NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division

CONTEST NUMBER 1

PART I *FALL, 2002* *CONTEST 1* *TIME: 10 MINUTES*

F02S1 A farmer went to a state fair. On Monday he bought 12 sheep and 4 goats for \$60. On Tuesday he bought some sheep for \$24 and on Wednesday he bought some goats for \$30. He bought 3 more sheep on Tuesday than goats on Wednesday. The cost of a sheep and a goat was the same on all three days. If $\$s$ and $\$g$ are the prices of a sheep and a goat, respectively, compute the ordered pair (s, g).

F02S2 Quadrilateral $TRIO$ is inscribed in a circle and has $m\angle O = 120°$, $TO = IO$, $TR + IR = 7$, and $\frac{TR}{IR} = 6$. Compute the area of quadrilateral $TRIO$.

PART II *FALL, 2002* *CONTEST 1* *TIME: 10 MINUTES*

F02S3 If Kevin, Jack, & Lawrence work for the same amount of time, Kevin will get 50% more work done than Jack and 25% more work done than Lawrence. If Kevin can do a job in 37 days, compute the number of days it will take to finish the job if the three of them work together.

F02S4 Compute: $\sin 10° \sin 30° \sin 45° \sin 50° \sin 60° \sin 70°$.

PART III *FALL, 2002* *CONTEST 1* *TIME: 10 MINUTES*

F02S5 A dealer sold 100 books for \$100, selling some for \$2, some for \$1 and the rest for 50 cents apiece. Compute the greatest number of two-dollar books he could have sold.

F02S6 Compute the number of strings of length eight, consisting only of A's and/or B's (for instance: BAAABBBB, AAAAAAAA, ...), that contain at least three consecutive A's or at least four consecutive B's.

ANSWERS:

F02S1 (3,6)

F02S2 $\frac{49\sqrt{3}}{12}$

F02S3 15

F02S4 $\frac{\sqrt{6}}{64}$

F02S5 33

F02S6 147

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division

CONTEST NUMBER 2

PART I	*FALL, 2002*	*CONTEST 2*	*TIME: 10 MINUTES*

F02S7 Five distinct integers are selected from $\{1,2,3,...,10\}$. Compute the probability that the median of the 5 selected integers is 7.

F02S8 Compute the least positive integer k such that $\frac{k-11}{7k+6}$ is a non-zero fraction *not* in simplest form.

PART II	*FALL, 2002*	*CONTEST 2*	*TIME: 10 MINUTES*

F02S9 How many four letter arrangements, with no repetition of letters, have K as its second letter and the arrangement is in alphabetical order from left to right. (AKMS is an example)

F02S10 If $\tan A_1 \tan A_2 \cdots \tan A_{10} = 1$, compute the maximum value of $\sin A_1 \sin A_2 \cdots \sin A_{10}$.

PART III	*FALL, 2002*	*CONTEST 2*	*TIME: 10 MINUTES*

F02S11 Compute the number of subsets of a set with ten distinct elements that have three or more elements.

F02S12 Compute the total number of solutions, in nonnegative integers, to the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 21$, given that $0 \le x_1 \le 3$, $1 \le x_2 \le 3$, and $x_3 \ge 15$.

ANSWERS:		
	F02S7	$\frac{5}{28}$
	F02S8	94
	F02S9	1050
	F02S10	$\frac{1}{32}$
	F02S11	968
	F02S12	106

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division

CONTEST NUMBER 3

PART I ***FALL, 2002*** ***CONTEST 3*** ***TIME: 10 MINUTES***

F02S13 Compute the smallest positive integer with exactly 12 divisors.

F02S14 A rectangle has integral sides, m and n. The interior of the rectangle is divided into unit squares by lines parallel to the sides of the rectangle. The number of unit squares that have at least one side which is part of the rectangle is equal to the number of unit squares that have no sides as part of the rectangle. Find all possible values of mn.

PART II ***FALL, 2002*** ***CONTEST 3*** ***TIME: 10 MINUTES***

F02S15 If $\log(x-2y)-\log x=\log y-\log(x-2y)$, compute $\frac{y}{x}$.

F02S16 A right circular cone has a slant height of 3 and a base of radius 1. The sphere containing the height of the cone as its diameter intersects the surface of the cone in a circle whose radius is x. Compute x.

PART III ***FALL, 2002*** ***CONTEST 3*** ***TIME: 10 MINUTES***

F02S17 Lawrence and Andrei roll a pair of unfair dice. The probability of rolling a 4 on Lawrence's die is 3/8, and the probability of rolling a 1, 2, 3, 5, or 6 on Lawrence's die is 1/8 each. The probability of rolling a 3 on Andrei's die is 1/2, and the probability of rolling a 1, 2, 4, 5, or 6 on Andrei's die is 1/10 each. What is the probability of 7 appearing as the sum of the two numbers when the dice are rolled?

F02S18 Let $f(x)=x^6+x^5+x^4+x^3+x^2+x+1$. Compute the remainder when $f(x^7)$ is divided by $f(x)$.

ANSWERS:

F02S13	60
F02S14	48, 60
F02S15	1/4
F02S16	8/9
F02S17	1/4
F02S18	7

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division

CONTEST NUMBER 4

Part I ***Fall, 2002*** ***Contest 4*** ***Time: 10 Minutes***

F02S19 Aneley is thinking of an integer such that 3 times that number minus 14, is greater than twice the number plus 33; and twice the number plus 26 is greater than 4 times the number minus 72. Compute Aneley's number.

F02S20 Compute all positive integers $x < 100$ such that $x^2 + x - 16$ is a multiple of 101.

PART II ***FALL, 2002*** ***CONTEST 4*** ***TIME: 10 MINUTES***

F02S21 Two circles have the same center. The radius of the larger circle is $\frac{\sqrt{10\pi}}{\pi}$. The smaller circle has half the area of the larger. Compute the area of an equilateral triangle inscribed in the smaller circle.

F02S22 Quadrilateral ABCD is inscribed in circle O.

If AB = 60, BC = 39, CD = 25, and DA =52, compute the radius of circle O:

PART III ***FALL, 2002*** ***CONTEST 4*** ***TIME: 10 MINUTES***

F02S23 The area of an acute triangle is 84 and two of the sides have lengths 13 and 14. Compute the length of the third side.

F02S24 Compute all values of x, which satisfy: $3^{2x+1} - x3^{x+1} - 3^x - 6x^2 - 7x - 2 = 0$.

ANSWERS:

F02S19	48
F02S20	20, 80
F02S21	$\frac{15\sqrt{3}}{4\pi}$
F02S22	65/2 or $32\frac{1}{2}$ or 32.5
F02S23	15
F02S24	0, -1, 1

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division

CONTEST NUMBER 5

PART I	*FALL, 2002*	*CONTEST 5*	*TIME: 10 MINUTES*

F02S25 The sum of the ages of a man and his wife are 6 times the sum of the ages of their children. Two years ago, the sum of their ages were 10 times the sum of the ages of their children at that time. Six years from now, the sum of their ages will be 3 times the sum of the ages of their children at that time. How many children do they have?

F02S26 Compute: $\dfrac{1}{\sin 10°} - \dfrac{\sqrt{3}}{\cos 10°}$.

PART II	*FALL, 2002*	*CONTEST 5*	*TIME: 10 MINUTES*

F02S27 Compute all positive integers n for which $5^n + 5^5 - 5^4$ is a perfect square.

F02S28 Compute all values of n such that: $\sqrt[n]{17\sqrt{5}+38} + \sqrt[n]{17\sqrt{5}-38} = \sqrt{20}$.

PART III	*FALL, 2002*	*CONTEST 5*	*TIME: 10 MINUTES*

F02S29 Compute the number of ways \$1.00 can be paid with nickels and dimes with the condition that the total number of coins is a prime number.

F02S30 The numbers $\tan 1°$, $\tan 3°$, $\tan 5°$, ..., $\tan 89°$ are written on a board. At each step, any two numbers, A and B, are erased and replaced with the number $\dfrac{1+A+B-AB}{1-A-B-AB}$. At the end, there will be only one number left. Compute the number.

ANSWERS:

F02S25	3
F02S26	4
F02S27	5
F02S28	3, -3
F02S29	4
F02S30	1

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior A Division Solutions   Contest 1

F02S1 $12s + 4g = 60$, $\frac{24}{s} = \frac{30}{g} + 3$, or $24g = 30s + 3sg$, or $32g = 40s + 4sg$

$8(60 - 12s) = 40s + s(60 - 12s)$, $12s^2 - 196s + 480 = 0$, so $(3s-40)(s-3) = 0$

Notice that if $s = 40/3$, g would be negative, so the ordered pair is **(3,6)**

F02S2 $TR = 6$, $IR = 1$. The area of $\Delta TRI = \frac{1}{2}(6)(1)\sin 60° = \frac{3\sqrt{3}}{2}$. By Law of Cosines,

$TI^2 = 6^2 + 1^2 - 2(6)(1)\cos 60° = 31$. Since $TI = \sqrt{31}$ is the measure of the base of the isosceles ΔTIO with a vertex angle of measure $120°$, the height of ΔTIO is $\frac{\sqrt{31}}{2}\tan 30° = \frac{\sqrt{93}}{6}$, so the area of

$\Delta TIO = \frac{1}{2}\sqrt{31}\frac{\sqrt{93}}{6} = \frac{31\sqrt{3}}{12}$. Area of quadrilateral $TRIO = \frac{3\sqrt{3}}{2} + \frac{31\sqrt{3}}{12} = \mathbf{\frac{49\sqrt{3}}{12}}$.

F02S3 Let K, J, and L be, respectively, the rates at which Kevin, Jack and Lawrence work. Let the total amount of work be W.

$K = \frac{3J}{2} = \frac{5L}{4}$, $\frac{W}{K} = 37$, $J + K + L = \frac{2K}{3} + K + \frac{4K}{5} = \frac{37K}{15}$, $\frac{W}{J+K+L} = 37 \cdot \frac{15}{37} = 15$

F02S4 The values of $\sin 30°$, $\sin 45°$, and $\sin 60°$ are well known and so, with this in mind, that part of the product is effectively calculated. So the problem becomes to calculate

$E = \sin 10° \sin 50° \sin 70° = \sin 10° \cos 20° \cos 40°$. Keeping in mind the double angle formula for $\sin 2x = 2\sin x \cos x$, we introduce a "catalyst" into the product:

$E\cos 10° = \cos 10° \sin 10° \cos 20° \cos 40° = \frac{1}{2}\sin 20° \cos 20° \cos 40°$

$= \frac{1}{4}\sin 40° \cos 40° = \frac{1}{8}\sin 80° = \frac{1}{8}\cos 10°$. Since $\cos 10°$ is obviously not equal to 0, we have $E = \frac{1}{8}$.

Finally, our entire answer is $\frac{1}{2} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \frac{1}{8} = \mathbf{\frac{\sqrt{6}}{64}}$.

F02S5 Let T, O, and H be the numbers of \$2, \$1 and \$0.50 books, respectively. Then $T+H+O =$ 100 and $2T + H + .5O = 100$. $T - .5H = 0$, $T+H \le 100$, $H = 2T$, and the maximum T is **33**.

F02S6 Let us first count the cases where four consecutive B's occur. We will split this into cases depending on where the four consecutive B's start. If they start in position one, we have $2^4 = 16$ possibilities. If they start in position two, three, four, or five, we have $2^3 = 8$ possibilities each, for a total of 48 possibilities of having four consecutive B's. Now we will count the cases where there are three consecutive A's, again splitting it into cases depending on where the three A's in a row first occur. If they start in position one, there are $2^5 = 32$ possibilities. If they start in position two, three, or four, there are $2^4 = 16$ possibilities. If they start in position five, there are 14 possibilities (since you cannot have a AAA as the first 3, since that case already got counted in the beginning). And if they start in the 6th position, there are 13 cases, since the first 4 positions cannot contain 3 consecutive A's, that is, it cannot be any of AAAB, AAAA, or BAAA. Therefore, there are 107 cases with three consecutive A's. Now from these $107 + 48 = 155$ cases, we must subtract those cases which contain both four consecutive B's and three consecutive A's. There are 8 such cases, as can be easily verified. Hence, the answer is $155 - 8 = \mathbf{147}$.

New York City Interscholastic Mathematics League

Senior A Division Solutions Contest 2

F02S7 There is a total of ${}_{10}C_5$ ways of picking 5 numbers out of 10. There are a total of ${}_6C_2$ ways of choosing 2 numbers from the first 6, ${}_3C_2$ ways of choosing 2 numbers from the last 3, and only 1 way of choosing the 7. So the probability that 7 is the median of the 5 numbers is $\frac{{}_6C_2 \cdot 1 \cdot {}_3C_2}{{}_{10}C_5} = \mathbf{\frac{5}{28}}$

F02S8 $\frac{k-11}{7k+6}$ is not in simplest form iff its reciprocal is not in simplest form, that is iff $\frac{7k+6}{k-11} = \frac{7(k-11)+83}{k-11}$ is reducible. This occurs iff $\frac{83}{k-11}$ is reducible. Since 83 is prime, $k-11$ must be a multiple of 83, and cannot be 0. Hence $k-11 = 83$, and $\mathbf{k = 94}$.

F02S9 K is the 11th letter of the alphabet. Hence there are 10 choices for the first letter. We need 2 letters which come after k and we know what order they will be put in so there are $\binom{26-11}{2} = \binom{15}{2} = 105$ ways. Therefore the answer is (10)(105) = 1050.

F02S10 From $\tan A_1 \tan A_2 \cdots \tan A_{10} = 1$, we get that

$$\sin A_1 \sin A_2 \cdots \sin A_{10} = \cos A_1 \cos A_2 \cdots \cos A_{10} = \sqrt{\sin A_1 \cos A_1 \cdots \sin A_{10} \cos A_{10}}$$

$= \sqrt{2^{-10} \sin 2A_1 \sin 2A_2 \cdots \sin 2A_{10}} \le \sqrt{2^{-10}} = 2^{-5} = \mathbf{\frac{1}{32}}$, with equality if and only if all the $A_i = \frac{\pi}{4}$.

F02S11 $2^{10} - \binom{10}{0} - \binom{10}{1} - \binom{10}{2} = 1024 - 1 - 10 - 45 = \mathbf{968}$

F02S12 We can "give" x_3 the 15 it MUST have, and give x_2 the 1 it MUST have. Then the problem reduces to $x_1 + x_2 + x_3 + x_4 + x_5 = 5$, with the restriction that $x_1 \le 3$, and $x_2 \le 2$. If there were no restriction, there would be ${}_9C_4 = 126$. Amongst these, we must subtract those that violate $x_1 \le 3$, which means those that have $x_1 \ge 4$, and there are ${}_5C_1 = 5$ of those. We still have to subtract those that violate $x_2 \le 2$, which means those that have $x_2 \ge 3$, and there are ${}_6C_2 = 15$ of those. But now, we have to add back those that violate both $x_1 \le 3$ and $x_2 \le 2$, which means both of the following are true: $x_1 \ge 4$ and $x_2 \ge 3$, however, that clearly can't be. Thus the answer is $126 - 5 - 15 + 0 = \mathbf{106}$

New York City Interscholastic Mathematics League

Senior A Division Solutions — Contest 3

F02S13 If we have a positive integer $N = p_1^{e_1} p_2^{e_2} \cdots$, where each p_i divides N, then the number of divisors of N is $(e_1 + 1)(e_2 + 1)(e_3 + 1)\ldots$. $12 = (2^2)(3)$ and the smallest N that works is $(2^2)(3)(5) =$ **60**.

F02S14 $\frac{mn}{2} = 2m + 2n - 4$, and solving for m we get $m = 4 + \frac{8}{n-4}$. For m to be an integer, so must $\frac{8}{n-4}$. So n can take on the values 2, 3, 5, 6, 8, and 12, and the corresponding m values are 0, -4, 12, 8, 6, and 5. Since m and n must be positive, we have only 2 cases. Either one of m or n is 6, and the other is 8, or one of m or n is 5 and the other is 12. Thus there are two values of mn, namely **48 and 60.**

F02S15 First, notice that the equation makes sense iff $x, y > 0$. Using the properties of logarithms we obtain $(x-2y)^2 = xy \Leftrightarrow x^2 - 5xy + 4y^2 = 0$. Dividing by x^2 we obtain a quadratic equation in y/x that, produces 1/4 or 1 as possible candidates for y/x. Checking, we find **1/4** is the only valid answer.

F02S16 The height of the cone is $2\sqrt{2}$. Let V be the vertex of the cone, H be the foot of the altitude from the vertex, and O be the center of the circle. Let x be the radius of the circle and let $OH = y$, so $VO = 2\sqrt{2} - y$. We know that in a cross-section of the cone the tangent of the angle between the slant height and the base is $2\sqrt{2}$. By similar triangles from the cross section we have $\frac{2\sqrt{2} - y}{x} = 2\sqrt{2}$. Also, by examining the greater circle of the cross section, we have, by the power of point theorem: $x^2 = (2\sqrt{2} - y)y$. Thus $y = \frac{x}{2\sqrt{2}}$, and $x^2 = \left(2\sqrt{2} - \frac{x}{2\sqrt{2}}\right)\frac{x}{2\sqrt{2}}$. $x = \frac{8-x}{8}$ so $x = \frac{8}{9}$.

F02S17 Let (x, y) represent an x on Lawrence's die and a y on Andrei's die. A sum of seven can occur in 6 ways: (6, 1), (5, 2), (4, 3), (3, 4), (2, 5), (1, 6), and the probabilities of each, respectively, are $\frac{1}{80}, \frac{1}{80}, \frac{15}{80}, \frac{1}{80}, \frac{1}{80}$, and $\frac{1}{80}$. Thus the probability of rolling a sum of 7 is $\frac{20}{80} = \frac{1}{4}$.

F02S18 (Solution 1) Using the division algorithm we have $f(x^7) = f(x)Q(x) + R(x)$, where $\deg(R(x)) \le 5$. Since $f(x)(x-1) = x^7 - 1$, the 6 zeros of $f(x)$ are none other than 6 complex seventh roots of unity; so if α is a zero of $f(x)$, then $\alpha^7 = 1$. Therefore, $f(\alpha^7) = f(1) = 7$. On the other hand, $f(\alpha^7) = f(\alpha)Q(\alpha) + R(\alpha)$, $7 = R(\alpha)$, and this holds for 6 distinct values of α. But the polynomial $R(x) - 7$ of degree less than 6 can vanish at 6 places only if $R(x) - 7$ is identically $0 \Rightarrow$ $\boldsymbol{R(x) = 7}$ for all x.

(Solution 2) $f(x^7) = x^{42} + x^{35} + x^{28} + x^{21} + x^{14} + x^7 + 1$

$$f(x)\cdot(x-1) = (x-1)(x^6 + x^5 + x^4 + x^3 + x^2 + x + 1) = x^7 - 1$$

$$f(x)\cdot(x-1)(x^{35} + 2x^{28} + 3x^{21} + 4x^{14} + 5x^7 + 6) = x^{42} + x^{35} + x^{28} + x^{21} + x^{14} + x^7 - 6 = f(x^7) - 7$$

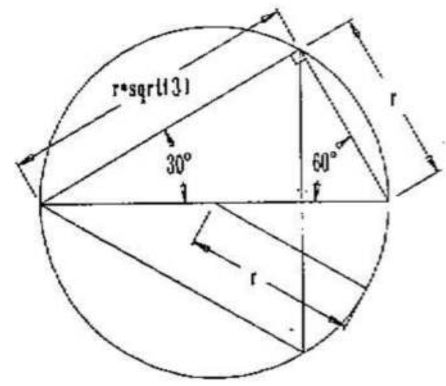
The remainder is 7.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division Solutions Contest 4

F02S19 Let A be Aneley's integer. $3A - 14 > 2A + 33$, so $A > 47$. $2A + 26 > 4A - 72$, so $A < 49$, the only integer between 47 and 49 is 48, so A = **48.**

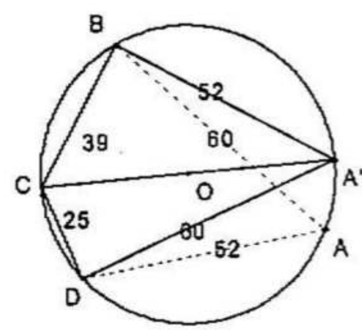
F02S20 **(Solution 1)** Notice that 101 is a prime. We have to solve $x^2 + x - 16 \equiv 0 \pmod{101}$. This is equivalent to $x^2 + 102x \equiv 16 \pmod{101}$, and $x^2 + 102x + 2601 \equiv 2617 \pmod{101}$.
$(x+51)^2 \equiv 2617 \equiv 900 \pmod{101}$. We can take the square root (because 101 is prime) of both sides to get the dual equalities: $x + 51 \equiv \pm 30 \pmod{101}$. Thus $x \equiv -81 \equiv 20 \pmod{101}$ or $x \equiv -21 \equiv 80 \pmod{101}$. Now we see that the answer to the question is **20, 80.**

(Solution 2) Let $x = t + 20$. Then, $(t+20)^2 + t + 20 - 16 = t^2 + 41t + 404$ is divisible by 101. But this only occurs if $t^2 + 41t = t(t+41)$ is divisible by 101. Clearly, this only occurs when $t = 0$, or $t = 60$, that is when x = **20 or 80.**



F02S21 Let r = radius of smaller circle, then $r^2 = 5/\pi$. The side length of an equilateral triangle inscribed in a circle is $\sqrt{3}$ times the radius of that circle. So the area of this particular triangle is $\frac{(\sqrt{3})^2 (5/\pi)\sqrt{3}}{4} = \frac{15\sqrt{3}}{4\pi}$.

F02S22 We reflect triangle ABD about the diameter perpendicular to $\overline{BD}$ so that B and D switch places. Then we see that $A'BC$ and $A'DC$ are right triangles by verifying the leg lengths, and thus $A'C$ is a diameter. The length of the radius is thus $\frac{\sqrt{60^2 + 25^2}}{2} = \frac{65}{2}$.



F02S23 Let X be the vertex between the two given sides.
$84 = \frac{1}{2}(13)(14)\sin X$, so $\sin X = \frac{12}{13}$. Since the angle X is acute,
$\cos X$ is the positive root of $\left(\frac{12}{13}\right)^2 + \cos^2 X = 1$, so $\cos X = \frac{5}{13}$. By the Law of Cosines,
$x^2 = 13^2 + 14^2 - 2(13)(14)\cos X = 225$, so the length of the third side is **15.**

F02S24 $3^{2x+1} - x3^{x+1} - 3^x - 6x^2 - 7x - 2 = 0 = (3 \cdot 3^x + 3x + 2)(3^x - 2x - 1)$

$3^x = 2x + 1$ has at most 2 roots since $2x+1$ is a line and 3^x is concave up.
The roots are **1** and **0.**
$3 \cdot 3^x = -3x - 2$ must have only 1 root, since $(3)3^x$ is increasing but $-3x-2$ is decreasing.
That root is **-1.** Therefore x = -1, 0, 1.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
Senior A Division Solutions   Contest 5

F02S25 Suppose they have n children and at present time, the sum of the children's ages is c. $10(c - 2n) = 6c - 4$ and $3(c + 6n) = 6c + 12$. Solving for n we get $n = \mathbf{3}$

F02S26 Let our expression equal E. $\dfrac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ} = E$, so

$$\frac{E}{4} = \frac{\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ}{2\sin 10^\circ \cos 10^\circ} = \frac{\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ}{2\sin 10^\circ \cos 10^\circ}.$$ Simplifying, $\dfrac{E}{4} = \dfrac{\sin 20^\circ}{\sin 20^\circ} = 1 \Rightarrow \boldsymbol{E = 4}$.

F02S27 We need to find all solutions in integers of $-5^4 + 5^5 + 5^n = m^2$. Rewriting, we have $5^n = m^2 - (5^5 - 5^4) = m^2 - 2500 = (m-50)(m+50)$. So $(m-50)$ and $(m+50)$ are both powers of 5. The only powers of 5 that differ by 100 are $5^2 = 25$ and $5^3 = 125$, so $n = 2 + 3 = \mathbf{5}$.

F02S28 Note that $17\sqrt{5} + 38 = \left(\sqrt{5}+2\right)^3$, and $17\sqrt{5} - 38 = \left(\sqrt{5}-2\right)^3 = \left(\dfrac{1}{\sqrt{5}+2}\right)^3$. Thus,

$\left(\sqrt{5}+2\right)^{\frac{3}{n}} + \left(\dfrac{1}{\sqrt{5}+2}\right)^{\frac{3}{n}} = \sqrt{20}$. Setting $t = \left(\sqrt{5}+2\right)^{\frac{3}{n}}$, $t + \dfrac{1}{t} = \sqrt{20}$, so $t = \dfrac{2\sqrt{5} \pm \sqrt{20-4}}{2} = \sqrt{5} \pm 2$. If $t = \left(\sqrt{5}+2\right)^{\frac{3}{n}} = \sqrt{5}+2$, $n = 3$. If $t = \left(\sqrt{5}+2\right)^{\frac{3}{n}} = \sqrt{5}-2$, $n = -3$. So the answers are $\boldsymbol{n = 3}$, and $\boldsymbol{n = -3}$.

F02S29 There are 11 ways of selecting the number of dimes (which fixes the number of nickels). They range from 0 dimes to 10 dimes (20 nickels to 0 nickels). Thus the number of coins can be anywhere from 20 to 10, inclusive. The number of prime numbers in [10, 20] is **4**.

F02S30 If $A = \tan x$ and $B = \tan y$, then $\dfrac{1 + A + B - AB}{1 - A - B - AB} = \dfrac{1 + \frac{A+B}{1-AB}}{1 - \frac{A+B}{1-AB}} =$

$$\frac{1 + \frac{\tan x + \tan y}{1 - \tan x \tan y}}{1 - \frac{\tan x + \tan y}{1 - \tan x \tan y}} = \frac{1 + \tan(x+y)}{1 - \tan(x+y)} = \tan(x + y + 45^\circ).$$ Therefore, if there are n numbers written on the board, the final number will be the tan(sum of the n numbers + (n-1)45°)

$$= \tan\left(\frac{45}{2}(1+89) + 44(45)\right)$$

$$= \tan 4005^\circ = \tan 45^\circ = 1$$