NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER ONE**

PART I *FALL, 1999* *CONTEST 1* *TIME: 10 MINUTES*

F99S1 In simplest terms compute the value of: $\dfrac{1^{3x+6}+(-1)^{2x-6}}{16^{-(2^{-2})}+27^{-(3^{-1})}}$

F99S2 If $x^2y = xy^2 + 30$ and $x^3y = xy^3 + 210$, compute $2x^3 - 21x^2 + 49x$.

PART II *FALL, 1999* *CONTEST 1* *TIME: 10 MINUTES*

F99S3 If $f(a) + f(b) = f(ab)$ for all a, b, compute $f(1999)$.

F99S4 The lengths of the bases of an isosceles trapezoid are 18 and 30, and the length of a diagonal is 26. Compute its area.

PART III *FALL, 1999* *CONTEST 1* *TIME: 10 MINUTES*

F99S5 Compute all x that satisfy $\dfrac{(x^2-11x+28)(x^2-11x+30)}{\sqrt{x-5}} = 0.$

F99S6 A circle is inscribed in a right triangle so that the point of tangency divides the hypotenuse into two segments whose lengths have a product of 1999. Compute the area of the triangle, in simplest terms.

ANSWERS:

F99S1	$\frac{12}{5}$
F99S2	-30
F99S3	0
F99S4	240
F99S5	4, 6, 7
F99S6	1999

New York City Interscholastic Math League

Senior A Division | **Contest Number Two**

Part I *Fall, 1999* *Contest 2* *Time: 10 Minutes*

F99S7 The nation of Sapfu has its own monetary unit, the tarfu, which is written in base z, $z \neq 10$. Shopping in the Sapfu Mall, Mr. Fu-Fu buys a shirt for 43 tarfus and a jacket for 44 tarfus. His change from a 122-tarfu bill is 24 tarfus. Compute z.

F99S8 If $x^3y = xy^3 + 60$ and $x^3y^2 = x^2y^3 + 48$, compute $\frac{1}{x} + \frac{1}{y}$.

Part II *Fall, 1999* *Contest 2* *Time: 10 Minutes*

F99S9 Statements **A** and **B** are independent of each other. The probability that **A** is true is $\frac{\sqrt{3}}{4}$. The probability that **B** is true is $\frac{1}{\sqrt{3}}$. Compute the probability that $\mathbf{A} \to \mathbf{B}$ is true.

F99S10 There exists three non congruent rectangles such that the area of each can be doubled by increasing both the integral length and integral width by 3. If c and d represent the width and length respectively of the three original rectangles, compute the three ordered pairs (c, d), where c < d.

Part III *Fall, 1999* *Contest 2* *Time: 10 Minutes*

F99S11 If $\log 24 = x$ and $\log 250 = y$, express $\log \sqrt{6}$ in terms of x and y, in simplest terms, and without logarithms.

F99S12 The lengths of two adjacent sides of a parallelogram are 9 and 7. The lengths of its diagonals are 14 and x. Compute x.

Answers:		
	F99S7	9
	F99S8	$\frac{5}{4}$
	F99S9	$\frac{5-\sqrt{3}}{4}$
	F99S10	(4,21), (5,12), (6,9)
	F99S11	$\frac{x+y-3}{2}$
	F99S12	8

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER THREE**

PART I *FALL, 1999* *CONTEST 3* *TIME: 10 MINUTES*

F99S13 If 75,1A4,6B2 is divisible by 264, what is the value of A and of B?

F99S14 Find both pairs of coordinates of the intersections of the graphs of $(x+4)^2+(y-3)^2=225$ and $(x-10)^2+(y-3)^2=169$.

PART II *FALL, 1999* *CONTEST 3* *TIME: 10 MINUTES*

F99S15 Three positive numbers form an arithmetic progression. If the smallest term is increased by 1, a geometric progression is formed. If the largest term of the arithmetic progression is increased by 2, a different geometric progression is formed. What is the arithmetic progression?

F99S16 Given right triangle ABC and Square ABED, with C outside the square. If $\angle C$ is a right angle, AC = 3, and BC = 4, compute CD.

PART III *FALL, 1999* *CONTEST 3* *TIME: 10 MINUTES*

F99S17 The average of n positive integers is n. If no two integers are equal, express in simplest terms the greatest value that any one integer can have, in terms of n.

F99S18 In circle 0, two tangents intersect a diameter at its endpoints. A third tangent intersects the circle at A and the other two tangents at B and C. If BO = $4\sqrt{2}$ and CO = $5\sqrt{3}$, compute the ratio $AB:AC$.

ANSWERS:

	F99S13	A = 2, B = 3
	F99S14	(5,15), (5,–9)
	F99S15	8, 12, 16
	F99S16	$\sqrt{58}$
	F99S17	$\frac{n^2+n}{2}$
	F99S18	32 : 75

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER FOUR**

PART I *FALL, 1999* *CONTEST 4* *TIME: 10 MINUTES*

F99S19 Mr. Meyers sells three types of kneepads: skimpy, regular, and luxury. Last year he sold 1999 skimpy kneepads at \$10 each, 3998 regular kneepads at \$16 each, and 5997 luxury kneepads at \$22 each. Compute the average price of these kneepads, in dollars.

F99S20 Compute the value of:
$1995 \cdot 1997^2 + 1997 \cdot 1999^2 + 1999 \cdot 1995^2 - 1995^2 \cdot 1997 - 1997^2 \cdot 1999 - 1999^2 \cdot 1995$.

PART II *FALL, 1999* *CONTEST 4* *TIME: 10 MINUTES*

F99S21 Given that $x \neq y$ and that $f(x,y) = k \cdot f(y,x)$ for all x and y. Compute all values of k.

F99S22 Point P is located in the interior of rectangle ABCD. Its distances to points A, B, C, and D are 3 cm, 5 cm, 7 cm, and x cm, respectively. Compute x.

PART III *FALL, 1999* *CONTEST 4* *TIME: 10 MINUTES*

F99S23 The average of n positive integers is n, where n is odd. No two integers are equal. If the smallest of these integers, s, is as large as possible, express s in terms of n and in simplest terms.

F99S24 Quadrilateral ABCD is inscribed in circle O. If $\tan B = x$, where $0 < B < \frac{\pi}{2}$, express in terms of x the value of $\sin A + \cos B - \sin C - \cos D$, in simplest terms.

ANSWERS:		
	F99S19	18
	F99S20	16
	F99S21	± 1
	F99S22	$\sqrt{33}$
	F99S23	$\frac{1}{2}(n+1)$
	F99S24	$\frac{2}{\sqrt{x^2+1}}$ or $\frac{2\sqrt{x^2+1}}{x^2+1}$

New York City Interscholastic Math League

Senior A Division | **Contest Number Five**

Part I *Fall, 1999* *Contest 5* *Time: 10 Minutes*

F99S25 Andy's and Brandy's ages are in a ratio of 2:3. Brandy's and Candy's ages are in a ratio of 4:5. The sum of Andy's, Brandy's, and Candy's ages is 105 years. How old is Candy, in years?

F99S26 Compute the largest value of n such that 2^n is a factor of 100!.

Part II *Fall, 1999* *Contest 5* *Time: 10 Minutes*

F99S27 In base b, $\frac{1}{4}$ and $.\overline{15}$ represent the same number. Compute b.

F99S28 In a right triangle the hypotenuse has length c and the altitude to that hypotenuse has length h. If θ is the measure of the smallest angle of the triangle, express in terms of c and h the value of $\cos 2\theta$ in simplest terms.

Part III *Fall, 1999* *Contest 5* *Time: 10 Minutes*

F99S29 Compute all values of x for which $\left(x-2\sqrt{x}-6\right)^2+\left(x-2\sqrt{x}-6\right)+1=7$.

F99S30 In trapezoid ABCD, the bases are $\overline{AB}$ and $\overline{DC}$, and diagonals $\overline{AC}$ and $\overline{BD}$ meet at E. A line FG is drawn through point E and is parallel to $\overline{AB}$ such that point F is on $\overline{AD}$ and point G is on $\overline{BC}$. If AB = 5 and DC = 20, compute FG.

Answers:

	F99S25	45
	F99S26	97
	F99S27	7
	F99S28	$\sqrt{\frac{c^2-4h^2}{c^2}}$ or equivalent.
	F99S29	9,16
	F99S30	8

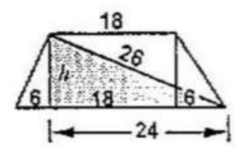
SOLUTIONS

F99S1 ANSWER: $\frac{12}{5}$. $\frac{1^{3x+6}+(-1)^{2x-6}}{16^{-(2^{-2})}+27^{-(3^{-1})}}=\frac{1+1}{16^{-\frac{1}{4}}+27^{-\frac{1}{3}}}=\frac{2}{\frac{1}{2}+\frac{1}{3}}=\frac{12}{5}$.

F99S2 ANSWER: -30. $x^2y=xy^2+30 \Rightarrow x^2y-xy^2=30 \Rightarrow xy(x-y)=30$. Also, $x^3y=xy^3+210 \Rightarrow x^3y-xy^3=210 \Rightarrow xy(x-y)(x+y)=210$. By substitution, $(30)(x+y)=210 \Rightarrow y=7-x$. The required expression has the variable x only, so substitute again: $x^2y=xy^2+30 \Rightarrow x^2(7-x)-x(7-x)^2=30 \Rightarrow 2x^3-21x^2+49x=-30$.

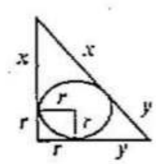
F99S3 ANSWER: 0. Suppose $a=0$. Then $f(a)+f(b)=f(ab) \Rightarrow f(0)+f(b)=f(0) \Rightarrow f(b)=0$ for all b. [Similarly, if $b=0$, then $f(a)=0$ for all a.] Thus, $f(1999)=0$.

F99S4 ANSWER: 240. Draw two altitudes, as shown in the diagram. Then the segments of the larger base are 6, 18 and 6. A right triangle is formed whose hypotenuse has length 26 and whose legs have length 24 and h. Then from the 5-12-13 Pythagorean Triple, $h=10$. Therefore the area of the trapezoid is $\frac{1}{2}(10)(18+30)=240$.



F99S5 ANSWER: 4, 6, 7 Multiply both sides by $\sqrt{x-5}$. Then $(x^2-11x+28)(x^2-11x+30)=(x-4)(x-5)(x-6)(x-7)=0 \Rightarrow x=4, 5, 6, 7$.
However $x=5$ does not check in the original equation. Therefore the solution is $x=4, 6, 7$.

F99S6 ANSWER: 1999. From the diagram, $(x+r)^2+(y+r)^2=(x+y)^2 \Rightarrow x^2+2xr+r^2+y^2+2yr+r^2=x^2+2xy+y^2 \Rightarrow r^2+xr+yr=xy \Rightarrow r^2+xr+yr+xy=2xy \Rightarrow (r+x)(r+y)=2xy$. But the area of the triangle is $(\frac{1}{2})(r+x)(r+y)$, which equals xy. Thus, any circle inscribed in a right triangle divides the hypotenuse into two segments whose product is equal to the area of the triangle. Since the product of the segments of the hypotenuse is given as 1999, the area of the triangle is also 1999.



SOLUTIONS

F99S7 **ANSWER: 9.** Because 4 is a digit in base z, $z > 4$. Since in the ones column $3 + 4 + 4$ ends in a 2, the base is 9 or a factor of 9. However, 9 is the only factor greater than 4, so $z = 9$. Thus 1 is carried to the second column. Then (to check): $1 + 4 + 4 + 2 = 11$ (in base 10) which is 12 in base 9.

F99S8 **ANSWER: $\frac{5}{4}$.** Divide $x^3y - xy^3 = 60$ by $x^3y^2 - x^2y^3 = 48$:
$\frac{60}{48} = \frac{x^3y - xy^3}{x^3y^2 - x^2y^3} = \frac{xy(x-y)(x+y)}{x^2y^2(x-y)} = \frac{x+y}{xy} = \frac{1}{x} + \frac{1}{y}$. Thus $\frac{1}{x} + \frac{1}{y} = \frac{60}{48} = \frac{5}{4}$.

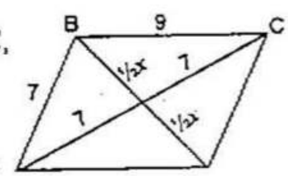
F99S9 **ANSWER:** $\frac{5-\sqrt{3}}{4}$. Set up the truth table as at the right. The probability that A implies B is a true statement is $\frac{3}{12} + \frac{4\sqrt{3}-3}{12} + \frac{15-7\sqrt{3}}{12} = \frac{15-3\sqrt{3}}{12}$ *or* $\frac{5-\sqrt{3}}{4}$. Alternately, $1 - \frac{3\sqrt{3}-3}{12} = 1 - \frac{\sqrt{3}-1}{4} = \frac{5-\sqrt{3}}{4}$.

A	B	A → B
T	T	T : $\frac{\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{3} = \frac{3}{12}$
T	f	f : $\frac{\sqrt{3}}{4} \cdot \frac{3-\sqrt{3}}{3} = \frac{3\sqrt{3}-3}{12}$
f	T	T : $\frac{4-\sqrt{3}}{4} \cdot \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}-3}{12}$
f	f	T : $\frac{4-\sqrt{3}}{4} \cdot \frac{3-\sqrt{3}}{3} = \frac{15-7\sqrt{3}}{12}$

F99S10 **ANSWER: (4,21), (5,12), (6,9).** For original length ℓ and width w, $(\ell + 3)(w + 3) = 2\ell w \Rightarrow \ell w - 3\ell - 3w - 9 = 0 \Rightarrow \ell w - 3\ell - 3w + 9 = 18 \Rightarrow (\ell - 3)(w - 3) = 18$. But $18 = 1 \cdot 18 = 2 \cdot 9 = 3 \cdot 6$. Then: the factor pair (1,18) produces "before and after" areas of $4 \times 21 = 84$ and $7 \times 24 = 168$, (2,9) produces $5 \times 12 = 60$ and $8 \times 15 = 120$, and (3,6) produces $6 \times 9 = 54$ and $9 \times 12 = 108$.

F99S11 **ANSWER: $\frac{1}{2}(x + y - 3)$** $x + y = \log 24 + \log 250 = \log(24 \cdot 250) = \log 6000 = \log 6 + \log 1000$. Then $\log 6 = x + y - 3$. Finally, $\log\sqrt{6} = \frac{1}{2}\log 6 = \frac{1}{2}(x + y - 3)$.

F99S12 **ANSWER: 8. METHOD 1:** Using Stewart's Theorem with ΔABC, $7^2 \cdot 7 + 9^2 \cdot 7 = 14(\frac{1}{2} \cdot x)^2 + 14 \cdot 7 \cdot 7$. Solving yields $x = 8$.
METHOD 2: In any parallelogram whose side-lengths are a and b, and whose diagonal-lengths are d_1 and d_2, $d_1^2 + d_2^2 = 2(a^2 + b^2)$.
Thus, $x^2 + 14^2 = 2(7^2 + 9^2)$. Solving yields $x = 8$.



SOLUTIONS

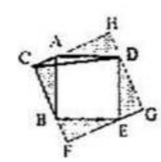
F99S13 **ANSWER: A=2, B=3.** Since $264 = 2^3 \cdot 3 \cdot 11$, any number divisible by 264 is divisible by 8, 3, and 11. Use the test of divisibility for 11: the difference between (7+1+4+B) and (5+A+6+2) = B – A – 1 is also divisible by 11. Thus, A = B – 1. Use the test of divisibility for 8: since 8|1000, 8|75,1A4,000. Then 8|6B2 $\Rightarrow$ B = 3 or 7 $\Rightarrow$ A = 2 or 6. Use the test of divisibility for 3: Since 3|(7+5+1+A+4+6+B+2) $\Rightarrow$ 3|(25+A+B) $\Rightarrow$ 3|(1+A+B), test (2,3) and (6,7): 3|(1+2+3) [yes!] and 3|(1+6+7) [no]. Only A=2 and B=3 works. The number is 75,124,632.

F99S14 **ANSWER: (5,15) and (5,–9).** **METHOD 1** — The line of centers of the two circles is horizontal. The radii are 13 and 15, and the distance between the centers is 14, thus forming the well-known 13-14-15 triangle. This triangle can be split into two right triangles whose side-lengths are 9, 12, 15, and 5, 12, 13. Thus the altitude to side 14 meets that side at 9 units to the right of (–4,3), which is (5,3). The length of the altitude is 12, so the circles intersect at (5,3 ± 12) = (5,15) and (5,–9).

METHOD 2 — Subtract the two given equations to get $(x+4)^2 - (x-10)^2 = 15^2 - 13^2$ $\Rightarrow$ $x = 5$. Then $9^2 + (y-3)^2 = 225$ $\Rightarrow$ $(y-3)^2 = 144$ $\Rightarrow$ $y - 3 = \pm 12$ $\Rightarrow$ $y = 3 \pm 12$

F99S15 **ANSWER: 8, 12, 16.** let the terms of the AP be $b - d$, b, and $b + d$, where d represents the common difference. Then: $\frac{(b-d)+1}{b} = \frac{b}{b+d}$ $\Rightarrow$ $b = d^2 - d$. Also $\frac{b-d}{b} = \frac{b}{(b+d)+2}$ $\Rightarrow$ $b = \frac{1}{2}(d^2 + 2d)$. Equating, $d^2 - d = \frac{1}{2}(d^2 + 2d)$ $\Rightarrow$ $d = 4$ or 0. But substituting, $d = 0$ in the first GP produces 1 = 0, and therefore is impossible. Then $d = 4$ produces $b = 4^2 - 4 = 12$ and the original terms are 8, 12, and 16.

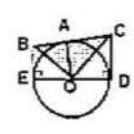
F99S16 **ANSWER: $\sqrt{58}$.** **METHOD 1** — AB = 5 = AD. In right triangle CAB, sin CAB = .8. In triangle CAD, the law of cosines produces $CD^2 = 3^2 + 5^2 - 2 \cdot 3 \cdot 5 \text{ Cos CAD} = 34 - 30\cos(\text{CAB} + 90) = 34 - 30(-\sin \text{CAB}) = 34 - 30(-.8) = 58$. Then $CD = \sqrt{58}$.



METHOD 2 — Replicate right triangle ACB as shown. Then CFGH is also a square. $CD^2 = CH^2 + HD^2 = 7^2 + 3^2 = 58$. Thus $CD = \sqrt{58}$.

F99S17 **ANSWER: $\frac{1}{2}(n^2 + n)$.** The sum of n numbers which average n is n^2. To maximize one integer, minimize each of the others. Thus the sum of the others is $1 + 2 + 3 + \ldots + (n-1) = \frac{1}{2}(n^2 - n)$. Hence, the largest possible integer is $n^2 - \frac{1}{2}(n^2 - n) = \frac{1}{2}(n^2 + n)$.

F99S18 **ANSWER: 32:75.** In quadrilateral BCDE, ∠B and ∠C are supplementary. Draw $\overline{AO}$, $\overline{BO}$, and $\overline{CO}$. ΔBEO ≅ ΔBAO and ΔCDO ≅ ΔCAO by SSS. Thus, ∠EBA and ∠DCA are bisected, making ∠OBA and ∠OCA complementary. Then ΔOBC is a right triangle and OA is the altitude to its hypotenuse. Ergo, $OB^2 = BA \cdot BC$ and $OC^2 = CA \cdot CB$. Divide the equations: $OB^2 : OC^2 = BA : CA$. Hence, $AB:AC = (4\sqrt{2})^2 : (5\sqrt{3})^2 = 32:75$.



SOLUTIONS

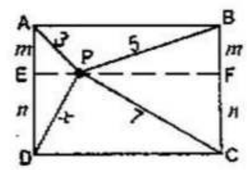
F99S19 **ANSWER: 18.** Let $x = 1999$. Then Mr. Meyers' sales were: skimpy — \$$10x$, regular — $16(2x) = 32x$, and luxury — $22(3x) = 66x$ for a total of $108x$ dollars. The average sale is $108x \div 6x = 18$. Observe that the value of x does not affect the average price.

F99S20 **ANSWER: 16.** Let a = 1995, b = 1997, c = 1999. Note that b − a = 2, c − b = 2 and c − a = 4. Then $ab^2 + bc^2 + ca^2 - a^2b - b^2c - c^2a = (ab^2 - a^2b) + (bc^2 - b^2c) + (ca^2 - c^2a) = ab(b-a) + bc(c-b) - ca(c-a) = 2ab + 2bc - 4ca = (2ab - 2ac) + (2bc - 2ac) = 2a(b-c) + 2c(b-a) = -4a + 4c = 4(c-a) = 16$.

Many methods of factoring this polynomial exist. How many ways can you find?

F99S21 **ANSWER: ±1.** For all x, and all y, $f(x,y) = k \cdot f(y,x) = k \cdot k \cdot f(x,y)$. Thus $f(x,y) = k^2 \cdot f(x,y) \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$.

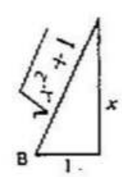
F99S22 **ANSWER: $\sqrt{33}$.** Draw $\overline{EPF} \parallel \overline{AB}$. Then $\overline{EF} \perp \overline{AD}, \overline{BC}$. By the Pythagorean Theorem, $EP^2 = 9 - m^2 = x^2 - n^2$, and $PF^2 = 25 - m^2 = 49 - n^2$. Subtract equations: $9 - 25 = x^2 - 49 \Rightarrow x = \sqrt{33}$.



F99S23 **ANSWER: $\frac{n+1}{2}$.** To maximize the smallest quantity, minimize all other quantities. Then all terms must be consecutive integers, and the middle term (since n is odd) must be n.

Each of $\frac{n-1}{2}$ terms is less than the average and each of $\frac{n-1}{2}$ terms is more than the average. Thus the value of the smallest term is $n - \frac{n-1}{2} = \frac{n+1}{2}$.

F99S24 **ANSWER: $\frac{2}{\sqrt{x^2+1}}$** Since angles A and C are supplementary, sin C = sin A. Since angles B and D are supplementary, cos D = − cos B. Then sin A + cos B − sin C − cos D = sin A + cos B − sin A + cos B = 2 cos B. Since tan B = $x = \frac{x}{1}$ and ∠B is acute, cos B = $\frac{1}{\sqrt{x^2+1}}$ and sin A + cos B − sin C − sin D = 2 cos B = $\frac{2}{\sqrt{x^2+1}}$.



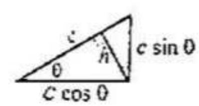
SOLUTIONS

F99S25 **ANSWER: 45.** The LCD $(3,4) = 12$, so let $12x$ represent Brandy's age. Then $8x$ and $15x$ represent Andy's and Candy's ages, respectively $\Rightarrow$ $8x + 12x + 15x = 105$, so $x = 3$. Then Candy is $15 \cdot 3 = 45$ years old.

F99S26 **ANSWER: 97.** First, 100! contains $\left[\frac{100}{2}\right] = 50$ multiples of 2. But each multiple of 4 has an additional factor of 2, so 100! contains $\left[\frac{100}{4}\right] = 25$ additional multiples of 4. Similarly, each multiple of 8 has .. still another factor of 2, so 100! contains $\left[\frac{100}{8}\right] = 12$ additional multiples of 8. Similarly, 100! contains $\left[\frac{100}{16}\right] = 6$ additional multiples of 16, $\left[\frac{100}{32}\right] = 3$ additional multiples of 32, and $\left[\frac{100}{64}\right] = 1$ additional multiple of 64. Then 100! has a total of $50 + 25 + 12 + 6 + 3 + 1 = 97$ factors of 2. Thus, 2^{97} is a factor of 100!, but 2^{98} is not, so $n = 97$,

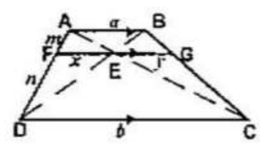
F99S27 **ANSWER: 7.** In base b, $.\overline{15} = \frac{1}{b} + \frac{5}{b^2} + \frac{1}{b^3} + \frac{5}{b^4} + \cdots = \frac{b+5}{b^2} + \frac{b+5}{b^4} + \frac{b+5}{b^6} + \cdots = \frac{\frac{b+5}{b^2}}{1 - \frac{1}{b^2}} = \frac{b+5}{b^2-1}$. $\frac{b+5}{b^2-1} = \frac{1}{4} \Rightarrow$ $b = 7$ or -3. Since $b > 0$, $b = 7$.

F99S28 **ANSWER: $\sqrt{\frac{c^2-4h^2}{c^2}}$ or equivalent.** The area of the triangle is $\frac{1}{2}(ch)$. Since the length of the hypotenuse is c and the measure of the smallest angle is θ, the lengths of the legs are $c\cos\theta$ and $c\sin\theta$. Then the area $= \frac{1}{2}(c\sin\theta)(c\cos\theta) = \frac{1}{2}(ch)$ $\Rightarrow$ $2\sin\theta\cos\theta = \sin 2\theta = \frac{2h}{c}$. Thus, since $\sin^2 2\theta + \cos^2 2\theta = 1$, $\cos 2\theta = \sqrt{1-\left(\frac{2h}{c}\right)^2} = \sqrt{\frac{c^2-4h^2}{c^2}}$.



F99S29 **ANSWER: 9, 16.** Let $a = x - 2\sqrt{x} - 6$. Therefore $a^2 + a + 1 = 7$ $\Rightarrow$ $a = 2, -3$. Then: $x - 2\sqrt{x} - 6 = 2$ or $x - 2\sqrt{x} - 6 = -3$. The former leads to $(\sqrt{x} - 4)(\sqrt{x} + 2) = 0$ $\Rightarrow$ $\sqrt{x} = 4$ (accept) or $\sqrt{x} = -2$ (reject). The latter leads to $(\sqrt{x} - 3)(\sqrt{x} + 1) = 0$ $\Rightarrow$ $\sqrt{x} = 3$ (accept) or $\sqrt{x} = -1$ (reject). Thus $x = 16$ or 9.

F99S30 **ANSWER: 8** Since $\Delta AFE \sim \Delta ADC$, $\frac{x}{b} = \frac{m}{m+n}$. Since $\Delta DFE \sim \Delta DAB$, $\frac{x}{a} = \frac{n}{m+n}$. Add equations: $\frac{x}{a} + \frac{x}{b} = \frac{m}{m+n} + \frac{n}{m+n} = \frac{m+n}{m+n} = 1$, so $\frac{1}{a} + \frac{1}{b} = \frac{1}{x}$. Similarly, $\frac{1}{a} + \frac{1}{b} = \frac{1}{y}$. Thus, for bases 5 and 20, $FG = x + y = 4 + 4 = 8$.



January 20, 2000

Dear Math Team Coach,

Enclosed is your copy of the Fall, 1999 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

Division	Question	Correct answer
Senior A	F99S1	Add "x is an integer"
	F99S5	Accept 6, 7 or 4, 6, 7
	F99S21 was eliminated	If f(x, y) = 0 then k = any number..

Have a great spring term!

Sincerely yours,
Richard Geller
Secretary, NYCIML