



New York City
Interscholastic
Mathematics
League

SENIOR B DIVISION

CONTEST NUMBER ONE

FALL 2000

PART I: 10 minutes

NYCIML Contest One

FALL 2000

F2000SB1. In equilateral triangle ABC , \overline{DE} is drawn parallel to \overline{BC} so that D is on \overline{AB} and E is on \overline{AC} . If $BC = 10$ and $DB = 2$, compute the area of trapezoid $DECB$.

F2000SB2. A fair die is rolled twice. Compute the probability that the result of the first roll exceeds the result of the second roll.

PART II: 10 minutes

NYCIML Contest One

Fall 2000

F2000SB3. A regular hexagon is inscribed in a circle. If the area of the hexagon is $54\sqrt{3}$, compute the area of the circle.

F2000SB4. If ${}_NC_1$, ${}_NC_2$ and ${}_NC_3$ form an arithmetic progression, compute N .

PART III: 10 minutes

NYCIML Contest One

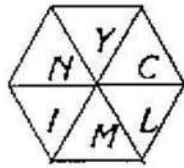
Fall 2000

F2000SB5. The sum of all but one of the angles of a convex polygon is 3624° . Compute the number of sides of the polygon.

F2000SB6. Three different integers are randomly chosen from $\{1, 2, 3, 4, 5, 6, 7\}$. Compute the probability that the sum of these three integers is odd.

ANSWERS:

1. $9\sqrt{3}$
2. $\frac{5}{12}$
3. 36π
4. 7
5. 23
6. $\frac{16}{35}$



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CONTEST NUMBER TWO

FALL 2000

PART I: 10 minutes

NYCIML Contest Two

FALL 2000

F2000SB7. Two successive increases of 25 % are equivalent to one increase of $N\%$. Compute N .

F2000SB8. Compute N if $4\sqrt{N} - 4 = N$.

PART II: 10 minutes

NYCIML Contest Two

Fall 2000

F2000SB9. Compute the area of an isosceles trapezoid with sides 6, 9, 9, 10.

F2000SB10. Compute the number of positive integral factors of 143,143.

PART III: 10 minutes

NYCIML Contest Two

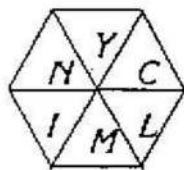
Fall 2000

F2000SB11. Compute the sum of all integers between 100 and 200 inclusive, that are divisible by 7.

F2000SB12. A game is won when a fair die is rolled three times and the third roll is the sum of the first two. If Sam wins the game, compute the probability that he rolled a 1 at least once.

ANSWERS:

- 7. 56.25
- 8. 4
- 9. $8\sqrt{77}$
- 10. 18
- 11. 2107
- 12. $\frac{3}{5}$



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CONTEST NUMBER THREE

FALL 2000

PART I: 10 minutes

NYCIML Contest Three

FALL 2000

F2000SB13. Compute the sum of the digits of all integers from 101 to 200 inclusive.

F2000SB14. Compute the number of ounces of water that must be added to 15 ounces of a 40 % acid solution to make it a 30 % acid solution.

PART II: 10 minutes

NYCIML Contest Three

Fall 2000

F2000SB15. Compute the number of positive integers N , $N < 1000$, for which $\log_8 N$ is rational.

F2000SB16. Compute the sum $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \dots + \frac{1}{19 \cdot 20}$.

PART III: 10 minutes

NYCIML Contest Three

Fall 2000

F2000SB17. Compute all ordered pairs of integers (x, y) such that $x + y = xy$.

F2000SB18. The base of a pyramid is an equilateral triangle with sides of length 6. The lateral edges of the pyramid are of length 10. Compute the volume of the pyramid.

ANSWERS:

13. 1001

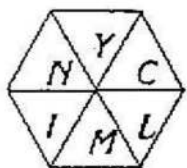
14. 5

15. 10

16. $\frac{19}{20}$

17. $(0, 0), (2, 2)$

18. $6\sqrt{66}$



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CONTEST NUMBER FOUR

FALL 2000

PART I: 10 minutes

NYCIML Contest Four

FALL 2000

F2000SB19. Compute the ordered pair (x, y) which is the solution of
 $x^2 - 8x + y^2 + 6y = -25$.

F2000SB20. If $\log 2 = a$, express $\log \frac{5}{16}$ in terms of a , with no logarithms.

PART II: 10 minutes

NYCIML Contest Four

Fall 2000

F2000SB21. Compute r so that ${}_{12}C_r$ is a maximum.

F2000SB22. The vertices of triangle ABC are $A(-1, -2)$, $B(2, 4)$, and $C(6, 2)$.
Compute the length of the altitude from B to \overline{AC} .

PART III: 10 minutes

NYCIML Contest Four

Fall 2000

F2000SB23. Compute all values of x that are solutions of $3 \cdot 9^{2x} - 10 \cdot 9^x + 3 = 0$.

F2000SB24. Solve for all positive acute angles x : $2 \csc x = \tan 72^\circ + \tan 18^\circ$.

ANSWERS:

19. $(4, -3)$

20. $1-5a$

21. 6

22. $\frac{6\sqrt{65}}{13}$

23. $\frac{1}{2}, -\frac{1}{2}$

24. 36



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CONTEST NUMBER FIVE

FALL 2000

PART I: 10 minutes

NYCIML Contest Five

FALL 2000

F2000SB25. If $i = \sqrt{-1}$, compute $(1+i)^{20}$.

F2000SB26. Working alone, John can paint a room in 8 hours. After he has been working for 3 hours, Dave helps him and together they complete the job in 2 more hours. Compute how long it would have taken Dave to paint the room alone.

PART II: 10 minutes

NYCIML Contest Five

Fall 2000

F2000SB27. Compute, $\sin\left(\text{Arc sin } \frac{3}{5} + \text{Arc cos } \frac{5}{13}\right)$ where *Arc sin* and *Arc cos* denote principal value.

F2000SB28. Compute the number of ways four men and four women can be seated at a circular table if the men and women alternate around the table. (Two seatings are identical only when each person has the same right neighbor.)

PART III: 10 minutes

NYCIML Contest Five

Fall 2000

F2000SB29. Compute the length of a diagonal of a cube with edge of length 4.

F2000SB30. If 796, 1157, and 1594 are divided by the positive integer q , they all leave a positive remainder r . Compute r .

ANSWERS:

25. -1024

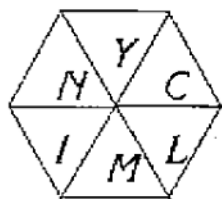
26. $5\frac{1}{3}$

27. $\frac{63}{65}$

28. 144

29. $4\sqrt{3}$

30. 17



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CONTEST NUMBER ONE

FALL 2000

SOLUTIONS

F00SB1. $\text{Area}(\triangle ABC) = \frac{10^2}{4}\sqrt{3} = 25\sqrt{3}$ $\text{Area}(\triangle ADE) = 16\sqrt{3}$
 $\text{Area}(DECB) = 25\sqrt{3} - 16\sqrt{3} = 9\sqrt{3}$

F00SB2 There are 36 possible results, of which 6 are the same result on both dice.
 15 of the remaining 30 have the first roll exceeding the second roll.
 $\frac{15}{36} = \frac{5}{12}$

F00SB3. The radius is equal to the side of the hexagon. $54\sqrt{3} = \frac{3}{2}s^2\sqrt{3}$ $s^2 = 36$
 $s = 6$ $A = 36\pi$.

F00SB4 ${}_NC_2 - {}_NC_1 = {}_NC_3 - {}_NC_2$ $\frac{N(N-1)}{2} - N = \frac{N(N-1)(N-2)}{6} - \frac{N(N-1)}{2}$
 $N^3 - 9N^2 + 14N = 0$ $N(N-2)(N-7) = 0$ $N = 7$.

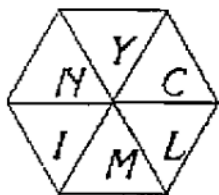
OR

Construct a Pascal's triangle until the second, third, and fourth terms of a row form an arithmetic progression. This occurs in the seventh row.

F00SB5. The sum of the angles is the next multiple of 180° greater than 3624° .
 This is 3780° . $3780^\circ = (N-2)180^\circ$ $N = 23$

F00SB6. Either all three integers are odd or one is odd and the other two are even.

$\frac{{}_4C_3 + 4 \cdot {}_3C_2}{{}_7C_3} = \frac{4 + 4 \cdot 3}{35} = \frac{16}{35}$



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CONTEST NUMBER TWO

FALL 2000

SOLUTIONS

F00SB7. $\frac{5}{4}\left(\frac{5}{4}x\right) = \frac{25}{16}x = 1\frac{9}{16}x \quad \frac{9}{16} = 56.25\%.$

F00SB8. $N = 4\sqrt{N} - 4 \quad N + 4 = 4\sqrt{N} \quad (N + 4)^2 = 16N \quad N^2 - 8N + 16 = 0 \quad N = 4.$

F00SB9. $A = \frac{1}{2}\sqrt{77}(6 + 10) = 8\sqrt{77}$

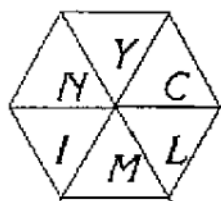
F00SB10. $143143 = 143 \cdot 1001 = 13 \cdot 11 \cdot 7 \cdot 11 \cdot 13 = 7^1 11^2 13^2.$ Adding one to each exponent and multiplying, the number of factors is $2 \cdot 3 \cdot 3 = 18.$

F00SB11. $7 \cdot 15 = 105$ is the smallest. $7 \cdot 28 = 196$ is the largest. There are 14 numbers. $S = \frac{14}{2}(105 + 196) = 7(301) = 2107.$

F00SB12. There are 15 possibilities:

1 1 2	2 1 3	3 1 4	4 1 5	5 1 6
1 2 3	2 2 4	3 2 5	4 2 6	
1 3 4	2 3 5	3 3 6		
1 4 5	2 4 6			
1 5 6				

Of these 15 possibilities, 9 of $\frac{9}{15} = \frac{3}{5}$ them have 1 at least once.



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CONTEST NUMBER THREE

FALL 2000

SOLUTIONS

F00SB13. There are 100 numbers. In the tens and units places, there are 10 of each digits. $10(0+1+2+3+\dots+9) = 450$. In the hundreds place, there are 99 ones and 1 two. $2(450) + 99 + 2 = 1001$.

F00SB14. There are .4(15) ounces of acid in the original solution. That equals .3(15+x) ounces of acid in the diluted solution. $.4(15) = .3(15+x)$ $x=5$.

F00SB15. $\log_2 N$ will be rational if N is any power of 2. There are 10 possible values: 1, 2, 4, 8, 16, 32, 64, 128, 256, and 512.

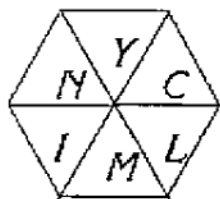
F00SB16. $\frac{1}{N(N+1)} = \frac{1}{N} - \frac{1}{N+1}$ $S = \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{19} - \frac{1}{20}$. All terms cancel out except for $\frac{1}{1} - \frac{1}{20} = \frac{19}{20}$.

F00SB17. $xy - x - y + 1 = 1$ $(x-1)(y-1) = 1$ Either $x-1 = y-1 = -1$ or $x-1 = y-1 = 1$. These give the solutions (0,0) and (2,2).

F00SB18. $V = \frac{1}{3}Ah$. The area of the base = $\frac{6^2}{4}\sqrt{3} = 9\sqrt{3}$. The height is one leg of a right triangle, which has hypotenuse of 10 and the other leg is the distance from the center of the equilateral triangle to the vertex of the triangle. This distance is $2\sqrt{3}$, since it is $\frac{2}{3}$ the altitude of an equilateral triangle with side 6.

$$(2\sqrt{3})^2 + h^2 = 10^2 \rightarrow h^2 = 88 \rightarrow h = \sqrt{88}$$

$$V = \frac{1}{3}(9\sqrt{3})(\sqrt{88}) \rightarrow V = 6\sqrt{66}$$



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CONTEST NUMBER FOUR

FALL 2000

SOLUTIONS

F00SB19. $x^2 - 8x + 16 + y^2 + 6y + 9 = -25 + 25 = 0. (x-4)^2 + (y+3)^2 = 0. (4, -3)$

F00SB20.

$$\log \frac{5}{16} = \log 5 - \log 16 = \log \frac{10}{2} - \log 2^4 = \log 10 - \log 2 - 4 \log 2 = 1 - a - 4a = 1 - 5a$$

F00SB21. As we can see by examining the middle number of any even power row in Pascal's triangle, the maximum value will be when $r = \frac{1}{2}n$. Therefore, $r=6$.

F00SB22. Since $AB = \sqrt{45}$, $BC = \sqrt{20}$, $AC = \sqrt{65}$, the triangle is a right triangle with hypotenuse \overline{AC} . The area is

$$\frac{1}{2} \sqrt{45} \sqrt{20} = 15 \rightarrow \frac{1}{2} h \sqrt{65} = 15 \rightarrow h = \frac{30}{\sqrt{65}} = \frac{30\sqrt{65}}{65} = \frac{6\sqrt{65}}{13}$$

Let $y = 9^x \Rightarrow 3 \cdot y^2 - 10y + 3 = 0 \Rightarrow (3y-1)(y-3) = 0 \Rightarrow y = \frac{1}{3}$ or $y = 3 \Rightarrow$

F00SB23.

$$9^x = \frac{1}{3} \text{ or } 9^x = 3 \Rightarrow 3^{2x} = 3^{-1} \text{ or } 3^{2x} = 3^1 \Rightarrow x \in \left\{ \frac{1}{2}, -\frac{1}{2} \right\}$$

F00SB24.

$$\begin{aligned} \frac{2}{\sin x} &= \frac{\sin 72}{\cos 72} + \frac{\sin 18}{\cos 18} = \\ &= \frac{\sin 72 \cos 18 + \sin 18 \cos 72}{\cos 72 \cos 18} = \frac{\sin 72 \cos 18 + \sin 18 \cos 72}{\sin 18 \cos 18} \\ &= \frac{\sin 90}{\frac{1}{2} \sin 36} = \frac{2}{\sin 36} = 2 \csc 36 \rightarrow x = 36. \end{aligned}$$



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CONTEST NUMBER FIVE

FALL 2000

SOLUTIONS

F00SB25. $(i+1)^2 = 1 + 2i + i^2 = 2i$

$$(i+1)^{20} = (2i)^{10} = 2^{10} i^{10} = 1024(-1) = -1024.$$

F00SB26. John completes $\frac{5}{8}$ of the job and Dave completes $\frac{2}{x}$ of the job.

$$\frac{5}{8} + \frac{2}{x} = 1 \rightarrow x = 5\frac{1}{3}.$$

F00SB27.

$$\sin\left(\text{Arc sin } \frac{3}{5} + \text{Arc cos } \frac{5}{13}\right) = \sin \text{Arc sin } \frac{3}{5} \cos \text{Arc cos } \frac{5}{13} + \sin \text{Arc cos } \frac{5}{13} \cos \text{Arc sin } \frac{3}{5} =$$
$$\frac{3}{5} \cdot \frac{5}{13} + \frac{12}{13} \cdot \frac{4}{5} = \frac{63}{65}.$$

F00SB28. Seat the men first. Once the first man sits down, there are $3!$ ways the other men can sit. There are now $4!$ ways that the women can sit.
 $(3!)(4!) = 144.$

F00SB29. The diagonal of a cube is the hypotenuse of a right triangle whose legs are an edge of the cube and the diagonal of a side.

$$4^2 + (4\sqrt{2})^2 = x^2 \rightarrow x = 4\sqrt{3}$$

F00SB30. since all the divisions leave the same remainder, q must divide the difference between any two numbers. $1157-796 = 361 = 19(19).$
 $1594-1157=437=19(23).$ Therefore, $q=19$ and dividing any of the numbers by 19, we get $r=17.$