SOPH FROSH DIVISION **CONTEST NUMBER ONE** **SPRING 2004**
PART I: 10 minutes **NYCIML Contest One** **Spring 2004**

S04SF1. A grocer mixed x pounds of coffee worth 80 cents a pound with y pounds of coffee worth 95 cents a pound, and sold the mixture at 88 cents a pound. He received as much for the mixture as he would have if he had sold the two grades of coffee separately. Find the ratio x:y.

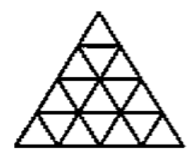
S04SF2. The lengths of the sides of a triangle are 5, 6, and x. If the area of the triangle is also x, compute the greatest possible value of x.

PART II: 10 minutes **NYCIML Contest One** **Spring 2004**

S04SF3. Two boxes of candy and three roses cost \$33. Four boxes of candy and seven roses cost \$71. Compute the cost of a box of candy and a dozen roses.

S04SF4. On the mythical island of Kora-Kora the only coins in circulation are 17-cent, 21-cent, and 28-cent. Assuming stores have all the coins they need to make change, what is the least number of coins you must have with you in order to be able to purchase a single 1-cent newspaper, and receive correct change?

PART III: 10 minutes **NYCIML Contest One**
Spring 2004



S04SF5. Compute the number of rhombuses in the diagram. (The figure is composed of 16 small equilateral triangles.)

S04SF6. AB% of CDE is 400, where AB and CDE represent integers (not products). Compute all possible two-digit integers AB.

ANSWERS:
S04SF1. **7:8 or 7/8**
S04SF2. $4\sqrt{2}+\sqrt{21}$
S04SF3. **\$69**
S04SF4. **2**
S04SF5. **21**
S04SF6. **50, 64, 80**

SOPH FROSH DIVISION **CONTEST NUMBER TWO** **SPRING 2004**

PART I: 10 minutes **NYCIML Contest Two** **Spring 2004**

S04SF7. Maria's average bowling score through Friday is 177. During the weekend, she bowls a 199, raising her average by 1 point. What would she have to bowl in the next game to raise her average by another point?

S04SF8. If 2^n divides 80!, compute the maximum integral value of n.

PART II: 10 minutes **NYCIML Contest Two** **Spring 2004**

S04SF9. At a given moment a set of three bells ring together. The first bell is rung every 55 seconds, the second every 60 seconds, and the third every 65 seconds. In how many seconds will the bells next ring together?

S04SF10. The diagonals of an isosceles trapezoid are perpendicular, and their point of intersection divides each into segments in the ratio 2:5. If the height of the trapezoid is 10, compute its area.

PART III: 10 minutes **NYCIML Contest Two** **Spring 2004**

S04SF11. Compute the units digit of 2004^{2004}.

S04SF12.

$$\begin{cases} ab+cd=5 \\ ac+bd=17 \\ ad+bc=25 \\ a^2+b^2+c^2+d^2=75 \end{cases}$$

Compute all possible values of $a+b+c+d$.

ANSWERS:

S04SF7. **201**

S04SF8. **78**

S04SF9. **8580**

S04SF10. **100**

S04SF11. **6**

S04SF12. **±13**

SOPH FROSH DIVISION **CONTEST NUMBER THREE** **SPRING 2004**

PART I: 10 minutes **NYCIML Contest Three** **Spring 2004**

S04SF13. Moe, Larry, and Curly own a pie bakery. Moe owns 60% of the bakery, and Larry owns twice as much as Curly. Express Curly's part of the bakery as a fraction in lowest terms.

S04SF14. A palindrome is a number that reads the same backwards and forward, such as 2662. Compute the number of positive integer palindromes less than 2004.

PART II: 10 minutes **NYCIML Contest Three** **Spring 2004**

S04SF15. In Marie Curie High School, each of the math, chemistry and physics clubs has 75 members. Any two clubs have 35 members in common, and 15 students are members in all three clubs. How many students of Marie Curie High School are members of at least one of the three clubs?

S04SF16. Find the four digit perfect square, of which the first two digits are identical and the last two digits are also identical.

PART III: 10 minutes **NYCIML Contest Three** **Spring 2004**

S04SF17. When a polynomial is divided by $2x+4$ the quotient is $x-3$ and the remainder is 17. Express the polynomial in simplest form.

S04SF18. An equilateral triangle has perimeter $18\sqrt{3}$. Point P is inside the triangle so that the distances from P to two of the sides are 5 and 3. Compute the distance from P to the third side.

ANSWERS:

S04SF13. $\frac{2}{15}$

S04SF14. **119**

S04SF15. **135**

S04SF16. **7744**

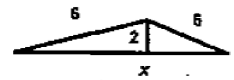
S04SF17. $2x^2-2x+5$

S04SF18. **1**

SOPH FROSH DIVISION **CONTEST NUMBER ONE SOLUTIONS** **SPRING 2004**

S04SF1. $80x + 95y = 88(x + y)$
$80x + 95y = 88x + 88y$
$7y = 8x$
$x{:}y = 7{:}8$

S04SF2. Let h be the altitude to the side of length x.



$x = \dfrac{(x)(h)}{2}$, so $h = 2$. By Pythagorean Theorem,
$x = \sqrt{36-4} + \sqrt{25-4} = 4\sqrt{2} + \sqrt{21}$

S04SF3. Let c be the cost of a box of candy, and r be the cost of a rose. $2c + 3r = 33$, $4c + 7r = 71$, so $c = 9$, $r = 5$, and a dozen roses and a box of candy cost **\$69**.

S04SF4. If you pay with **two (2)** 28-cent coins, you can receive two 17-cent and a 21-cent coin in change.

S04SF5. Let the side of the big triangle be 4. There are six rhombi with side 1 and one rhombus with side 2 in each of the three possible orientations. $7 \times 3 = \mathbf{21}$

S04SF6. $\dfrac{AB}{100}(CDE) = 400$, so $(AB)(CDE) = 40000 = 2^6 \times 5^4$. Since CDE has three digits, $40 < AB$. The only possible values of AB are **50, 64, and 80.**

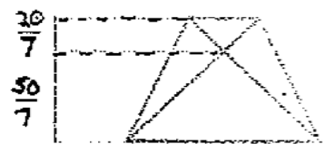
SOLUTIONS

S04SF7. Let n be the number of games she bowled through Friday. $\frac{177n+199}{n+1}=178$, so $n=21$. She must score p points, where $\frac{178(22)+p}{23}=179$, so $p=\mathbf{201}$.

S04SF8. 40 numbers contribute at least one factor of 2, 20 numbers contribute at least another, 10 more contribute at least a third, 5 more contribute a fourth, 2 contribute a fifth, and finally the number 64 contributes a sixth factor.
$40+20+10+5+2+1=\mathbf{78}$

S04SF9. The answer is the least common multiple of 55, 60 and 65.
$55=5(11)$, $60=2^2(3)(5)$, $65=5(13)$
Therefore the least common multiple is $2^2(3)(5)(11)(13)=\mathbf{8580}$.

S04SF10. By similar triangles, the heights of the two isosceles right triangles are $\frac{20}{7}$ and $\frac{50}{7}$, making the lengths of their hypotenuses (which are also bases of the trapezoid) $\frac{40}{7}$ and $\frac{100}{7}$, respectively. The area of the trapezoid is $\frac{1}{2}\left(\frac{40}{7}+\frac{100}{7}\right)(10)=\mathbf{100}$



S04SF11. $2004^{2004}=\left(2004^2\right)^{1002}$. Since 2004^2 has a units digit 6, any of its powers will have **6** as its units digit too.

S04SF12.

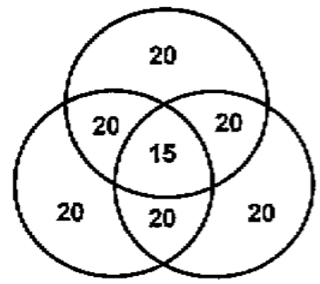
$$(a+b+c+d)^2=a^2+b^2+c^2+d^2+2ab+2cd+2ac+2bd+2ad+2bc$$
$$=75+2(5+17+25)=169\text{, so } a+b+c+d=\mathbf{\pm 13}.$$

SOLUTIONS

S04SF13. Curly owns $\frac{1}{3}$ of $\frac{4}{10}$ of the bakery. $\frac{4}{10}\left(\frac{1}{3}\right)=\mathbf{\frac{2}{15}}$

S04SF14. There are 9 1-digit and 9 2-digit palindromes, 90 3-digit palindromes, 10 4-digit palindromes that are less than 2000, and 2002 is also a palindrome. $9+9+90+10+1=\mathbf{119}$

S04SF15. Any two clubs share $35-15=20$ members that are not a member of the third club. This leaves each club $75-15-20-20=20$ members that do not belong to any other club. Among the clubs there are $6(20)+15=\mathbf{135}$ students total.



S04SF16. If the required number is N, then 11 must divide N. If a prime divides a perfect square, the square of the prime must also divide the perfect square, so $N = 121x^2$ for some integer x. Since $N < 10000 = 100^2$, x must be less than 10. Trying x = 9, 8, etc., we find that x = 8 is the only solution, and $N = 121(64) = \mathbf{7744}$.

S04SF17. $P(x)=(2x+4)(x-3)+17=\mathbf{2x^2-2x+5}$

S04SF18. Given ANY point inside an equilateral triangle, if the distances to the sides of the triangle are a, b, and c, $a+b+c=h$, where h is the height of the triangle. (see diagram) If the perimeter is $18\sqrt{3}$, the side is $6\sqrt{3}$, and the height is 9. The third distance is $9-5-3=\mathbf{1}$.