = 7{:}x$, and $x = \mathbf{35/3}$.

S05J17.

$42+35+30-11-9-10+x=80 \rightarrow x=\mathbf{3}$.



S05J18. Let r be the radius of one of the circles. Then, since the center of such a circle is on the line y = x, we have $(x-r)^2+(y-r)^2=r^2$. Since (3, 2) is a point on the circle, the equation becomes $(3-r)^2+(2-r)^2=r^2$, or $r^2-10r+13=0$. We want the sum of the roots of this equation, which is **10**.