New York City
Interscholastic
Mathematics
League

JUNIOR DIVISION **CONTEST NUMBER ONE** **SPRING 2000**

PART I: 10 minutes **NYCIML Contest One** **Spring 2000**

S00J1. On the island of Geen, 50 bananas = 20 coconuts, 30 coconuts = 12 fish and 100 fish = 1 radio. Compute how many bananas = 1 radio.

S00J2. A fair six sided die is rolled 5 times. Compute the probability that the number rolled on each throw is higher than the previous one.

PART II: 10 minutes **NYCIML Contest One** **Spring 2000**

S00J3. The Harvard chess club has 22 members and at each meeting each member plays one game against each other member. How many games are played at each meeting?

S00J4. A triangle has sides 13, 14 and 15. The length of the altitude to one side is an integer. Compute the length of this altitude.

PART III: 10 minutes **NYCIML Contest One** **Spring 2000**

S00J5. Arthur looks in his bank and sees that he has quarters and dimes in the ratio of 3:2. All together, he has $23.75. If the number of quarters is Q and the number of dimes is D, compute (Q,D).

S00J6. A line is drawn through the point (4,0) and tangent to a unit circle with center at (1,0). Compute the absolute value of the y-intercept of the line.

ANSWERS:

J1. 625

J2. $\frac{1}{1296}$

J3. 231

J4. 12

J5. (75,50)

J6. $\sqrt{2}$

New York City
Interscholastic
Mathematics
League

JUNIOR DIVISION | **CONTEST NUMBER TWO** | **SPRING 2000**

PART I: 10 minutes | **NYCIML Contest Two** | **Spring 2000**

S00J7. If Deanna gets 88 on her next math test her average will be 87 for all her math tests. If she gets 78 on her next math test her average will be 85 for all her math tests. Compute the number of math tests she has taken.

S00J8. Danny grows and picks a 100-pound watermelon that he calculates is 99% water. He dries it out in the sun for a day and then realizes that the watermelon is now 98% water. Compute the new weight of the watermelon.

PART II: 10 minutes | **NYCIML Contest Two** | **Spring 2000**

S00J9. Carlos drives from Appleville to Bananatown, a distance of 200 miles. He then drives from Bananatown to Carrotville, a distance of 120 miles. He then drives from Carrotville to Datetown and knows that this distance is 1/5 the total distance for his drives. Compute the total number of miles that Carlos drove.

S00J10. Compute all values of m for which one of the solutions of $x^2+(2m+1)x+m^2+2m+3=0$ is twice the other.

PART III: 10 minutes | **NYCIML Contest Two** | **Spring 2000**

S00J11. Find all integers x such that $x^{(x+1)^2}=x^{16}$.

S00J12. Points A, B, and C are chosen on a unit circle so that there exist congruent circles, each with radius r, with centers at A, B, and C, that are pairwise externally tangent. Compute r.

ANSWERS:
J7. 4
J8. 50
J9. 400
J10. -5
J11. –5,-1,0,1,3
J12. $\frac{\sqrt{3}}{2}$

New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER THREE** | **SPRING 2000**

PART I: 10 minutes | **NYCIML Contest Three** | **Spring 2000**

S00J13. Nine years from now, Sam will be twice as old as he was three years ago. Compute Sam's present age.

S00J14. Compute the difference between the sum of the positive integral factors of 66 and the sum of the positive integral factors of 70.

PART II: 10 minutes | **NYCIML Contest Three** | **Spring 2000**

S00J15. Jennifer goes to the store and buys a bag of jellybeans. She meets Melanie on the way home and she gives Melanie ½ her jelly beans and two more. She now meets Sue and she gives Sue ½ her jelly beans and two more. She now meets Eddie and she gives Eddie ½ her jelly beans and two more. When she gets home she has only 2 jellybeans left. Compute the number of jellybeans she bought at the store.

S00J16. In right triangle ABC side $\overline{AB}$ is the hypotenuse. The perpendicular from C to the angle bisector of angle BAC meets $\overline{AB}$ at P. If the measure of angle ABC is $40°$, compute the measure of angle ACP.

PART III: 10 minutes | **NYCIML Contest Three** | **Spring 2000**

S00J17. The units digit of the difference of the squares of two integers is 3. Compute the units digit of the sum of the squares of the two integers.

S00J18. A square has **opposite** vertices at (6,9) and (10,3). Compute the coordinates of the other two vertices.

ANSWERS:
J13. 15
J14. 0
J15. 44
J16. 65°
J17. 5
J18. (5,4), (11,8)

New York City
Interscholastic
Mathematics
League

JUNIOR DIVISION | **CONTEST NUMBER ONE SOLUTIONS** | **SPRING 2000**

S00J1. 1 radio=100 fish=250 coconuts = **625** bananas.

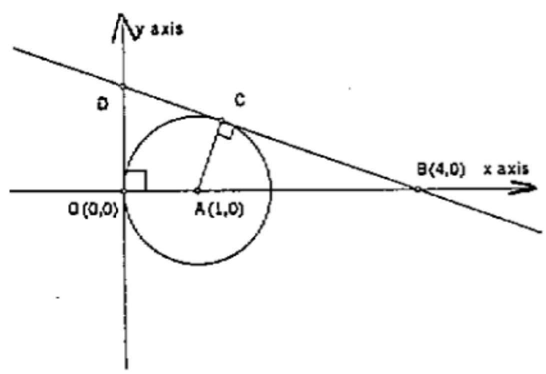
S00J2. There are 6^5 possible results of the throws. The five numbers rolled must be increasing, so we can leave out any one of 1,2,3,4,5,6. Thus the probability is $\frac{6}{6^5}=\frac{1}{1296}$.

S00J3. Each game corresponds to a pair of club members. There are ${}_{22}C_2$=**231** pairs.

S00J4. Heron's formula gives 84 as the area of the triangle. Thus, $\frac{1}{2}bh=84$, therefore bh=168. The only side length that divides 168 is 14 and the height to this side is **12**.

S00J5. Arthur can arrange his coins in stacks of 3 quarters and 2 dimes. Each stack is worth $0.95, and so Arthur will have 25 such stacks, or **75 quarters and 50 dimes**. Or, let 3X be the number of quarters and 2X be the number of dimes, 75X+20X=2375, 95X=2375, X=25.

S00J6. ΔABC and ΔDBO are similar, so $\frac{AC}{CB}=\frac{DO}{OB}$. OB=4, AC=1, AB=3, so CB=$2\sqrt{2}$. This gives DO=$\sqrt{2}$, and that is the value of the y-intercept. The other tangent will have a y-intercept of $-\sqrt{2}$, and the absolute value of either is $\sqrt{2}$.



New York City
Interscholastic
Mathematics
League

JUNIOR DIVISION | **CONTEST NUMBER TWO SOLUTIONS** | **SPRING 2000**

S00J7. Let S be the sum of Deanna's test scores, N be the number of tests Deanna took. $\frac{S+88}{N+1}=87$, $\frac{S+78}{N+1}=85$, $S+88=87(N+1)$, $S+78=85(N+1)$, $2(N+1)=10$, and $N=$**4**.

S00J8. 99 lbs of the original watermelon was water, 1 lb was non-water. Let W be the new weight of the water in the watermelon. $0.98\bullet(W+1)=W$, so $W=49$. The new weight of the watermelon is 49+1=**50**.

S00J9. Let x be the distance from Carrotville to Datetown. The total distance is $200+120+x \rightarrow x=\frac{1}{5}(200+120+x) \rightarrow 5x=x+320 \rightarrow x=80$ and the total distance is **400 miles.**

S00J10. Let a and $2a$ be the solutions to the equation. $a+2a=-(2m+1)$, $a=\frac{-(2m+1)}{3}$.
$a\bullet 2a=2a^2=m^2+2m+3$, $2(\frac{-(2m+1)}{3})^2=(m^2+2m+3)$, $m^2+10m+25=0$, so $m=$ **-5.**

S00J11. Checking for obvious solutions first, we find that $x=0$, $x=1$, $x=-1$ all work. For other values of x, equality will hold only if $(x+1)^2=16$, giving $x=3$ and $x=-5$ as the other answers. Thus there are five solutions: **-5, -1, 0, 1, 3.**

S00J12. The centers of the three circles are the vertices of an equilateral triangle with side $2r$. This triangle has a median $r\sqrt{3}$. The intersection of the medians marks the center of the unit circle, and is two-thirds of the way along each median from the vertex. Thus $\frac{2}{3}r\sqrt{3}=1$, and $r=\frac{\sqrt{3}}{2}$.

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER THREE SOLUTIONS** **SPRING 2000**

S00J13. Let a be Sam's age now. $a+9=2(a-3)$, $a=$**15.**

S00J14. $66=2\cdot3\cdot11$, so the sum of its positive integral factors is $(2+1)\cdot(3+1)\cdot(11+1)=144$. $70=2\cdot5\cdot7$, so the sum of its positive integral factors is $(2+1)\cdot(5+1)\cdot(7+1)=144$. The difference is **0**.

S00J15. This problem can be solved algebraically by letting the number of jellybeans purchased be X. The final equation to solve would be $\frac{x}{8}-\frac{7}{2}=2, \rightarrow x=44$. Working backwards is an alternate method of solution. Since she had 2 jellybeans when she got home, we can add two and double and see that she had 8 when she met Eddie. Continuing in this manner gives us the solution of **44**.

S00J16. Let Q be the point of intersection of the angle bisector of angle BAC and the perpendicular to it from C. Since angle ACB is a right angle, the measure of angle BAC is $50°$. This makes the measure of angle CAQ=$25°$, and since CAQ is a right triangle, the measure of angle ACP =**65°**.

S00J17. . Perfect squares can have unit digits 0, 1, 4, 5, 6, or 9. Those pairs that have a difference of 3 are, 4 with 1, and 9 with 6. Either combination would cause the units digit of the sum of the squares to be **5**.

S00J18. The midpoint of the segment connecting the vertices, (8,6), is the center of the square. To go from (8,6) to (6,9) you have to go 2 units left and 3 units up. To go from (8,6) to (10,3) you have to go 2 units right and 3 units down. Therefore the other two vertices will be located at 2 units up and 3 units right, and at 2 units down and 3 units left. They are **(11,8)** and **(5,4).**