New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER ONE** **FALL 1999**

PART I: 10 minutes **NYCIML Contest One** **Fall 1999**

F99J1. Compute the units digit of 3^{1999}.

F99J2. Sue can do a job in 7 hours working alone and Joy can do the same job working alone in 6 hours. They decide to do the job together, but after 2 hours they have a fight and Joy leaves. Compute the number of hours it takes Sue to finish the job.

PART II: 10 minutes **NYCIML Contest One** **Fall 1999**

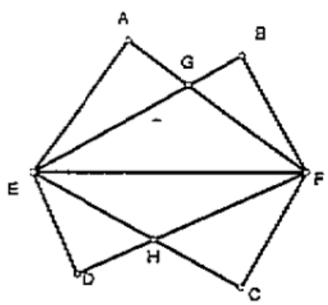
F99J3. How many zeros will there be at the end of $30!$?

F99J4. Circles of radii 6, 8 and 42 are pairwise externally tangent. Compute the area of the triangle having the centers of the circles as its vertices.

PART III: 10 minutes **NYCIML Contest One** **Fall 1999**

F99J5. $\begin{aligned} 7x+11y&=9 \\ 11x+7y&=13 \end{aligned}$ Compute $x+y$.

F99J6. Right triangles AEF, BEF, CEF and DEF are all similar and have hypotenuse $\overline{EF}$. If $AE=3$ and $EF=5$, compute the area of octagon AGBFCHDE.



ANSWERS:

J1. 7
J2. 8/3
J3. 7
J4. 336
J5. 11/9
J6. 117/8

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JUNIOR DIVISION **CONTEST NUMBER TWO** **FALL 1999**

PART I: 10 minutes **NYCIML Contest Two** **Fall 1999**

F99J7. On her latest math quiz, Sonia received a score of 84 and this raised her average from 71 to 72. What score will she need on the next quiz to raise her average to 73?

F99J8. A number is a palindrome if it is a positive integer that reads the same backwards or forwards. For example, 236632 is a palindrome. How many four-digit integers are palindromes if the sum of their digits is 14?

PART II: 10 minutes **NYCIML Contest Two** **Fall 1999**

F99J9. Toni can choose as many as she wants (or none) from the following toppings for her pizza: meatballs, onions, garlic, pepperoni, sausage, mushrooms. She will however, never choose pepperoni and sausage together. How many different types of pizza can she order?

F99J10. The digits in a 14 digit integer N can be arranged to form exactly 1000 other 14 digit integers. If there is only one possible answer and none of the digits is 0, how many different digits does N have?

PART III: 10 minutes **NYCIML Contest Two** **Fall 1999**

F99J11. The nth term of the sequence 36, 20, 12, 8, 6, 5, ... can be written as $2^{x-n}+y$. Find (x, y).

F99J12. In triangle ABC, AB = 6, AC= 7, and BC=9. $\overline{AD}$ and $\overline{BE}$ are altitudes to $\overline{BC}$ and $\overline{AC}$ respectively. Compute $BE-AD$.

ANSWERS:

J7. 86

J8. 7

J9. 48

J10. 2

J11. (6,4)

J12. $\frac{8\sqrt{110}}{63}$

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JUNIOR DIVISION **CONTEST NUMBER THREE** **FALL 1999**

PART I: 10 minutes **NYCIML Contest Three** **Fall 1999**

F99J13. The digits 1 through 5 are multiplied in pairs to form all possible different products that are not perfect squares. Compute the sum of these products.

F99J14. How many minutes after 4:00 p.m. will the hands of the clock be 12 degrees apart for the second time?

PART II: 10 minutes **NYCIML Contest Three** **Fall 1999**

F99J15. N is a two-digit number that is three times the sum of its digits. Find N.

F99J16. Circle C_1 is inscribed in equilateral triangle ABC. Circle C_2 is circumscribed about triangle ABC. Compute the ratio of the length of the radius of circle C_2 to the length of the radius of circle C_1.

PART III: 10 minutes **NYCIML Contest Three** **Fall 1999**

F99J17. On February 3, 1900, four people in the town of Whoville who loved math, held a meeting. The mayor gave any person who attended a math lover's meeting $1 for each time they attend. New meetings were held each week on the same day and time and each math lover brought 2 new math lovers. On March 2, 1900 how many dollars had been given out? (Hint: A century year is a leap year only if it is divisible by 400)

F99J18. From a standard deck of cards, 5 cards are drawn without replacement. The probability that there is exactly one pair and no three or four of a kind can be written as $\frac{p}{q}$ where p and q are relatively prime. Find (p, q).

ANSWERS:
J13. 81
J14. 24
J15. 27
J16. 2 or 2:1
J17. 160
J18. (352,833)

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JUNIOR DIVISION **CONTEST NUMBER ONE** **FALL 1999**

SOLUTIONS

F99J1. $3^0 = 1,\ 3^1 = 3,\ 3^2 = 9,\ 3^3 = 27$ (last digit 7), $3^4 = 81$ (last digit 1), so the powers of 3 go in cycles of 1,3,9,7. 1999 leaves a remainder of 3 when divided by 4, so the answer is **7.** ($1999 \equiv 3 \bmod 4$)

F99J2. Sue does 1/7 of the job in 1 hour and Joy does 1/6 of the job in 1 hour. Working together for 2 hours, they do 2/6 + 2/7 = 13/21 of the job and 8/21 remain.

$$\frac{\frac{1}{7}}{1\text{ hr.}} = \frac{\frac{8}{21}}{x\text{ hrs.}} \rightarrow \frac{1}{7}x = \frac{8}{21} \rightarrow x = \frac{8}{3}\text{ hrs.}$$

F99J3. We need only see how many of the factors are divisible by five, as each 5 will match an even number and give a multiple of 10.
30! has **7** (two 5's in 25)

F99J4. The sides of the triangle are the sum of 2 radii from different circles, 14, 48 and 50. This is a right triangle as each side is double the sides of a 7-24-25 right triangle. The area $= \frac{1}{2} \bullet 14 \bullet 48 = \mathbf{336}$**.** (or use Heron's formula)

F99J5. Adding the equations, we get $18x + 18y = 22 \rightarrow x + y = \frac{22}{18} = \frac{11}{9}$

F99J6. In triangle AEF, $\overline{EF}$ is the hypotenuse, so A is the right angle. Therefore AF=4. Similar right triangles with a common hypotenuse are congruent, so BF=CF=DE=AE=3 and BE=CE=DF=AF=4.
Area (AGBFCHDE)=Area (AFCE)+Area (BFDE)-Area (EGFH)=24-Area (EGFH)
Let O be the midpoint of $\overline{EF}$. Triangle EOG is a right triangle similar to triangle EBF.

Since EO=5/2, then GO=15/8 $\left(\frac{GO}{\frac{5}{2}} = \frac{3}{4} \rightarrow GO = \frac{15}{8}\right)$ and

Area (EGFH)=4 • Area (EOG)=75/8
Area (AGBFCHDE)= **117/8.**

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JUNIOR DIVISION **CONTEST NUMBER TWO SOLUTIONS** **FALL 1999**

F99J7. Let n be the number of quizzes included in Sonia's math quiz average of 71.

Then $\dfrac{71n+84}{n+1} = 72$ and $n = 12$.

To raise her average to 73 on her 14th quiz, Sonia must get a score of $(14)(73) - (13)(72) = 86$.

F99J8. If the sum of the four digits is 14, the sum of the first two must be 7. Thus we can have 1,6 or 2,5 or 3,4 in either order and 7,0 in only one order. Thus there are 7 numbers.

F99J9. There are 2^4 (number of subsets) ways of choosing from meatballs, onions, garlic and mushroom toppings. To each choice you can add sausage, pepperoni or neither. This yields $3 \bullet 2^4 = \mathbf{48}$.

F99J10. $\dfrac{14!}{a! \bullet b! \bullet c! \ldots} = 1001$.

$1001 = 13 \bullet 11 \bullet 7$. $\dfrac{14 \bullet 13 \bullet 12 \bullet 11 \bullet 10 \bullet 9 \bullet 8 \bullet 7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1}{a! \bullet b! \bullet c! \ldots} = 13 \bullet 11 \bullet 7$

We multiply both sides by $a! \bullet b! \bullet c! \ldots$ and divide by $13 \bullet 11 \bullet 7$ and get $14 \bullet 12 \bullet 10 \bullet 9 \bullet 8 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = a! \bullet b! \bullet c! \ldots$ $14 = 7 \bullet 2$ so we have, $10! \bullet 2 \bullet 12 = 10! \bullet 24 = 10! \bullet 4!$ Therefore there are **2** different digits.

F99J11. $2^{x-1} + y = 36$ $\quad 2^{x-2} + y = 20$ $\quad 2^{x-1} - 2^{x-2} = 16$ factoring $2^{x-2}(2-1) = 16$

$2^{x-2} = 16 \rightarrow x = 6 \rightarrow y = 36 - 2^5 = 4$. **(6,4)**

F99J12. Heron's formula gives us the area of triangle ABC to be $2\sqrt{110}$.

Thus by $A = \frac{1}{2}bh$, $BE = \dfrac{4\sqrt{110}}{7}$ and $AD = \dfrac{4\sqrt{110}}{9}$. So $BE - AD = \dfrac{8\sqrt{110}}{63}$

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JUNIOR DIVISION **CONTEST NUMBER THREE** **FALL 1999**

SOLUTIONS

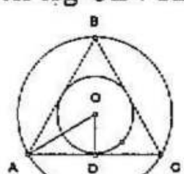
F99J13. The possible products are 2,3,5,6,8,10,12,15,20. Their sum is **81**.

F99J14. The angle between the hands changes at a rate of $1°$ every $\frac{2}{11}$ minutes. The hands start out $120°$ apart, with the angle decreasing. The second time the angle between the hands is $12°$ will come after a change of $132°$, which will occur $\frac{2}{11} \bullet 132 = \mathbf{24}$ minutes after 4.

F99J15. Let N be represented as 10a+b. Thus, 10a+b=3(a+b) and 7a=2b. Since a,b are integers, a=2, b=7 and the number is **27**.

F99J16.

Let D be the midpoint of $\overline{AC}$. Triangle ADO is a 30-60-90 triangle, with hypotenuse $\overline{AO}$ and shorter leg $\overline{OD}$. The ratio sought is $\frac{AO}{OD}$=**2.**



F99J17. By March 2, 4 meetings have been held. Each meeting held 3 times the attendance of the previous meeting. The number of dollars given out was 4+12+36+108=**160**.

F99J18. There are ${}_{52}C_5$ ways to choose 5 cards. There are 13 possible values for the pair, each value yielding 6 possible pairs. There are ${}_{12}C_3$ possible choices of values for the remaining 3 cards, each value yielding 4 different cards. $\frac{p}{q} = \frac{13 \bullet 6 \bullet {}_{12}C_3 \bullet 4^3}{{}_{52}C_5} = \frac{352}{833}$

(p,q)=**(352,833)**