NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

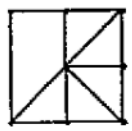
Sophomore-Freshman Division

CONTEST NUMBER 1

PART I ***FALL, 2002*** ***CONTEST 1*** ***TIME: 10 MINUTES***

F02SF1 Kevin has an average of 88 on his first 5 exams. If he wants a 90 average for the whole term (6 exams), compute the score he needs on his last exam?

F02SF2 Find the number of triangles that exist in the following figure.



PART II ***FALL, 2002*** ***CONTEST 1*** ***TIME: 10 MINUTES***

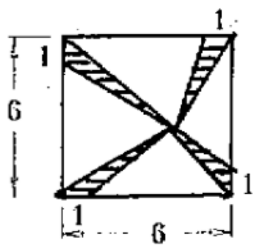
F02SF3 If $x+y=7$ and $xy=4$, Compute x^2+y^2?

F02SF4 Compute all values of x such that $\frac{4-10x+x^2}{4-10x-x^2}=\frac{3}{5}$

PART III ***FALL, 2002*** ***CONTEST 1*** ***TIME: 10 MINUTES***

F02SF5 How many different 4-digit positive even integers can be formed using the digits 1, 2, 3, 4 and 5, without repetition?

F02SF6 Find the ratio of the area of the shaded region to the area of the square.



ANSWERS:

F02SF1	100
F02SF2	9
F02SF3	41
F02SF4	2 and $\frac{1}{2}$
F02SF5	48
F02SF6	1:6

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Sophomore-Freshman Division

CONTEST NUMBER 2

PART I	***FALL, 2002***	***CONTEST 2***	***TIME: 10 MINUTES***

F02SF7 A number is 2003 more than 2003 times itself. Compute that number.

F02SF8 Compute the set of values of x that satisfies: $\dfrac{x+10}{x+8}-\dfrac{x+12}{x+10}=\dfrac{x+14}{x+12}-\dfrac{x+16}{x+14}$

PART II	***FALL, 2002***	***CONTEST 2***	***TIME: 10 MINUTES***

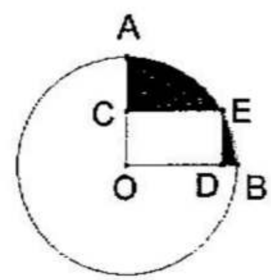
F02SF9 Aneley was hired by Springfield Elementary to paint room numbers in the new academic building. Each digit had to be painted separately, and she was paid 10 cents per digit. She painted room numbers 1 to 225. How much did she earn?

F02SF10 If $\left(a^2+c^2\right)^2=b\cdot d\cdot(b+d)^2=1$ and $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{a+c}{b+d}=0$. Find b^2-d^2.

PART III	***FALL, 2002***	***CONTEST 2***	***TIME: 10 MINUTES***

F02SF11 Two fair dice are rolled one at a time, compute the probability that the first roll is greater than the second?

F02SF12 Circle O has a radius of 4. $OCED$ is a rectangle. Compute the minimum area of the shaded region.



ANSWERS:

F02SF7	$-\dfrac{2003}{2002}$
F02SF8	-11
F02SF9	\$56.70
F02SF10	0
F02SF11	5/12
F02SF12	$4\pi-8$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Sophomore-Freshman Division
CONTEST NUMBER 3

PART I	*FALL, 2002*	*CONTEST 3*	*TIME: 10 MINUTES*

F02SF13 One positive integer is 6 more than another. The difference between their squares is 132. Compute the smaller number.

F02SF14 A barn is a 40 ft. by 40 ft. square. A goat is tied to one corner of the barn by a rope 50 ft. long. Over how many square ft. can the goat graze if the goat can only graze outside the barn?

PART II	*FALL, 2002*	*CONTEST 3*	*TIME: 10 MINUTES*

F02SF15 Compute the number of positive integers under 100 that have an odd number of factors.

F02SF16 The digits N, V, K, J, C, L, M, Y are all distinct. Evaluate V.

```
    N1V3K
     J4CK
+    2003
   ------
   LM1CYN
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PART III	*FALL, 2002*	*CONTEST 3*	*TIME: 10 MINUTES*

F02SF17 Compute the smallest two-digit integer that is the square of the sum of its digits.

F02SF18 The total area of the six faces of a rectangular box is 100 cm^2, and the total length of all its edges is 52 cm. Compute the length, in cm, of its internal diagonal.

ANSWERS:

F02SF13	8
F02SF14	1925π
F02SF15	9
F02SF16	8
F02SF17	81
F02SF18	$\sqrt{69}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division — CONTEST NUMBER 1

F02SF1 He needs a total of 540 points for a 90 average in 6 exams. On the first 5 exams, he has garnered $88 \cdot 5$ or 440 points. Therefore Kevin must achieve a 100 on his last exam!

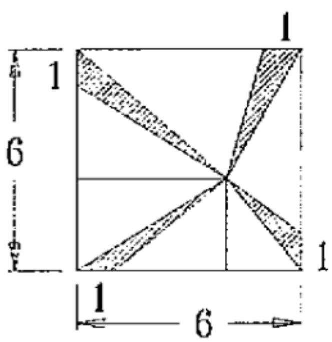
F02SF2 There are 5 of the "small" triangles, 2 of the "medium" triangles, and 2 of the "jumbo" triangles.

F02SF3 $(x+y)^2 = x^2 + 2xy + y^2$. Since this differs from what we are trying to compute by $2xy$, we subtract the $2xy$ from the expression. Hence, $7^2 - 2(4) = 49 - 8 = 41$.

F02SF4 Simplifying, we find that $20 - 50x + 5x^2 = 12 - 30x - 3x^2$, or $2x^2 - 5x + 2 = (2x-1)(x-2) = 0$, so $x = 2$ or $x = \frac{1}{2}$.

F02SF5 For the 4 – digit number to be even, the last digit must be either a 2 or a 4. Since there are no restrictions on the other digits, we see that there are 2 ways to pick the last digit, and $4 \cdot 3 \cdot 2$ ways to determine the other 3 digits. Therefore, there is a total of 48 numbers that could be formed.

F02SF6 We can draw altitudes to all the sides. We can quickly see that for each shaded area, there is an un-shaded area that is 5 times the area. Hence, the ratio is 1 : 6.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division

CONTEST NUMBER 2

F02SF7 Let's call the number x. $x-2003=2003x \Rightarrow 2002x=-2003 \Rightarrow x=-\dfrac{2003}{2002}$

F02SF8 For this type of question, it is efficient to do a change of variable. In this case, we will let $y=x+12$. $\dfrac{x+10}{x+8}-\dfrac{x+12}{x+10}=\dfrac{x+14}{x+12}-\dfrac{x+16}{x+14} \Rightarrow \dfrac{y-2}{y-4}-\dfrac{y}{y-2}=\dfrac{y+2}{y}-\dfrac{y+4}{y+2}$.

Combining fractions, we get

$$\frac{(y-2)^2-y(y-4)}{(y-4)(y-2)}=\frac{(y+2)^2-y(y+4)}{y(y+2)} \Rightarrow \frac{4}{(y-4)(y-2)}=\frac{4}{y(y+2)} \Rightarrow$$

$y^2+2y=y^2-6y+8 \Rightarrow 8y=8 \Rightarrow y=1$, and hence $x=-11$.

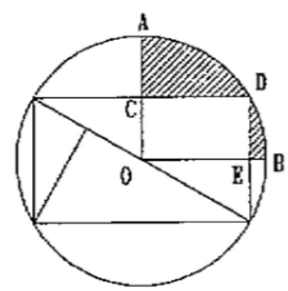
F02SF9 We see that there are 9 numbers that have 1 digit (1- 9), 90 numbers that have 2 digits (10 – 99), and 126 numbers that have 3 digits (100 – 225). $9\cdot1+90\cdot2+126\cdot3=567$ Therefore she earned \$56.70.

F02SF10 $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{a+c}{b+d}=0 \Rightarrow ad+bc=a+c=0$

$a=-c \quad and \quad b=d \quad \Rightarrow b^2-d^2=0$

F02SF11 Each roll has 6 possible outcomes; hence there are a total of 6 * 6 or 36 possible outcomes in the two rolls. We know that there are 6 ways for the two rolls to be exactly the same. If the two rolls are not the same, then one of the dice must have a higher number than the other. Since this property is symmetric (probability of roll 1 greater than 2 is equal to probability of roll 2 greater than one). There are (36 – 6)/2 possible ways to have roll 1 greater than roll 2, and hence the probability is 15/36 = 5/12.

F02SF12 First, we can see that to minimize the area of the shaded area is the same as maximizing the area of the rectangle. We can do this by extending the rectangle into all 4 quadrants of the circle as shown in the diagram. Now we can see that by maximizing this much bigger rectangle, we'll end up maximizing our original rectangle as well. The area of this much bigger triangle is the diameter times the height over two. Since the diameter is fixed, we have to make the height as big as possible, which is when it bisects the diameter. Hence we know that the big rectangle as well as the small rectangle must be squares. Therefore, the area of the shaded region is $\dfrac{\pi\cdot r^2}{4}-\dfrac{r^2}{2}=\dfrac{\pi-2}{4}\cdot 4^2=4\pi-8$



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Sophomore-Freshman Division

CONTEST NUMBER 3

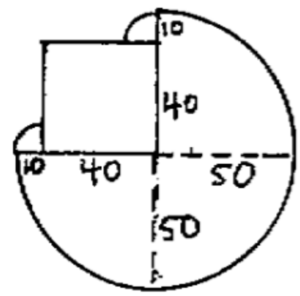
F02SF13 Let's call the bigger number x, hence the smaller number is x-6.

$x^2-(x-6)^2=x^2-(x^2-12x+36)=>12x-36=132$

$12x=168$ *so*, $x=14$

Hence, the smaller number is $14-6=8$.

F02SF14 $\frac{3}{4}\cdot 50^2\pi+\frac{2}{4}\cdot 10^2\pi=\mathbf{1925\pi}$



F02SF15 Integers usually have factors that are in pairs. $8= 2\cdot 4$. The only way an integer can have an odd number of factors is if it's a square of a number. There are 9 squares under 100. Hence the answer is 9.

F02SF16 We know that N must be 9, L must be 1 and M must be 0 very quickly due to the fact the answer is a 6 digit number. Also $1+J+2\geq 10$. Now we can use the fact that K must be either 3 or 8 and the fact that C appears more than once and try some numbers. V turns out to be 8.

F02SF17 Checking each two-digit perfect square, we find that **81** is the only one that works.

F02SF18 Let the edges be of lengths x, y, z, and let the internal diagonal have length d. Then we have $2xy+2yz+2xz=100$, $4x+4y+4z=52$, $x^2+y^2+z^2=d^2$. Using the second of these, we get $(x+y+z)^2=x^2+y^2+z^2+2xy+2xz+2yz=169$. Substituting the first and third above into the new equation, we get $d^2+100=169$, and $\boldsymbol{d}=\sqrt{69}$.