New York City
Interscholastic
Mathematics
League

SENIOR B DIVISION **CONTEST NUMBER ONE** **FALL 1999**

PART I: 10 minutes **NYCIML Contest One** **Fall 1999**

F99SB1. Compute $\log_2 8\sqrt{2}$ with no logarithms in the answer.

F99SB2. Solve for x: $2x+\sqrt{x}-1=0$

PART II: 10 minutes **NYCIML Contest One** **Fall 1999**

F99SB3. How many integers greater than 4000 can be formed using the digits 1, 3, 5, 7 without repetition?

F99SB4. Two circles with radii 3 and 12 are externally tangent. A line, not through the point of tangency of the two circles, is tangent to both circles. Compute the distance between the two tangent points on the line.

PART III: 10 minutes **NYCIML Contest One** **Fall 1999**

F99SB5. Two different integers are chosen from the integers between 1 and 20, inclusive. Compute the probability that their sum is odd.

F99SB6. How many obtuse triangles, whose sides have integral length, have two sides of length 9 and 12?

ANSWERS:
1. 7/2
2. 1/4
3. 12
4. 12
5. 10/19
6. 9

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SENIOR B DIVISION **CONTEST NUMBER TWO** **FALL 1999**

PART I: 10 minutes **NYCIML Contest TWO** **Fall 1999**

F99SB7. If $\frac{10^{30}}{20^{15}}$ is reduced to p^q, where p is a prime number, compute $p+q$.

F99SB8. Equilateral triangle ABC has sides of length 12. Point A is on a circle, $\overline{BC}$ is tangent to the circle at M, the midpoint of $\overline{BC}$. If D is the point of intersection of the circle and $\overline{AC}$, compute DC.

PART II: 10 minutes **NYCIML Contest TWO** **Fall 1999**

F99SB9. David, Jan, Arthur, and Zardosht are lined up randomly in a row. Compute the probability that David and Jan are next to each other.

F99SB10. Find the sum of all proper fractions whose denominator is less than or equal to 50. $\left(\frac{1}{2}+\frac{1}{3}+\frac{2}{3}+\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+...+\frac{49}{50}\right)$.

PART III: 10 minutes **NYCIML Contest TWO** **Fall 1999**

F99SB11. The sides of a triangle are 5, 12, 13. Compute the length of the shortest altitude in the triangle.

F99SB12. Compute the sum of the infinite series $\frac{1}{7}+\frac{2}{7^2}+\frac{3}{7^3}+...+\frac{n}{7^n}+...$

ANSWERS:
7. 20
8. 3
9. 1/2
10. 1225/2
11. 60/13
12. 7/36

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SENIOR B DIVISION **CONTEST NUMBER THREE** **FALL 1999**

PART I: 10 minutes **NYCIML Contest THREE** **Fall 1999**

F99SB13. In a group of 32 students, 17 students take French and 19 students take biology. If 11 students take both French and biology, how many take neither?

F99SB14. The equations $y = 2x^2$ and $y = -4x + k$ have exactly 1 common solution. Compute k.

PART II: 10 minutes **NYCIML Contest THREE** **Fall 1999**

F99SB15. Compute the smallest positive integer x for which $12250x$ is the square of an integer.

F99SB16. In a rhombus, one diagonal is twice the length of the other. If the area of the rhombus is A, express the perimeter of the rhombus in terms of A in simplest form.

PART III: 10 minutes **NYCIML Contest THREE** **Fall 1999**

F99SB17. Find the sum of the coefficients of the terms in the expansion of $(3x-1)^7$.

F99SB18. A triangle with sides 10, 12, and 12 is inscribed in a circle. Compute the radius of the circle.

ANSWERS:
13. 7
14. -2
15. 10
16. $2\sqrt{5A}$
17. 128
18. $\dfrac{72\sqrt{119}}{119}$

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SENIOR B DIVISION | CONTEST NUMBER FOUR | FALL 1999

PART I: 10 minutes **NYCIML Contest FOUR** **Fall 1999**

F99SB19. Compute the length of a chord which is 5 units from the center of a circle with radius 8.

F99SB20. If p and q are the roots of $3x^2-11x+4=0$, compute p^3q+pq^3.

PART II: 10 minutes **NYCIML Contest FOUR** **Fall 1999**

F99SB21. Compute the volume of a rectangular solid if the areas of three of its faces are 20, 25 and 80.

F99SB22. Compute the area of a regular hexagon that is circumscribed around a circle of radius 12.

PART III: 10 minutes **NYCIML Contest FOUR** **Fall 1999**

F99SB23. Compute $\sin 195^\circ$ in simplest radical form.

F99SB24. To the nearest second, what is the first time after midnight that the hands of the clock will be perpendicular?

ANSWERS:
19. $2\sqrt{39}$
20. 388/27
21. 200
22. $288\sqrt{3}$
23. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$
24. 12:16:22

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SENIOR B DIVISION	CONTEST NUMBER FIVE	FALL 1999

PART I: 10 minutes **NYCIML Contest FIVE** **Fall 1999**

F99SB25. A man can paint a room in 8 hours while his wife would take 6 hours to paint the same room. Compute the number of hours it will take them to paint the room if they work together.

F99SB26. Compute all possible ordered pairs of positive integers (x,y) such that $x^2 - y^2 = 45$.

PART II: 10 minutes **NYCIML Contest FIVE** **Fall 1999**

F99SB27. In triangle ABC, tan A=1/4, tan B=3/4. Compute tan C.

F99SB28. Compute the coefficient of x^4 in the expansion of $(x-2)^7$.

PART III: 10 minutes **NYCIML Contest FIVE** **Fall 1999**

F99SB29. What is the units digit of 37^{73}?

F99SB30. 3531_b is the square of 51_b where b is the base. Find b.

ANSWERS:
25. 24/7
26. (7,2), (9,6), (23,22)
27. –(16/13)
28. -280
29. 7
30. 7

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SENIOR B DIVISION **CONTEST NUMBER ONE** **FALL 1999**

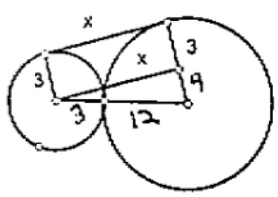
SOLUTIONS

F99SB1. Let $\log_2 8\sqrt{2} = x$. $2^x = 8\sqrt{2} = 2^{\frac{7}{2}} \rightarrow x = \frac{7}{2}$

F99SB2. Let $y = \sqrt{x} \rightarrow 2y^2 + y - 1 = 0 \rightarrow (2y-1)(y+1) = 0 \rightarrow y = -1, y = \frac{1}{2} \rightarrow x = \frac{1}{4}$ is the only solution.

F99SB3. There are 2 digits that can go in the thousands place. Once that is chosen, the remaining 3 can be arranged in 3! ways. 2x3!=12.

F99SB4. Draw a perpendicular from the center of the small circle to the radius of the larger as shown. $9^2 + x^2 = 15^2 \rightarrow x = 12$



F99SB5. One integer must be odd, the other even. The number of such pairs is 10x10=100. The number of all possible pairs is ${}_{20}C_2$, thus the probability is $\frac{10}{19}$.

F99SB6. Since c>a+b, the only possible triangle will have x, the third side $4 \le x \le 20$. In order to be obtuse, the largest side c must satisfy $c^2 > a^2 + b^2$. This will happen if the third side is 4, 5, 6, 7 or 16, 17 18, 19, 20. There are 9 values.

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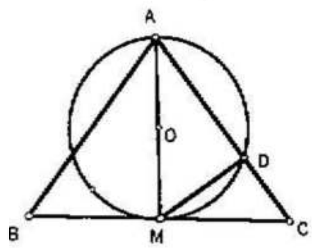
SENIOR B DIVISION **CONTEST NUMBER TWO** **FALL 1999**

SOLUTIONS

F99SB7. $\frac{10^{30}}{20^{15}} = \frac{2^{30}5^{30}}{2^{30}5^{15}} = 5^{15} \rightarrow p + q = 20$

F99SB8. Draw MD. $\angle$ADM=90° (inscribed in a semicircle), $\angle$C=60° MC=6, CD=3



F99SB9. Half of the time they are next to each other, thus the answer is ½. (you can examine the sample space)

F99SB10. Combining all fractions with like denominators, this becomes an arithmetic progression, first term $\frac{1}{2}$, common difference $\frac{1}{2}$, 49 terms.

$S = \frac{49}{2}\left(2 \bullet \frac{1}{2} + 48\frac{1}{2}\right) = \frac{49}{2}(25) = \frac{1225}{2}$

F99SB11. Since this is a right triangle, the shortest altitude will be the altitude to the hypotenuse. Since the area $= \frac{1}{2} \bullet 5 \bullet 12 = 30$, $\frac{1}{2}13h = 30 \rightarrow h = \frac{60}{13}$

F99SB12. Set this up as the sum of an infinite number of infinite series.

$\left(\frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots\right) + \left(\frac{1}{7^2} + \frac{1}{7^3} + \frac{1}{7^4} + \ldots\right) + \left(\frac{1}{7^3} + \frac{1}{7^4} + \ldots\right) + \ldots$ The sum of the first is $\frac{\frac{1}{7}}{1 - \frac{1}{7}} = \frac{1}{6}$. The sum of the second is $\frac{\frac{1}{49}}{1 - \frac{1}{7}} = \frac{1}{42}$ The total sum is $\frac{\frac{1}{6}}{1 - \frac{1}{7}} = \frac{7}{36}$

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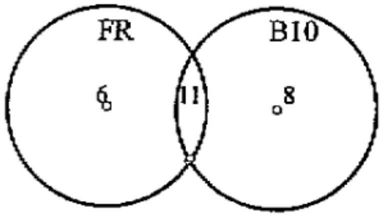
SENIOR B DIVISION **CONTEST NUMBER THREE** **FALL 1999**

SOLUTIONS

F99SB13. The easiest way to do this is with a Venn diagram



32-25=7

F99SB14. $y = 2x^2 \quad y = -4x + k \quad 2x^2 = -4x + k \quad 2x^2 + 4x - k = 0$ This will have 2 identical solutions if the discriminant is 0, 16+8k=0, k=-2

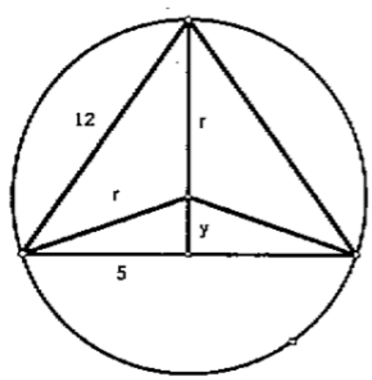
F99SB15. $12250 = 2^1 5^3 7^2$ To be a square all exponents must be even. $x = 2 \bullet 5 = 10$

F99SB16. Let one diagonal = 2x, the other 4x. The area = $\frac{1}{2} 2x \bullet 4x = 4x^2$ A side of the rhombus is $\sqrt{x^2 + 4x^2} = \sqrt{5x^2} = \sqrt{\frac{5A}{4}} = \frac{\sqrt{5A}}{2}$ Perimeter =4s= $\frac{4\sqrt{5A}}{2} = 2\sqrt{5A}$

F99SB17. The sum of the coefficients can be found by letting x=1. $(3 \bullet 1 - 1)^7 = 2^7 = 128$

F99SB18.

$$h = \sqrt{144 - 25} = \sqrt{119}$$
$$r^2 = y^2 + 25 = 25 + (h - r)^2 = 25 + h^2 - 2hr + r^2$$
$$2hr = 25 + h^2 = 25 + 119 = 144$$
$$r = \frac{144}{2h} = \frac{72}{\sqrt{119}} = \frac{72\sqrt{119}}{119}$$



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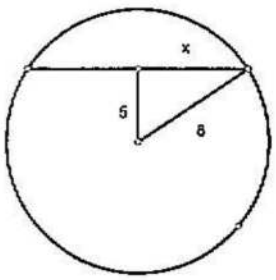
SENIOR B DIVISION **CONTEST NUMBER FOUR** **FALL 1999**

SOLUTIONS

F99SB19. $x = \sqrt{39}$ chord=$2\sqrt{39}$



F99SB20.

$$p^3q + pq^3 = pq(p^2 + q^2)$$

$$p^2 + q^2 = (p+q)^2 - 2pq = \left(\frac{11}{3}\right)^2 - 2\left(\frac{4}{3}\right) = \frac{121}{9} - \frac{8}{3} = \frac{97}{9}$$

$$pq(p^2 + q^2) = \frac{4}{3} \times \frac{97}{9} = \frac{388}{27}$$

F99SB21. Let the edges be a, b and c

ab=20 bc=25 ac=80

$a^2b^2c^2 = 4 \times 5 \times 5 \times 5 \times 16 \times 5$

$abc = 8 \times 5 \times 5 = 200$

F99SB22. Using a 30-60-90 triangle a side of the hexagon is $2 \times 4\sqrt{3} = 8\sqrt{3}$

Area of hexagon=$\frac{3}{2}s^2\sqrt{3} = \frac{3}{2}192\sqrt{3} = 288\sqrt{3}$

F99SB23. $\sin 195° = \sin(150 + 45)$ =sin150cos45+cos150sin45

$= \frac{1}{2} \bullet \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \bullet \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$

F99SB24. The minute hand moves 12 times as fast as the hour hand. If the minute hand moves 12x°, the hour hand moves x° 12x-x=90 11x=90 x=$\frac{90}{11}$ $12x = \frac{12 \bullet 90}{11}$

Since $6° = 1$ minute, the answer is $\frac{(90)(12)}{(11)(6)} = \frac{180}{11} = 16\frac{4}{11} = 16{:}22$ after 12

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SENIOR B DIVISION **CONTEST NUMBER FIVE** **FALL 1999**

SOLUTIONS

F99SB25. Let $\frac{x}{8}$ be the portion the man does, $\frac{x}{6}$ his wife.

$\frac{x}{8}+\frac{x}{6}=1 \rightarrow 7x=24 \rightarrow x=\frac{24}{7}$

F99SB26. (x+y)(x-y)=45. The pairs of numbers whose product is 45 are 1x45, 3x15, 5x9 These produce (23,22),(9,6) and (7,2)

F99SB27. $\tan(A+B)=\dfrac{\frac{1}{4}+\frac{3}{4}}{1-\frac{1}{4}\bullet\frac{3}{4}}=\dfrac{1}{\frac{13}{16}}=\dfrac{16}{13}$ C=180-(A+B) tanC=tan(180-(A+B))

=-tan(A+B) tanC=$\dfrac{-16}{13}$

F99SB28. The x^4 term is ${}_7C_4x^4(-2)^3=-280x^4$

F99SB29. The powers of 7 end in the digits 7,9,3,1 and repeat in cycles of 4. Since 73 is 1 more than a multiple of 4, $(37)^{73}$ ends with a 7.

F99SB30.

$3b^3+5b^2+3b+1=(5b+1)^2=25b^2+10b+1$

$3b^3-20b^2-7b=0$

$b(3b+1)(b-7)=0$

b=7 is the only possible answer.