New York City
Interscholastic
Mathematics
League

SENIOR B DIVISION **CONTEST NUMBER ONE** **SPRING 2000**

PART I: 10 minutes **NYCIML Contest One** **SPRING 2000**

S00SB1. A triangle with sides 3,4, and 5 is inscribed in a circle. Compute the area of the circle.

S00SB2. Compute the sum of the digits of the first 100 positive integers.

PART II: 10 minutes **NYCIML Contest One** **SPRING 2000**

S00SB3. Compute the number of integers in the solution set of $|5x-1|<17$.

S00SB4. Compute the infinite product $4^3 \bullet 4^1 \bullet 4^{\frac{1}{3}} \bullet 4^{\frac{1}{9}} \bullet 4^{\frac{1}{27}} \ldots$

PART III: 10 minutes **NYCIML Contest One** **SPRING 2000**

S00SB5. Compute how many ounces of pure acid should be added to 24 ounces of a 12% acid solution to make it a 15% acid solution?

S00SB6. Let $x = 59^4 + 4 \bullet 59^3 + 6 \bullet 59^2 + 4 \bullet 59 + 1$. How many positive integers are factors of x?

ANSWERS

1. $\frac{25\pi}{4}$
2. 901
3. 7
4. 512
5. 72/85
6. 225

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SENIOR B DIVISION **CONTEST NUMBER TWO** **SPRING 2000**

PART I: 10 minutes **NYCIML Contest TWO** **SPRING 2000**

S00SB7. If $8^x = \sqrt{32^y}$, compute $\frac{x}{y}$

S00SB8. Compute the number of diagonals in a convex polygon with 20 sides.

PART II: 10 minutes **NYCIML Contest TWO** **SPRING 2000**

S00SB9. Compute the remainder when 2^{2000} is divided by 7.

S00SB10. If $N = 15^3 + 21^3$, compute the largest prime factor of N

PART III: 10 minutes **NYCIML Contest TWO** **SPRING 2000**

S00SB11. If $\log_2(\log_2(\log_2(\log_2 x))) = 1$, compute x.

S00SB12. A diameter is drawn in a circle. One square is inscribed in a semicircle and another square is inscribed in the whole circle. Compute the ratio of the area of the smaller square to the area of the larger square.

ANSWERS:
7. 5/6
8. 170
9. 4
10. 13
11. 65536
12. 2/5

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SENIOR B DIVISION **CONTEST NUMBER THREE** **SPRING 2000**

PART I: 10 minutes **NYCIML Contest THREE** **SPRING 2000**

S00SB13. Change 408 base 10 to an equivalent base 2 number.

S00SB14. How many ordered pairs of positive integers (x, y) satisfy $10x+11y=1000$?

PART II: 10 minutes **NYCIML Contest THREE** **SPRING 2000**

S00SB15. The length of a rectangle is 4 times the width, and a diagonal is 7. Compute the area of the rectangle.

S00SB16. Compute all values of x: $\sqrt[3]{x} = \dfrac{2}{3+2\sqrt[3]{x}}$

PART III: 10 minutes **NYCIML Contest THREE** **SPRING 2000**

S00SB17. If [X] represents the greatest integer less than or equal to X, compute X:
X[X]=38

S00SB18. A point P is chosen in the interior of rectangle ABCD so that PA=3, PB=4, and PC=5. Compute PD.

ANSWERS:
13. 110011000
14. 9
15. 196/17
16. 1/8,-8
17. 19/3
18. $3\sqrt{2}$

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SENIOR B DIVISION **CONTEST NUMBER FOUR** **SPRING 2000**

PART I: 10 minutes **NYCIML Contest FOUR** **SPRING 2000**

S00SB19. If $\log_4 x = y$, express $\log_{16} x^2$ in terms of y with no logarithms.

S00SB20. There are 6 dimes and 5 quarters on a table. John takes 4 coins at random. Compute the probability that they are worth more than 80 cents.

PART II: 10 minutes **NYCIML Contest FOUR** **SPRING 2000**

S00SB21. If $x + \frac{1}{y} = 3$ and $y + \frac{1}{x} = 5$, compute $\frac{x}{y}$.

S00SB22. The medians to the legs of a right triangle are 6 and 8. Compute the length of the hypotenuse.

PART III: 10 minutes **NYCIML Contest FOUR** **SPRING 2000**

S00SB23. If 3!5!7!=x!, compute x.

S00SB24. The perimeter of an isosceles triangle is 30 and the altitude to the base is 5. Compute the area of the triangle.

ANSWERS:
19. y
20. 13/66
21. 3/5
22. $4\sqrt{5}$
23. 10
24. 100/3

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SENIOR B DIVISION **CONTEST NUMBER FIVE** **SPRING 2000**
PART I: 10 minutes **NYCIML Contest FIVE**

S00SB25. Each interior angle of a regular polygon measures $165°$. How many sides does it have?

S00SB26. The sides of a triangle are 2,9, and x, and the area of the triangle is x. Compute x.

PART II: 10 minutes **NYCIML Contest FIVE** **SPRING 2000**

S00SB27. How many points are common to the graphs $|x+1| = y$ and $x^2 = 16$?

S00SB28. Compute the smallest x for which [x]+[2x]+[3x]+[4x]+[5x]=19.
([X] represents the greatest integer less than or equal to X)

PART III: 10 minutes **NYCIML Contest FIVE** **SPRING 2000**

S00SB29. If $\frac{1}{11}, \frac{1}{x}$ and $\frac{1}{7}$ form an arithmetic progression, compute x.

S00SB30. Compute the largest x such that 18^x is a factor of 36!

ANSWERS:
25. 24
26. $\sqrt{77}$
27. 2
28. 7/5
29. 77/9
30. 8

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SENIOR B DIVISION **CONTEST NUMBER ONE** **SPRING 2000**

SOLUTIONS

S00SB1. Since it is a right triangle, the hypotenuse is equal to the diameter.

$$A = \pi\left(\frac{5}{2}\right)^2 = \frac{25\pi}{4}$$

S00SB2. If they are listed vertically, from 00 to 99, there are 100 numbers, 10 of each digit. 10(1+2+....+9)=450 for each column. 900 + the one for 100=901.

S00SB3. $-17<5x-1<17$ $-16<5x<18$ $-\frac{16}{5}<x<\frac{18}{5}$ x=-3,-2,-1,0,1,2,3. 7 integers

S00SB4. The product = $4^{3+1+\frac{1}{3}+\ldots}$ Using $S=\frac{a}{1-r}$ $3+1+\frac{1}{3}+\frac{1}{9}\ldots\ldots=\frac{3}{1-\frac{1}{3}}=\frac{9}{2}$

$$4^{\frac{9}{2}}=512$$

S00SB5. Let x= amount of acid to add .12(24)+x=.15(24+x) 288+100x=360+15x
x=72/85

S00SB6. $x=(59+1)^4=60^4=(2^2\bullet 3\bullet 5)^4=2^8\bullet 3^4\bullet 5^4$. Since the number of factors is the product of one more than each exponent, $9\times 5\times 5=225$.

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SENIOR B DIVISION **CONTEST NUMBER TWO** **SPRING 2000**

SOLUTIONS

S00SB7. $2^{3x} = 2^{\frac{5y}{2}} \to 3x = \frac{5y}{2} \to 6x = 5y \to \frac{x}{y} = \frac{5}{6}$

S00SB8. Each point has a diagonal from every other point except itself and the two adjacent points. Since diagonal $\overline{AB}$ is the same as diagonal $\overline{BA}$, $n = \frac{20(20-3)}{2} = 170$

S00SB9. $2^3 \equiv 1 \pmod 7$ (leaves a remainder of 1 when divided by 7)
Taking both sides to the power 666, $(2^3)^{666} \equiv (1)^{666} \pmod 7 \to 2^{1998} \equiv 1 \pmod 7$
$(2^{1998})2^2 \equiv 2^2 \pmod 7 \equiv 4 \pmod 7$

S00SB10. Factoring the sum of 2 cubes, $15^3 + 21^3 = (15+21)(15^2 - 15 \bullet 21 + 21^2) =$
$36 \bullet 351 = 2^2 \bullet 3^2 \bullet 3^3 \bullet 13$

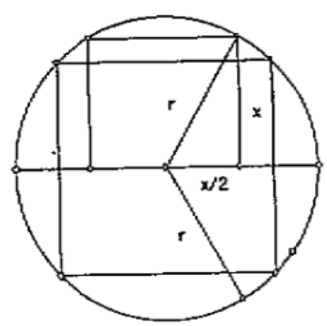
S00SB11. $\log_2(\log_2(\log_2(\log_2 x))) = 1$
$\log_2(\log_2(\log_2 x)) = 2$
$\log_2(\log_2 x) = 4$
$\log_2 x = 16$
$x = 2^{16} = 65536$

S00SB12. Area of big square $= \frac{1}{2}(2r)^2 = 2r^2$

Area of small square $= x^2$

$x^2 + (\frac{x}{2})^2 = r^2 \quad \frac{5x^2}{4} = r^2$

$x^2 = \frac{4r^2}{5} \quad \text{ratio} = \frac{\frac{4r^2}{5}}{2r^2} = \frac{2}{5}$



New York City Interscholastic Mathematics League

SENIOR B DIVISION **CONTEST NUMBER THREE** **SPRING 2000**

SOLUTIONS

S00SB13. 408=256+128+16+8 The number is 110011000 or divide 408 by 2 continuously and take the remainders in reverse order.

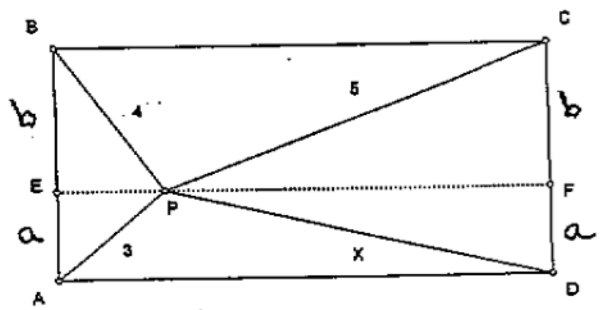
S00SB14. $x=\dfrac{1000-11y}{10}\rightarrow$ y must be a multiple of 10
Y=10,20,30…80,90 9 values

S00SB15. Let x=width $A=4x \bullet x=4x^2$ $(4x)^2+x^2=49$ $17x^2=49$
Multiply by $\dfrac{4}{17}$, $4x^2=\dfrac{196}{17}$

S00SB16. Let $y=\sqrt[3]{x}$ $y=\dfrac{2}{3+2y}$ $2y^2+3y=2$ $2y^2+3y-2=0$
$(2y-1)(y+2)=0$ $y=\dfrac{1}{2}, y=-2$ $x=\dfrac{1}{8}, x=-8$

S00SB17. The number must be between 6 and 7. [x]=6 6x=38 $x=\dfrac{38}{6}=\dfrac{19}{3}$

S00SB18. Through P, draw a line $\overline{EF}$ parallel to $\overline{AD}$. PF$=5^2-b^2=x^2-a^2$
PE$=4^2-b^2=3^2-a^2$ Subtracting, $9=x^2-9$, $x^2=18$, $x=3\sqrt{2}$



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SENIOR B DIVISION **CONTEST NUMBER FOUR** **SPRING 2000**

SOLUTIONS

S00SB19. $\log_4 x = y \quad 4^y = x \quad$ squaring: $4^{2y} = x^2 \quad 16^y = x^2 \quad \log_{16} x^2 = y$

S00SB20. Either 4 quarters or 3 quarters and a dime will work.

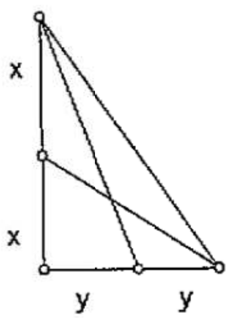
$$\frac{{}_5C_4 + {}_5C_3 \bullet 6}{{}_{11}C_4} = \frac{5 + 10 \bullet 6}{330} = \frac{65}{330} = \frac{13}{66}$$

S00SB21. $x + \frac{1}{y} = 3 \quad xy + 1 = 3y \quad y + \frac{1}{x} = 5 \quad xy + 1 = 5x$ subtracting,

$3y = 5x \rightarrow \frac{x}{y} = \frac{3}{5}$

S00SB22. $(2x)^2 + y^2 = 6^2 \quad x^2 + (2y)^2 = 8^2 \quad 5x^2 + 5y^2 = 100 \quad x^2 + y^2 = 20$

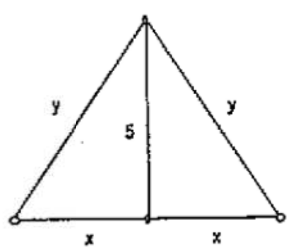
$4x^2 + 4y^2 = 80$ hypotenuse $= \sqrt{80} = 4\sqrt{5}$



S00SB23. 3!5!7!=3x2x5x4x3x2x7x6x5x4x3x2=10! when terms are rearranged

S00SB24. 2x+2y=30 x+y=15 $x^2 + 5^2 = (15 - x)^2 \quad x^2 + 25 = 225 - 30x + x^2$

30x=200 $x = \frac{20}{3}$ $A = \frac{1}{2} \bullet \frac{40}{3} \bullet 5 = \frac{100}{3}$



New York City Interscholastic Mathematics League

SENIOR B DIVISION **CONTEST NUMBER FIVE** **SPRING 2000**

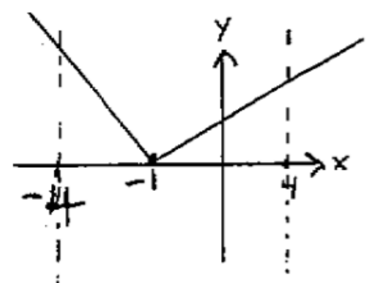
SOLUTIONS

S00SB25. Each interior angle measures $\frac{(n-2)180}{n}$, $\frac{(n-2)180}{n}=165$

165n=180n-360, 15n=360, n=24

S00SB26. $A=\frac{1}{2}abSinC$ $x=\frac{1}{2}2xSinC$ SinC=1, C=90° $x^2+2^2=9^2$ $x=\sqrt{77}$

S00SB27. The graphs intersect in 2 points.



S00SB28. Let f(x)=[x]+[2x]+[3x]+[4x]+[5x]

f(1)=15 $f(1\frac{1}{4})=17$ $f(1\frac{2}{5})=19$ $f(1\frac{1}{5})=16$ $f(1\frac{1}{3})=18$

S00SB29. $\frac{1}{x}$ is the average of $\frac{1}{11}$ and $\frac{1}{7}$. $\frac{\frac{1}{11}+\frac{1}{7}}{2}=\frac{18}{154}=\frac{9}{77}$ $x=\frac{77}{9}$

S00SB30. Since $18=2\bullet 3^2$, the number of eighteens will be half the number of threes, since there will be more than enough twos. In 36!, there are 12 multiples of 3, of which 4 are multiples of 3 squared, and one 3 cubed. There are then 1x8+2x3+1x3=17 threes, and 17/2 or 8 eighteens.