New York City
Interscholastic
Mathematics
League

JUNIOR DIVISION | **CONTEST NUMBER ONE** | **FALL 2003**

PART I: 10 minutes | **NYCIML Contest One** | **Fall 2003**

F03J1. Express in simplest form: $\dfrac{\sqrt{\sqrt{48}-\sqrt{3}}}{\sqrt{\sqrt{27}-\sqrt{12}}}$

F03J2. Two pumps are simultaneously emptying a pool. Working alone, the first pump can empty the pool in 4 hours, while, the second pump, working alone, can empty the pool in 6 hours. If it is raining at a constant rate that would fill an empty pool in 12 hours, compute the number of hours it will take for the two pumps to empty the (initially filled) pool?

PART II: 10 minutes | **NYCIML Contest One** | **Fall 2003**

F03J3. Among three consecutive positive integers, one is a multiple of 5, another is a multiple of 7, and the remaining one is a multiple of 9. Compute the smallest possible sum of these three integers.

F03J4. Compute the largest possible area of a triangle inscribed in a circle with area 3π.

PART III: 10 minutes | **NYCIML Contest One** | **Fall 2003**

F03J5. Compute the sum of the numerator and the denominator of the fraction obtained when $0.\overline{727272}$ is written in lowest terms.

F03J6. In ΔABC, $AB=5$, $AC=4$, and $BC=3$. Points D and E are chosen on $\overline{AC}$ so that $AD=CE=x$. If $BD:BE=3:2$, compute x.

ANSWERS:

F03J1. $\sqrt{3}$ F03J4. $\dfrac{9\sqrt{3}}{4}$

F03J2. 3 F03J5. 19

F03J3. 165 F03J6. $\dfrac{3\sqrt{39}-16}{5}$

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER TWO** **FALL 2003**

PART I: 10 minutes **NYCIML Contest Two** **Fall 2003**

F03J7. A fair six-sided die has its faces numbered 2, 3, 5, 7, 11, and 13. If two of these dice are rolled simultaneously, compute the number of different possible sums that can be rolled.

F03J8. Compute the number of ordered pairs (x, y) of positive integers that satisfy: $x^2 + 6x = y^2 + 4$.

PART II: 10 minutes **NYCIML Contest Two** **Fall 2003**

F03J9. If Kevin gets a 92 on his final exam, his average will be 105% of what it would be if he were to get a 70. Compute the sum of Kevin's test grades before the final exam.

F03J10. In ΔABC, $AB = 7$, $BC = 5$, and $CA = 8$. The bisector of angle B divides the bisector of angle A into two segments. Compute the ratio of the larger of these segments to the smaller.

PART III: 10 minutes **NYCIML Contest Two** **Fall 2003**

F03J11. The sum of the first n positive odd integers is greater than 2003. Compute the smallest possible value for n.

F03J12. A certain clock is running fast, and it is observed that the hour and minute hand meet exactly every 64 minutes. If the clock shows the correct time at noon, compute the number of minutes until it shows 3:00 p.m.

ANSWERS:
F03J7. 17
F03J8. 1
F03J9. 370
F03J10. 3 : 1
F03J11. 45
F03J12. 176

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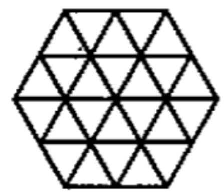
JUNIOR DIVISION | **CONTEST NUMBER THREE** | **FALL 2003**
PART I: 10 minutes | **NYCIML Contest Three** | **Fall 2003**

F03J13. Kevin recently noticed that he will be x years old in the year x^2. In what year was Kevin born?

F03J14. Solve, in simplest form, for all values of x: $\frac{5x^2-15x}{x^2+4}=x-3$

PART II: 10 minutes | **NYCIML Contest Three** | **Fall 2003**

F03J15. Compute the total number of triangles in the diagram at the right.



F03J16. Joe is on a "random walk" in Lattice Forest. He randomly picks one of the four directions north, south, east, and west, and walks one mile in that direction. He repeats this process two additional times. Compute the probability that after walking 3 miles, he is an integral number of miles from where he began.

PART III: 10 minutes | **NYCIML Contest Three** | **Fall 2003**

F03J17. Compute the number of distinct arrangements of the letters LEVEL.

F03J18. In ΔXYZ, $XY=10$ and $XZ=15$. Points A and B are chosen on sides $\overline{XY}$ and $\overline{XZ}$, respectively, so that $AX:BX=1:3$. If $\overline{AB}$ intersects the median from X to $\overline{YZ}$ at T, compute the ratio $AT:TB$.

ANSWERS:

F03J13. 1980
F03J14. 1, 3, and 4

F03J15. 38

F03J16. 5/8
F03J17. 30

F03J18. 1:2 or 1/2

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JUNIOR DIVISION **CONTEST NUMBER ONE** **FALL 2003**

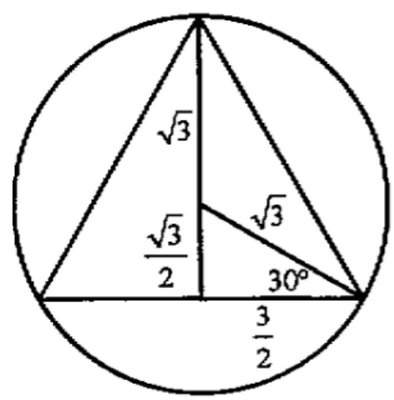
SOLUTIONS

F03J1. $\dfrac{\sqrt{\sqrt{48}-\sqrt{3}}}{\sqrt{\sqrt{27}-\sqrt{12}}}=\sqrt{\dfrac{4\sqrt{3}-\sqrt{3}}{3\sqrt{3}-2\sqrt{3}}}=\sqrt{\dfrac{3\sqrt{3}}{\sqrt{3}}}=\sqrt{3}$

F03J2. If the size of the pool is k units, the pumps empty $\frac{k}{4}$ and $\frac{k}{6}$ units per hour and the rain adds $\frac{k}{12}$ units per hour. Since the water is being taken out of the pool at a rate of $\frac{k}{4}+\frac{k}{6}-\frac{k}{12}=\frac{k}{3}$ units per hour, it will take **3** hours for the pool to empty.

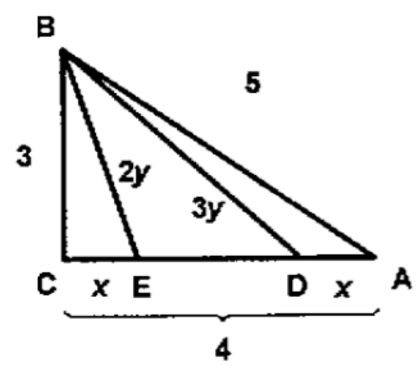
F03J3. It is probably most useful to look at the multiples of 9. List small multiples of 9 and look for nearby multiples of 5 and 7. The smallest multiple of 9 that satisfies the conditions of the problem is 54, and $54+55+56=\mathbf{165}$

F03J4. The radius of the circle is $\sqrt{3}$. The triangle with the largest area will be the equilateral triangle. By drawing the 30-60-90 triangle as in the diagram, we find the area of the triangle is $\dfrac{(3)\left(\dfrac{3\sqrt{3}}{2}\right)}{2}=\mathbf{\dfrac{9\sqrt{3}}{4}}$.



F03J5. $0.727272...=\frac{72}{99}=\frac{8}{11}$. The sum is **19**.

F03J6. Let BE and BD be $2y$ and $3y$, respectively. Since $x^2+3^2=(2y)^2$ and $(4-x)^2+3^2=(3y)^2$, we have $9(x^2+9)=4(x^2-8x+25)$, or $5x^2+32x-19=0$, so $x=\mathbf{\dfrac{3\sqrt{39}-16}{5}}$, the positive solution, is the answer.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
JUNIOR DIVISION **CONTEST NUMBER TWO** **FALL 2003**
SOLUTIONS

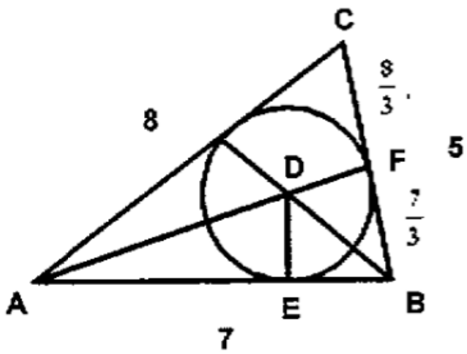
F03J7. There are 5 odd totals possible, each corresponding to a 2 and an odd number being rolled. Every even number from 4 to 26 can be a sum, for 12 more totals. Altogether there are **17** totals possible.

F03J8. $x^2+6x=y^2+4 \to x^2+6x+9-y^2=13 \to (x+3)^2-y^2=13$

Factoring a difference of two squares, we obtain $(x+3+y)(x+3-y)=13$. Since x and y are positive and 13 is prime, we must have $x+3+y=13$ and $x+3-y=1$. This yields $x=4$, $y=6$, for only **1** solution.

F03J9. Let S be the sum of Kevin's grades so far and let n be the number of tests he took. We have $\frac{S+92}{n+1}=(1.05)\frac{S+70}{n+1}$, or $100S+9200=105S+7350$, yielding $S=\mathbf{370}$.

F03J10. Let D be the incenter of the triangle. Let E be the point of tangency of the circle on side $\overline{AB}$. Let F be the intersection of the bisector of angle A and the opposite side.



$DE=r=\frac{A}{s}=\frac{\sqrt{10(5)(3)(2)}}{10}=\sqrt{3}$

$AE=\frac{AB+AC-BC}{2}=5$

$AD=\sqrt{\sqrt{3}^2+5^2}=2\sqrt{7}$.

The length of the entire bisector of angle A can be found from the bisector formulas. $BF=BC\frac{AB}{AB+AC}=\frac{7}{3}$, $CF=BC\frac{AC}{AB+AC}=\frac{8}{3}$.

$AF=\sqrt{(AB)(AC)-(BF)(CF)}=\sqrt{56-\frac{56}{9}}=\frac{8\sqrt{7}}{3}$, so $DF=\frac{8\sqrt{7}}{3}-2\sqrt{7}=\frac{2\sqrt{7}}{3}$.

The ratio $AD:DF=\mathbf{3:1}$.

F03J11. The sum of the first n odd positive integers equals n^2, so we must find the smallest n such that $n^2>2003$. $44^2=1936$, $45^2=2025$, so $n=\mathbf{45}$.

F03J12. A regular clock's hands pass each other 11 times in a 12 hour span, or once every $\frac{12}{11}$ hours. The fast clock's hand pass each other every $\frac{16}{15}$ hours. Thus the clock shows $\frac{12}{11}$ hours after only $\frac{16}{15}$ hours have passed. The clock will show 3 hours after $\frac{16}{15}\div\frac{12}{11}\times 3=\frac{44}{15}$ hours, or after **176** minutes.

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JUNIOR DIVISION **CONTEST NUMBER THREE** **FALL 2003**

SOLUTIONS

F03J13. The two closest squares to 2003 are $45^2 = 2025$ and $46^2 = 2116$. Only in the first case would Kevin be alive today. Kevin was born in $2025 - 45 = \mathbf{1980}$.

F03J14. Either $x = \mathbf{3}$ or we can divide both sides by $x-3$ obtaining $\frac{5x}{x^2+4} = 1$ This simplifies to $x^2 - 5x + 4 = 0$, or $(x-4)(x-1) = 0$ yielding additional roots **1** and **4.**

F03J15. Call the side of a small triangle *a*. We have, counting both orientations, 24 triangles with side *a*, 12 triangles of side 2*a*, and 2 triangles of side 3*a*, for a total of **38**.

F03J16. The only way Joe can be a non-integer distance away is if he walks one mile in one direction and 2 miles in a perpendicular direction, in some order. We can pick the first direction in 4 ways, the second in 2, and the order of the miles in 3, so there are 24 walks that leave Joe at a non-integer distance. Since there are $4^3 = 64$ walks total, 40 walks leave him at an integer distance, for a probability $\frac{40}{64} = \mathbf{\frac{5}{8}}$.

F03J17. Permutations with repetitions yields $\frac{5!}{(2!)(2!)(1!)} = \mathbf{30}$.

F03J18. Draw the median $\overline{ZN}$, and let C be the intersection of the two medians. Note that $XZ : XN = 15 : 5 = XB : XA$. This means that triangles XAB and XNZ are similar, and so $\overline{AB} \parallel \overline{NZ}$. Thus $AT : TB = NC : CZ = \mathbf{1:2}$, because in any triangle if the three medians are drawn, they will intersect at a point that is one of the trisection points of each of the medians.