NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER ONE

PART I *SPRING, 2000* *CONTEST 1* *TIME: 10 MINUTES*

S00S1 If $A = 2^x + 2^{-x}$ and $B = 2^x - 2^{-x}$, compute the value of $A^2 - B^2$.

S00S2 Equilateral triangle ABC has sides of length 2. Unit circles with centers A, B and C are drawn. The area in the interior of triangle ABC but exterior to the three circles may be expressed in $a + b\pi$ form. Compute the ordered pair (a, b).

PART II *SPRING, 2000* *CONTEST 1* *TIME: 10 MINUTES*

S00S3 WY2K, a radio station, is holding a guessing contest. A number is chosen from 1 to 2000 inclusive. Listeners may guess only once. The D.J. tells the listeners if the guess was high, low or just right. Future guesses may be made using this information. What is the fewest number of guesses needed to guarantee the correct guess?

S00S4 k ! terminates with exactly 496 zeros. Find the largest k that satifies this condition.

PART III *SPRING, 2000* *CONTEST 1* *TIME: 10 MINUTES*

S00S5 Compute all real value(s) of x that satisfy:

$$|x^2 + 7x + 4| = x + 3$$

S00S6 Square MATH, with MA = 4, has a circle inscribed. A smaller circle is drawn tangent to the inscribed circle and sides MA and AT. Compute the maximum radius of the smaller circle.

ANSWERS:

S00S1	4
S00S2	$(\sqrt{3}, -\frac{1}{2})$
S00S3	11
S00S4	1999
S00S5	$x = -1, -3 + 2\sqrt{2}$
S00S6	$6 - 4\sqrt{2}$

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER TWO

PART I *SPRING, 2000* *CONTEST 2* *TIME: 10 MINUTES*

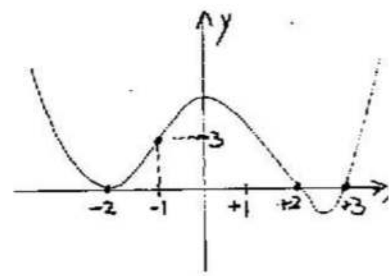
S00S7 Two fair dice are rolled. If the dice show the same values then one value is the tens' digit and the other the units' digit of a two-digit number. If the dice show different values then the larger value is the tens' digit and the smaller value the units' digit of a two-digit number. Compute the average of all possible two-digit numbers.

S00S8 $\sqrt{43 - 15\sqrt{8}} = a + b\sqrt{c}$ and $a > 0$. Compute the ordered triple of integers (a, b, c) that satisfies this equation if $a > c$.

PART II *SPRING, 2000* *CONTEST 2* *TIME: 10 MINUTES*

S00S9 Find the value(s) of k such that $2(2000^2 - 1999^2 + 1998^2 - 1997^2 + \ldots + 2^2 - 1^2) = k^2 + k$

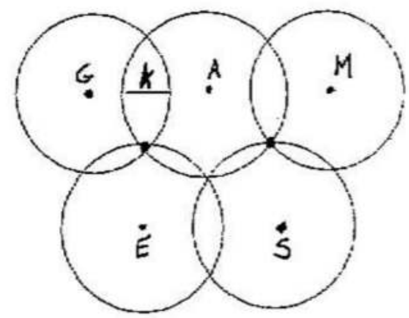
S00S10 A fourth order polynomial that has the following graph may be written as $aX^4 + bX^3 + cX^2 + dX + e$. Compute $a + b + c + d + e$.



PART III *SPRING, 2000* *CONTEST 2* *TIME: 10 MINUTES*

S00S11 $\angle A$ and $\angle B$ are both acute angles. Measured in degrees, $m\angle A = x^2$ and $m\angle B = 3x + 80$. Compute all value(s) of x such that $\sin A = \cos B$.

S00S12 Five congruent circles, with radius r, are linked. The radii overlap a distance of k. GA = AM = MS = ES = EA = GE = AS. Compute k / r.



ANSWERS:

S00S7	46
S00S8	(5,-3,2)
S00S9	2000,-2001
S00S10	9/2
S00S11	-5, 2
S00S12	$2 - \sqrt{3}$

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER THREE

PART I *SPRING, 2000* *CONTEST 3* *TIME: 10 MINUTES*

S00S13 Compute $\dfrac{1999^3 - 1000^3 - 999^3}{1999\,(1000)\,(999)}$

S00S14 Sin $x = a$. Express sin $3x$ in terms of a.

PART II *SPRING, 2000* *CONTEST 3* *TIME: 10 MINUTES*

S00S15 Square DEFG, with sides of length 2, has unit circles centered on each vertex. The area inside the square but outside the circles may be expressed in $a + b\pi$ form. Compute the ordered pair (a, b).

S00S16 On a 30 question multiple choice test with 5 choices, a student gets 5 pts for every correct choice, 2 pts for each question omitted and 0 pts per wrong choice. Toby Ann Aimetaker is sure of 10 answers but wants to score 100 or more points. How many questions should she guess on to have the greatest chance of scoring at least 100?

PART III *SPRING, 2000* *CONTEST 3* *TIME: 10 MINUTES*

S00S17 $1999! = 1 \times 2 \times 3 \times \ldots 1998 \times 1999$ and $1999!!! = 1 \times 3 \times 5 \ldots 1997 \times 1999$
If $1999!!! = \dfrac{1999!}{2^a \times a!}$, compute a.

S00S18 The perimeter of a sector of a circle is f feet. Compute the number of feet in the radius, in terms of f, that will maximize the area of the sector.

ANSWERS:

S00S13	3
S00S14	$3a - 4a^3$
S00S15	(4, -1)
S00S16	5
S00S17	999
S00S18	$r = f / 4$ feet

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER FOUR

PART I *SPRING, 2000* *CONTEST 4* *TIME: 10 MINUTES*

S00S19 $\dfrac{(1999)^4 + 4\times(1999)^3 + 6\times(1999)^2 + 4\times(1999) + 1}{(45^2 - 5^2)} = 2000^n$. Compute n.

S00S20 Two rectangles are similar. The first rectangle's area is cut in half. The new rectangle's area is cut in half and so on (infinitely). The second rectangle has an area equal to the sum of the area of the first rectangle and the areas of all the halved rectangles. The larger rectangle's length is k times the smaller rectangle's length. Compute k.

PART II *SPRING, 2000* *CONTEST 4* *TIME: 10 MINUTES*

S00S21 A Tic-Tac-Toe board is a square board containing 3 rows of 3 boxes (9 boxes total). A player wins if they can get 3 boxes in a row vertically, horizontally or diagonally. A chicken randomly chooses 3 boxes out of the nine. Compute the probability that the chicken wins.

S00S22 Three vertices of parallelogram ABCD are (-1, 3), (3, 2) and (1, 0). The fourth vertex of the parallelogram is point D. The three possible coordinates for point D are the vertices of a triangle. Compute the area of the triangle.

PART III *SPRING, 2000* *CONTEST 4* *TIME: 10 MINUTES*

S00S23 In Cube NEWYORKA, each edge has a length of 2. The cube has 8 unit spheres each centered on a vertex of the cube. The volume enclosed by the cube but outside the spheres may be expressed in $a + b\pi$ form. Compute the ordered pair (a, b).

S00S24 Liz, Ralph and Tommy are playing a game where they toss a fair die. Liz wins if she tosses an even number, Ralph wins if he tosses a 3 or 5, and Tommy wins if he tosses a 1. The die is rotated from Liz to Ralph to Tommy to Liz to Ralph to Tommy to Liz, etc. until one player wins. Compute the probability that Tommy wins.

ANSWERS:

S00S19	3
S00S20	$\sqrt{2}$
S00S21	2 / 21
S00S22	20
S00S23	(8, -4 / 3)
S00S24	1 / 13

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER FIVE

PART I | ***SPRING, 2000*** | ***CONTEST 5*** | ***TIME: 10 MINUTES***

S00S25 Compute the value of the following : $\sum_{k=1}^{1000}(\log_2 2k) - \sum_{k=1}^{1000}(\log_2 k)$

S00S26 Express the sum of the following series in simplest form.
$\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \frac{5}{243} + \ldots + \frac{n}{3^n} + \ldots$

PART II | ***SPRING, 2000*** | ***CONTEST 5*** | ***TIME: 10 MINUTES***

S00S27 In a small town, called Bullseye, the avenues are concentric circles. Between two such avenues is 2000 π square yards of land. If each of the avenues is an integral distance from the center, compute the number of possible distances that may exist between the two avenues.

S00S28 Compute all real value(s) of k, such that the system of equations below has no solution.

$$kX + 3Y + 3Z = 1000$$
$$3X + kY + 3Z = 900$$
$$3X + 3Y + kZ = 100$$

PART III | ***SPRING, 2000*** | ***CONTEST 5*** | ***TIME: 10 MINUTES***

S00S29 Only one n-sided polygon can have a smallest interior angle of 100° with each successive angle 10° greater than its predecessor. Compute n.

S00S30 Let $x = 3^a$. Compute all integral values of a, that satisfy:
$(\log_9 x^2 - 3)^2 + (\log_{27} x^3 - 9)^2 = (\log_3 x^2 - 12)^2$.

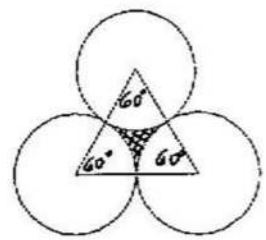
ANSWERS:

S00S25	1000
S00S26	3 / 4
S00S27	6
S00S28	k= 3, -6
S00S29	8
S00S30	a = 3, 9

SOLUTIONS

S00S1 **Answer:** 4 . **Method I:** Let $A + B = 2*2^{x}$ and $A - B = 2*2^{-x}$ then
$A^2 - B^2 = (A+B)(A-B) = 4(2^{x})(2^{-x}) = 4$.
Method II: Without loss of generality, since x can be any value, choose $x = 0$, then $A = 2$ and $B = 0$. $\therefore A^2 - B^2 = 4$.

S00S2 **Answer:** $(\sqrt{3}, -½)$.
The area of an equilateral triangle $= s^2\sqrt{3}/4$.
The side has a length of 2. $\therefore A_{triangle} = \sqrt{3}$
The area unshaded $= 3 A_{sector} = A_{semicircle} = ½\pi r^2 = ½\pi$
$A_{shaded} = A_{triangle} - A_{semicircle}$
$\therefore \text{Area}_{Shaded} = \sqrt{3} - \pi/2$



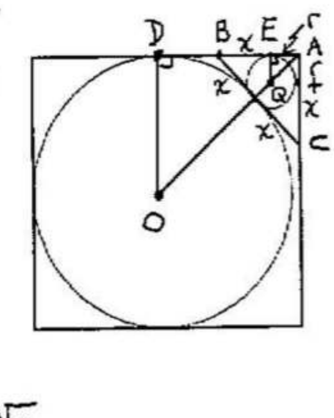
S00S3 **Answer:** 11. With each successive guess, the strategy is to split the remaining numbers in half. If x = the number of guesses then 2^x numbers can be separated into two equal or near equal halves. So $2^x > 2000$ will guarantee that we guess the correct number. $x = 11$ is the least integral value.

S00S4 **Answer:** 1999. The number of zeros at the end of a number is determined by the number of factors of 10 (or 2*5). It is obvious that there are more 2s than 5s in k!. $\therefore$ the number of 5s will limit the # of zeros. 5! has 1 zero, 25! has 6 zeros, 125! has 31 zeros, 625! has 156 zeros. To attain 496 zeros, we need 3*625! (468 zeros) 0*125! (0 zeros) 4*25! (24 zeros) and 4*5! (4 zeros) $\therefore 3*625 + 4*25 + 4*5 = 1995$. We can add 4 more before the next factor of 5 would be added at 2000 $\therefore$ 1999.

S00S5 **Answer:** $x = -1, -3 + 2\sqrt{2}$. If $|x^2 + 7x + 4| = x + 3$ then either

$x^2 + 7x + 4 = x + 3$ or $-(x^2 + 7x + 4) = x + 3$
$\therefore x^2 + 6x + 1 = 0$ $\qquad \therefore x^2 + 8x + 7 = 0$
$x = -3 \pm 2\sqrt{2}$ $\qquad x = -1, -7$
$|x^2 + 7x + 4| \geq 0$ so $x + 3 \geq 0$, $\therefore$ reject $x = -3 - 2\sqrt{2}$ and -7.

S00S6 **Answer:** $r = 6 - 4\sqrt{2}$. **Method I** From right triangle ABC, $(x + r)^2 + (x + r)^2 = (2x)^2$ D is the midpoint of MA $\therefore AD = 2x + r = 2$. Solving the two equations simultaneously gives us two answers $r = 6 \pm 4\sqrt{2}$ we must reject $r = 6 + 4\sqrt{2}$ because $0 < r < 2$. $\therefore r = 6 - 4\sqrt{2}$.



Method 2: Draw OD and QE. In isosceles right $\triangle$ODA, $OD = 2$ so OA is $2\sqrt{2}$. Similarly in similar $\triangle$QEA, $QA = r\sqrt{2}$. Then $OA = 2 + r + r\sqrt{2} = 2\sqrt{2}$.
Thus, $r(1 + \sqrt{2}) = 2\sqrt{2} - 2$. Solving for r yields $r = 6 - 4\sqrt{2}$.

SOLUTIONS

S00S7 **Answer:** 46.The possibilities include 11, 21, 22, 31, 32, 33, 41, 42, 43, 44, 51, 52, 53, 54, 55, 61, 62, 63, 64, 65, and 66. Therefore the total is 966 in 21 possible rolls. The average roll is 966 / 21 which reduces to 46.

S00S8 **Answer:** (5, -3, 2). $\sqrt{43 - 15\sqrt{8}} = a + b\sqrt{c}$ so square both sides

$a^2 + b^2c + 2ab\sqrt{c} = 43 - 15\sqrt{8}$

thus $a^2 + b^2c = 43$ and $2ab\sqrt{c} = -15\sqrt{8}$ so c is even and $a^2 \le 43$

Since c is even, a must be odd because a^2 + even number = 43, a may be 1, 3 or 5

If $a = 1$ then $b^2c = 42$ which yields nonintegral values for b and c.

If $a = 3$ then $b^2c = 34$ which yields nonintegral values for b and c.

Only $a = 5$ works, giving $b = -3$ and $c = 2$.

S00S9 **Answer:** 2000 and -2001.

$2(2000^2 - 1999^2 + 1998^2 - 1997^2 + ... + 2^2 - 1^2) = k^2 + k$

$2((2000^2 - 1999^2) + (1998^2 - 1997^2) + ... + (2^2 - 1^2)) = k^2 + k$

$2((2000 + 1999)(2000 - 1999) + ... + (2 + 1)(2 - 1)) = k^2 + k$

$2(2000 + 1999 + 1998 + ... + 3 + 2 + 1) = 2(2000 * 2001 / 2) = 2000 * 2001$

$\therefore 2000 * 2001 = k^2 + k = k(k + 1)$ so $k = 2000$ or -2001

S00S10 **Answer:** 9 / 2.

$F(X) = a(X - r_1)(X - r_2)(X - r_3)(X - r_4)$, where r_1, r_2, r_3, r_4 are the roots of F

From the graph F has a double root at $X = -2$, and a single root at $X = 2, 3$.

$F(X) = a(X + 2)(X + 2)(X - 2)(X - 3)$

$F(-1) = a(1)(1)(-3)(-4) = 3$ so $a = 1/4$

Now find $a + b + c + d + e = F(1) = 1/4\,(3)(3)(-1)(-2) = 9/2$

S00S11 **Answer:** -5 and 2. Since sin A = cos B, A and B are complementary A + B = 90.

$\therefore x^2 + 3x + 80 = 90$ or $x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0 \therefore x = -5$ or 2

S00S12 **Answer:** $2 - \sqrt{3}$. Triangle AGE is equilateral.

$\angle$BGA is 30° because BG bisects $\sphericalangle$EGA

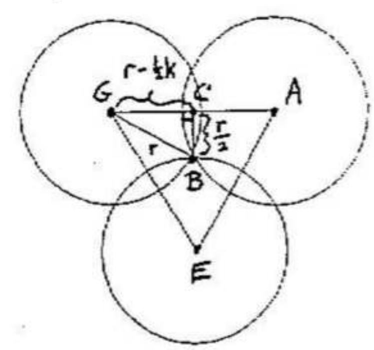
see symmetry of figure to right

From right triangle GBC, $(r - \frac{1}{2}k)^2 + (\frac{1}{2}r)^2 = r^2$

$\therefore r - \frac{1}{2}k = \sqrt{3}\,r/2$

Or $k = 2r - \sqrt{3}\,r = (2 - \sqrt{3})\,r$

$\therefore k/r = 2 - \sqrt{3}.$



NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER THREE

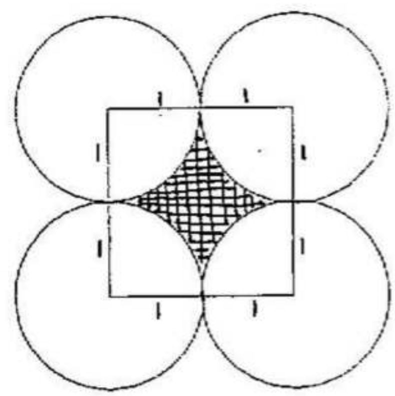
SOLUTIONS

S00S13 **Answer:** 3 . Let $x = 1000$ and $y = 999$ then the expression becomes

$$\frac{(x+y)^3 - x^3 - y^3}{(x+y)xy} = \frac{x^3 + y^3 + 3x^2y + 3xy^2 - x^3 - y^3}{x^2y + xy^2} = \frac{3x^2y + 3xy^2}{x^2y + xy^2} = 3.$$

S00S14 **Answer:** $3a - 4a^3$. Use the double angle formulas, $\sin 2X = 2 \sin X \cos X$, $\cos 2X = 1 - 2\sin^2 X$, also the Pythagorean identity $\sin^2 X + \cos^2 X = 1$ and $\sin(X+Y) = \sin X \cos Y + \cos X \sin Y$. Then $\sin 3X = \sin(2X + X) =$
$\sin 2X \cos X + \cos 2X \sin X = 2 \sin X \cos^2 X + (1 - 2\sin^2 X)\sin X$
$\therefore \sin 3X = 2a(1 - a^2) + (1 - 2a^2)a = 3a - 4a^3$

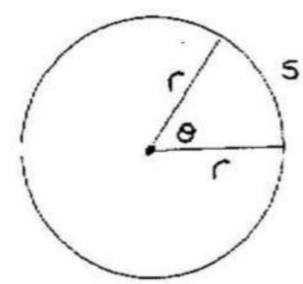
S00S15 **Answer:** (4, -1). The area of the square = s^2.
The side has a length of 2. $\therefore A_{square} = 4$
The area unshaded = $4 A_{sector} = A_{circle} = \pi r^2 = \pi$
$A_{shaded} = A_{square} - A_{circle}$
$\therefore$ Area $_{Shaded} = 4 - \pi$



S00S16 **Answer:** 5 . To break 100 points, the minimum number right is 14 (with 15 or 16 blank).
If you guess on 4, you must get all 4 correct.
Probability = $(1/5)^4 = 1/625$
If you guess on 5, then either all 5 right $(1/5)^5$ or 4 right and 1 wrong ${}_5C_1 (1/5)^4 (4/5)$
Probability = 21 / 3125.
If you guess on 6, then either all 6 right $(1/5)^6$ or 5 right and 1 wrong ${}_6C_1 (1/5)^5 (4/5)$
Probability = 5 / 3125. Four right and two wrong gives you a score of 98.
If you guess on 7, then either all 7 right $(1/5)^7$ or 6 right and 1 wrong ${}_7C_1 (1/5)^6 (4/5)$
Or 5 right and 2 wrong ${}_7C_2 (1/5)^5 (4/5)^2$. Probability < 21 / 3125
If you continue to guess, the probability decreases further.
$\therefore$ You should guess on 5 answers to have the highest probability of breaking 100.

S00S17 **Answer:** 999. $1999! = 1 \times 2 \times 3 \times \ldots 1998 \times 1999$ and $1999!!! = 1 \times 3 \times \ldots 1997 \times 1999$
$\therefore \quad 1999! = 1999!!! \times 2 \times 4 \ldots 1996 \times 1998 = 1999!!! \times 2^{999} \times (1 \times 2 \times 3 \times \ldots \times 999)$
Solving for $1999!!! = \dfrac{1999!}{2^{999} \times 999!}$ $\quad \therefore a = 999.$

S00S18 **Answer:** $r = f/4$ feet.
$P = r + r + s = f \;\therefore s = f - 2r$ and $s = r\theta$
$K = \frac{1}{2} r s = \frac{1}{2} r (f - 2r) = \frac{1}{2} f r - r^2$
The graph of K is an inverted quadratic in terms of r
$\therefore$ the max of K is at $r = -b/2a = f/4$ feet.



NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER FOUR

SOLUTIONS

S00S19 **Answer:** 3. $(x+1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$, let x = 1999 then
$(1999)^4 + 4\times(1999)^3 + 6\times(1999)^2 + 4\times(1999) + 1 = 2000^4$.
$45^2 - 5^2 = (45+5)(45-5) = 50 \times 40 = 2000$
thus $2000^4 / 2000 = 2000^3$

S00S20 **Answer:** $\sqrt{2}$. If $A = A_{\text{smaller rectangle}}$ = base * height or b * h
then $A_{\text{larger rectangle}} = (\alpha b)(\alpha h) = \alpha^2 b*h$ where α = constant of similitude
and $A_{\text{larger rectangle}} = A + \frac{1}{2}A + 1/4\,A + \ldots = A/(1 - \frac{1}{2}) = 2A$
giving us $\alpha^2 = 2$ and α is positive $\therefore \alpha = \sqrt{2}$.

S00S21 **Answer:** 2 / 21. There are 8 ways to win (3 rows, 3 columns and 2 diagonals).
The chicken can select 3 boxes ${}_9C_3$ or 84 ways.

S00S22 **Answer:** 20. The three points that would make ABCD a parallelogram are E (1, 5), F (-3, 1), and G (5, -1). A, B, C are the midpoints of sides EF, EG and FG respectively. The area of $\triangle$EFG is 4 times the area of $\triangle$ABC or 20.

S00S23 **Answer:** (8, -4 / 3). Each sphere shares 1 / 8 of its total volume with the cube. Since there are 8 spheres, we need to subtract the volume of the sphere from the volume of the cube. $\text{Volume}_{\text{cube}} = 2^3 = 8$ $\text{Volume}_{\text{sphere}} = 4(1)^3\pi/3 = 4\pi/3$
$\therefore \text{Volume}_{\text{cube}} - \text{Volume}_{\text{sphere}} = 8 - 4\pi/3$

S00S24 **Answer:** 1 / 13. P(T wins in 1st rd) = P (L loses)*P(R loses)* P(T wins) = $12/6^3$
P(T wins in 2nd rd) = P(all lose in 1 rd)* P (L loses)*P(R loses)* P(T wins) = $720/6^6$
P(T wins in 3rd rd) = (P(all lose in 1 rd))^2 * P (L loses)*P(R loses)* P(T wins) =
$43200/6^9$ or $60/6^3$ * P (T winning in 2nd rd)
P(T wins game) = $\frac{12}{6^3} + \frac{12(60)}{6^6} + \frac{12(60)^2}{6^9} + \ldots\ldots\ldots = \frac{12}{156} = \frac{1}{13}$

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | CONTEST NUMBER FIVE

SOLUTIONS

S00S25 **Answer:** 1000 . Log b - log a = log (b / a) so $\sum_{k=1}^{1000} (\log_2 2k) - \sum_{k=1}^{1000} (\log_2 k) = \sum_{k=1}^{1000} \log_2 2$.
Since $\log_2 2 = 1$, $\sum_{k=1}^{1000} \log_2 2 = \sum_{k=1}^{1000} 1 = 1000$.

S00S26 **Answer:** 3 / 4 . **Method I:**

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \dots + \frac{1}{3^n} + \dots = \frac{1/3}{1-1/3} = \frac{1}{2}$$
$$\frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \dots + \frac{1}{3^n} + \dots = \frac{1/9}{1-1/3} = \frac{1}{6}$$
$$\frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \dots + \frac{1}{3^n} + \dots = \frac{1/27}{1-1/3} = \frac{1}{18}$$
$$\frac{1}{81} + \frac{1}{243} + \dots + \frac{1}{3^n} + \dots = \frac{1/81}{1-1/3} = \frac{1}{54}$$
$$\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \frac{5}{243} + \dots + \frac{n}{3^n} + \dots =$$
$$\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} + \frac{1}{162} + \dots + \frac{1}{2*3^{n-1}} + \dots = \frac{1/2}{1-1/3} = \frac{3}{4}$$

Method II : Let $N = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \dots + \frac{n}{3^n} + \dots$

Let $3N = 1 + \frac{2}{3} + \frac{3}{9} + \frac{4}{27} + \dots$

$- \quad N = \frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \dots$

$2N = 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots = \frac{1}{1-1/3} = \frac{3}{2}$ $\therefore N = \frac{3}{4}$

S00S27 **Answer:** 6. $a^2\pi - b^2\pi = 2000\pi$ or $a^2 - b^2 = (a+b)(a-b) = 2000$
only a - b = 2, 4, 8 10, 20, and 40 yield integral answers for a, b and d (d = a-b).
Therefore the answer is 6.

S00S28 **Answer:** k = 3, -6. By summing the three equations ($kX + 3Y + 3Z = 1000$, $3X + kY + 3Z = 900$, and $3X + 3Y + kZ = 100$), we get $(k+6)(X+Y+Z) = 2000$ if $k = -6$ then there is no solution.
Also, if $k = 3$ there is no solution the equations are inconsistent.

S00S29 **Answer:** 8 . The sum of the interior angles may be summed two ways. First, by using $180(n-2)$ and second $100 + 110 + 120 + \dots + (100 + 10(n-1)) = 100n + 10n(n-1)/2$.
So $180(n-2) = 100n + 10n(n-1)/2$ or $180n - 360 = 100n + 5n^2 - 5n$ which simplifies to $n^2 - 17n + 72 = (n-8)(n-9) = 0$ so $n = 8$ or 9. 9 is rejected or the largest angle would be 180°.

S00S30 **Answer:** 3 or 9. $\log_9 X^2 = \log X^2 / \log 3^2 = 2\log X / 2\log 3 = \log_3 X = a$
$\therefore$ the equation becomes $(a-3)^2 + (a-9)^2 = (2a-12)^2$
If $p^2 + q^2 = (p+q)^2$, then either $p = 0$ or $q = 0$.
Since $a - 3 + a - 9 = 2a - 12$ either $a - 3 = 0$ or $a - 9 = 0$ so $a = 3$ or 9

July 13, 2000

Dear Math Team Coach,

Enclosed is your copy of the Spring, 2000 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Senior A	S00S3	11 (2000 was also accepted)
	S00S20	$\sqrt{2}$ ($\sqrt{3}$ was also accepted)

Have a great summer!
MATH IS # 1

Sincerely yours,

Richard Geller

Secretary, NYCIML