# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B DIVISION** **CONTEST NUMBER ONE**

**PART I: TIME 10 MINUTES** **SPRING 1999**

S99B1 Compute the number of diagonals that can drawn in a polygon with 12 sides.

S99B2 How many ounces of pure acid should be added to 12 ounces of a solution which is 30% acid to make the solution 40% acid?

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**PART II: TIME 10 MINUTES** **SPRING 1999**

S99B3 Compute the sum of the reciprocals of the roots of $2x^2 - 7x - 10 = 0$.

S99B4 In a circle with radius 8, two parallel chords are drawn on opposite sides of the center, each one 4 units from the center. Compute the area of the region in the circle between the two chords in terms of $\pi$.

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**PART III: TIME 10 MINUTES** **SPRING 1999**

S99B5 Compute $21^3 - 3(21)^2(19) + 3(21)(19)^2 - 19^3$

S99B6 Find the largest integral value of N for which $2^N$ is a factor of $50!$

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**ANSWERS**

| | | |
|---|---|---|
| 1. 54 | 3. $\frac{-7}{10}$ | 5. 8 |
| 2. 2 | 4. $32\sqrt{3} + \frac{64\pi}{3}$ | 6. 47 |

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B DIVISION** **CONTEST NUMBER TWO**

**PART I: TIME 10 MINUTES** **SPRING 1999**

S99B7 The 6 digit number 296,8x7 is divisible by 11. Compute x.

S99B8 A circle is inscribed in a right triangle with legs 3 and 4. Compute the radius of the circle.

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**PART II: TIME 10 MINUTES** **SPRING 1999**

S99B9 Solve for all real values of x:

$$\sqrt[3]{x} = \frac{6}{5 + \sqrt[3]{x}}$$

S99B10 Bill and Hillary take turns flipping a fair coin, with the first person to flip a head is the winner. If Bill flips the coin first, find the probability that he wins.

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**PART III: TIME 10 MINUTES** **SPRING 1999**

S99B11 Solve for x:

$$\log_{10} x = 2 - 2\log_{10} 2.$$

S99B12 An isosceles triangle has legs of length 2 and vertex angle 36° . Compute the length of the base.

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**ANSWERS**

| | | |
|---|---|---|
| 7. 5 | 9. 1, -216 | 11. 25 |
| 8. 1 | 10. $\frac{2}{3}$ | 12. $-1 + \sqrt{5}$ |

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B DIVISION** **CONTEST NUMBER THREE**

**PART I: TIME 10 MINUTES** **SPRING 1999**

S99B13 Compute the number of 2 digit prime numbers with exactly one digit equal to 7.

S99B14 In right triangle ABC, CD is the altitude to hypotenuse AB. If AC = 6 and DB = 5, compute the area of $\Delta ABC$.

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**PART II: TIME 10 MINUTES** **SPRING 1999**

S99B15 The first 4 terms of an arithmetic progression are a, x, b, 4x.
Compute $\frac{a}{b}$.

S99B16 Compute the number of digits in the number $4^{16}\,5^{20}$ when it is expanded.

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**PART III: TIME 10 MINUTES** **SPRING 1999**

S99B17 Solve for all real values of x:
$|x-3|+|x-5|=2$

S99B18 Find all values of x, $0° \le x \le 180°$ which satisfy
cos2xcos3x – sin2xsin3x = 0.

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## ANSWERS

| | | |
|---|---|---|
| 13. 8 | 15. $\frac{-1}{5}$ | 17. $3 \le x \le 5$ |
| 14. $9\sqrt{5}$ | 16. 24 | 18. 18, 54, 90, 126, 162 |

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B DIVISION** **CONTEST NUMBER FOUR**

**PART I: TIME 10 MINUTES** **SPRING 1999**

S99B19 Solve for x: $x^{-2/3} = \frac{16}{9}$

S99B20 Compute the number of 3 digit integers which have an odd number of positive integral factors.

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**PART II: TIME 10 MINUTES** **SPRING 1999**

S99B21 Four positive integers, when added 3 at a time, yield sums of 100, 94, 89 and 80. Find the 4 numbers.

S99B22 Compute the area of a triangle with integral sides whose perimeter is 8.

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**PART III: TIME 10 MINUTES** **SPRING 1999**

S99B23 Compute the 1999$^{th}$ digit to the right of the decimal point in the decimal expansion of $\frac{1}{13}$.

S99B24 Three dice are thrown and their sum is 6. Compute the probability that all 3 dice landed on 2.

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**ANSWERS**

| | | |
|---|---|---|
| 19. $\frac{27}{64}$ | 21. 21, 27, 32, 41 | 23. 0 |
| 20. 22 | 22. $2\sqrt{2}$ | 24. $\frac{1}{10}$ |

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B DIVISION** **CONTEST NUMBER FIVE**

**PART I: TIME 10 MINUTES** **SPRING 1999**

S99B25 If $\log_5(\log_4(\log_3 x))=0$, compute x.

S99B26 AB is the diameter of a circle with radius 4. Equilateral triangle ABC is drawn, and BC intersects the circle at D. Compute the length of AD.

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**PART II: TIME 10 MINUTES** **SPRING 1999**

S99B27 Find the x intercept of the line joining (-1,1) and (3,9).

S99B28 Compute $(1+\tan 5^\circ)(1+\tan 40^\circ)$.

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**PART III: TIME 10 MINUTES** **SPRING 1999**

S99B29 A regular polygon contains interior angles of $162^\circ$. Compute the number of sides of the polygon.

S99B30 Five different numbers are chosen from the set

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

Compute the probability that the middle number (third largest) is 5.

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**ANSWERS**

| | | |
|---|---|---|
| 25. 81 | 27. -3/2 | 29. 20 |
| 26. $4\sqrt{3}$ | 28. 2 | 30. $\frac{5}{21}$ |

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B SOLUTIONS SPRING, 1999 CONTEST ONE**

S99B1 A diagonal can be drawn from any vertex to any other vertex except itself and the two adjacent to it. $d = \frac{N(N-3)}{2} = 54$

S99B2 $.3(12) + N = .4(N+12)$
$36 + 10N = 4N + 48$
$N = 2$

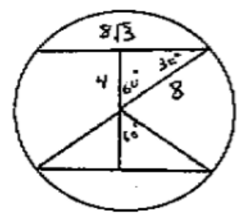
S99B3 $\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 r_2} = \frac{\frac{7}{2}}{\frac{-10}{2}} = \frac{-7}{10}$

S99B4 Since $30° - 60° - 90°$ triangles are formed each chord is $8\sqrt{3}$ and the area of each triangle is

$\frac{1}{2}(4)(8\sqrt{3}) = 16\sqrt{3}$

The area of each sector is $\frac{1}{6}(64\pi)$.

Total area is $2(16\sqrt{3}) + 2\left(\frac{32\pi}{3}\right) = 32\sqrt{3} + \frac{64\pi}{3}$




S99B5 The given number is $(21-19)^3 = 8$.

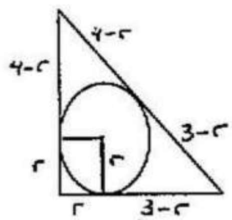
S99B6 There are 25 even numbers that are factors of 50!. 12 numbers are divisible by 4 and give a second factor of 2. 6 numbers are multiples of 8, 3 are multiples of 16, and 1 is a multiple of 32. Therefore, $25 + 12 + 6 + 3 + 1 = 47$.

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B SOLUTIONS SPRING, 1999 CONTEST TWO**

S99B7 $2 + 6 + x = 9 + 8 + 7 - 11$
$x = 5$

S99B8




$7 - 2r = 5$
$r = 1$

S99B9 Let $y = \sqrt[3]{x}$

$5y + y^2 = 6$ $\quad y^2 + 5y - 6 = 0$

$y = 1 \quad y = -6$

$\sqrt[3]{x} = 1 \quad x = 1 \quad \sqrt[3]{x} = -6 \quad x = -216$

S99B10 The probability is $\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}...$

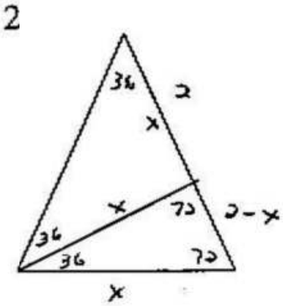
This is an infinite geometric progression.

$$S = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{2}{3}$$

S99B11 $\log_{10} x = \log_{10} 100 - \log_{10} 2^2 = \log_{10} \frac{100}{4} = \log_{10} 25$

$x = 25$

S99B12




Bisect one base angle.
Similar triangles are formed.

$\frac{2}{x} = \frac{x}{2-x} \quad x^2 = 4 - 2x$

$x^2 + 2x - 4 = 0$

$x = \frac{-2 \pm \sqrt{20}}{2}$

Reject the negative.

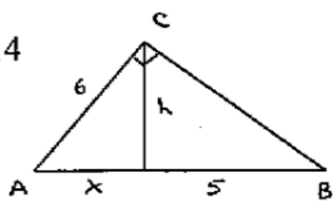
$x = -1 + \sqrt{5}$

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B SOLUTIONS SPRING, 1999 CONTEST THREE**

S99B13 The numbers are 17, 37, 47, 67, 71, 73, 79,97. 8 numbers in all.

S99B14




$$\frac{x}{6}=\frac{6}{x+5} \qquad x^2+5x-36=0$$

$$x = 4$$

$$\frac{4}{h}=\frac{h}{5} \qquad h=\sqrt{20}$$

$$A=\frac{1}{2}\sqrt{20}(9)=9\sqrt{5}$$

S99B15 Let the common difference be d. Then, $4x - x = 2d$ or $d = \frac{3x}{2}$.

$a=-\frac{x}{2}$. $b=\frac{5x}{2}$. $\frac{a}{b}=-\frac{1}{5}$.

S99B16 $4^{16}5^{20} = 2^{32}5^{20} = 2^{12}10^{20}$

$2^{12} = 4096$ which has 4 digits.
$2^{12}10^1$ has 5. $2^{12}10^2$ has 6. $2^{12}10^{20}$ has 24.

S99B17 If $x \geq 5$, equation becomes $x - 3 + x - 5 = 2$
$x = 5$
If $x \leq 3$, $3 - x + 5 - x = 2$, $x = 3$.
Between 3 and 5, $x - 3 + 5 - x = 2$
$2 = 2$ all x will work.

$3 \leq x \leq 5$

S99B18 $\cos(2x + 3x) = 0$
$\cos(5x) = 0$
This will work if $5x = 90$, $5x = 270$, $5x = 450$, etc. and
$x = 18, 54, 90, 126$, and $162$ in the given interval.

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B SOLUTIONS SPRING, 1999 CONTEST FOUR**

S99B19 $(x^{-2/3})^{-3/2} = \left(\frac{16}{9}\right)^{-3/2}$ $\quad x = \frac{27}{64}$

S99B20 Perfect squares, and only perfect squares, have an odd number of factors. Since the smallest 3 digit square is $100 = 10^2$ and the largest is $961 = 31^2$, there are 22 in all.

S99B21 $A + B + C = 100$ $\quad 3(A + B + C + D) = 363$

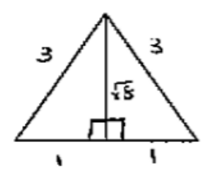
$A + B + D = 94$ $\quad A + B + C + D = 121$

$A + C + D = 89$ $\quad A = 41,\ B = 32,\ C = 27,\ D = 21$

$B + C + D = 80$

S99B22 The triangle must have sides 3, 3, 2.
(No other triangle fits the description.)

$$A = \frac{1}{2}(2)\sqrt{8} = \sqrt{8} = 2\sqrt{2}$$




S99B23 $\frac{1}{13} = .\overline{076923}$ $\quad$ Since $1999 \equiv 1 \pmod 6$
(leaves a remainder of 1 when divided by 6)
The decimal is the same as the first digit, which is 0.

S99B24 The only possibilities are {1, 2, 3} which has 6 ways of occurring, or {2, 2, 2} which has only 1 way, or {1, 1, 4} which has 3 ways.
Probability is $\frac{1}{1+3+6} = \frac{1}{10}$

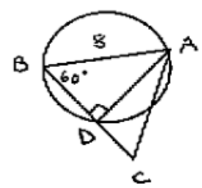
# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

**SENIOR B SOLUTIONS SPRING, 1999 CONTEST FIVE**

S99B25 $5^0 = \log_4(\log_3 x)$ $4^1 = \log_3 x$

$3^4 = x$ $x = 81$

S99B26




$AD = 4\sqrt{3}$

S99B27 The equation of the line is y – 9 = 2(x – 3) or y = 2x + 3

When y = 0, x = -3/2.

S99B28 $\tan 45 = 1 = \tan(40 + 5) = \dfrac{\tan 40 + \tan 5}{1 - \tan 40 \tan 5}$

$1 = \tan 40 + \tan 5 + \tan 5 \tan 40$

$(1 + \tan 5)(1 + \tan 40) = 1 + \tan 40 + \tan 5 + \tan 5 \tan 40 = 1 + 1 = 2$

S99B29 $162 = \dfrac{180(N-2)}{N}$ $18N = 360$ $N = 20$

S99B30 Since 5 is the third largest, 2 numbers must be picked from {1, 2, 3, 4} and 2 from {6, 7, 8, 9, 10}.

$P = \dfrac{{}_4C_2 \cdot {}_5C_2}{{}_{10}C_5} = \dfrac{6\ 10}{252} = \dfrac{5}{21}$