New York City Interscholastic Math League

Senior A Division | **Contest Number One**

Part I *Spring, 1998* *Contest 1* *Time: 10 Minutes*

S98S1 If my daughter will be n years old in the year n^2, in what year was she born?

S98S2 The lengths of the legs of a right triangle are m and n. Express the length of the altitude to the hypotenuse in simplest form, in terms of m and n.

Part II *Spring, 1998* *Contest 1* *Time: 10 Minutes*

S98S3 The roots of $x^3 - 5x^2 + 7x - 9 = 0$ are a, b, and c. The roots of $x^3 - px^2 + qx - r = 0$ are $a + b$, $b + c$, and $c + a$. Compute p.

S98S4 There are two times between 4 o'clock and 5 o'clock when the hands of a twelve-hour analogue (traditional) clock form an angle of exactly 60°. To the nearest second, compute either time.

Part III *Spring, 1998* *Contest 1* *Time: 10 Minutes*

S98S5 Given three consecutive odd positive integers greater than 10 such that the first integer is divisible by 3, the second integer is divisible by 5 and the third is divisible by 7. Compute the smallest possible value of the first integer.

S98S6 In ΔABC, points P, Q, and R lie in sides $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ respectively so that **AP:PB = BQ:QC = CR:RA = 3:7**. Compute the ratio of the areas of ΔPQR to ΔABC.

Answers:		
	S98S1	1980
	S98S2	$mn / (m^2 + n^2)^{1/2}$ or $mn(m^2 + n^2)^{1/2} / (m^2 + n^2)$
	S98S3	10
	S98S4	4:10:55 or 4:32:44
	S98S5	213
	S98S6	37:100

New York City Interscholastic Math League

Senior A Division **Contest Number Two**

Part I ***Spring, 1998*** ***Contest 2*** ***Time: 10 Minutes***

S98S7 In this question, ***PQR*** is defined as a three-digit integer whose nonzero digits are ***P***, ***Q***, and ***R***. Compute the value of (**ABC** + **ACB** + **BAC** + **BCA** + **CAB** + **CBA**) $\div (A + B + C)$.

S98S8 Compute the area of the region bounded by the graph of $|4x - 12| + |3y - 12| = 24$.

Part II ***Spring, 1998*** ***Contest 2*** ***Time: 10 Minutes***

S98S9 Compute the sum of all the coefficients in the expansion of

$$(11a^2bc - 7ab^2c - 5abc^2)^{8991}$$

S98S10 The roots of $x^3 - 5x^2 + 7x - 9 = 0$ are a, b, and c. The roots of $x^3 - px^2 + qx - r = 0$ are $a + b$, $b + c$, and $c + a$. Compute q.

Part III ***Spring, 1998*** ***Contest 2*** ***Time: 10 Minutes***

S98S11 A straight line makes an angle of 60° with the x-axis. Its x-intercept is 8, its y-intercept is negative and its equation is of the form $y = mx + k$. Compute the ordered pair (m, k).

S98S12 The first four terms of any arithmetic progression is represented by a, a + d, a + 2d, and a + 3d. Compute k in simplest terms of a and/or d, so that the following expression is the square of a polynomial:

$$(x + a)(x + a + d)(x + a + 2d)(x + a + 3d) + k$$

Answers:		
	S98S7	222
	S98S8	96
	S98S9	-1
	S98S10	32
	S98S11	$(\sqrt{3}, -8\sqrt{3})$
	S98S12	$k = d^4$

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION **CONTEST NUMBER THREE**

PART I *SPRING, 1998* *CONTEST 3* *TIME: 10 MINUTES*

S98S13 Maria must drive a certain distance in a certain time. If she drives at an average rate of 48 miles per hour, she will arrive $7\frac{1}{2}$ minutes early. If she drives at an average rate of 42 miles per hour, she will arrive 10 minutes late. Compute the number of minutes she needs to arrive exactly on time.

S98S14 Compute all three pairs of natural numbers whose squares differ by 200.

PART II *SPRING, 1998* *CONTEST 3* *TIME: 10 MINUTES*

S98S15 Compute the unit's digit in the expansion of $(1^{1998} + 9^{1998} + 9^{1998} + 8^{1998})^{1998}$.

S98S16 Given: $\arctan x + \arctan y = \arctan (x - y)$ for all values of x. If $y \neq 0$, express y in simplest form, in terms of x.

PART III *SPRING, 1998* *CONTEST 3* *TIME: 10 MINUTES*

S98S17 Compute all real values of x between 0 and 2π that satisfy $[\log (4x - 10)]^{4 \sin x + 2} = 1$.

S98S18 Compute both real values of x for which $(2x + 5)^{1/3} - (2x - 5)^{1/3} = 1$.

ANSWERS:	S98S13	130
	S98S14	(51,49),(27,23),(15,5) [Form not required]
	S98S15	9
	S98S16	$y = (x^2 + 2) / x$
	S98S17	$5, \frac{7\pi}{6}, \frac{11\pi}{6}$
	S98S18	$\pm\sqrt{13}$

New York City Interscholastic Math League

Senior A Division | **Contest Number Four**

Part I *Spring, 1998* *Contest 4* *Time: 10 Minutes*

S98S19 Given $32^{0n} \cdot 16^{1n} \cdot 8^{2n} \cdot 4^{3n} \cdot 2^{4n} \cdot 1^{5n} = 1024$. Compute n.

S98S20 What is the largest integer that leaves the same remainder upon dividing 1414, 1023, and 724?

Part II *Spring, 1998* *Contest 4* *Time: 10 Minutes*

S98S21 Janet has 100 socks of each color: red, white, blue, black, and shocking pink. She selects socks blindly one at a time and does not replace them. What is the minimum number of socks she must select in order to be certain that she has 12 pairs (a pair is two socks of the same color) without ever looking at them?

S98S22 A circle is inscribed in ΔABC so that the circle is tangent to side AB at P, side BC at Q and side CA at R. If $m\angle A = x$ and $m\angle B = y$, express $m\angle QPR$ in simplest form, in terms of x and y.

Part III *Spring, 1998* *Contest 4* *Time: 10 Minutes*

S98S23 Compute the irrational roots of:

$$(x^2 - 5x + 6)^2 - 5(x^2 - 5x + 6) + 6 = 0.$$

S98S24 Compute $\sum_{n=1}^{99} \tan(0.01n\pi - 0.5\pi)$.

Answers:

S98S19	½
S98S20	23
S98S21	28
S98S22	$\frac{1}{2}(x + y)$
S98S23	$\frac{1}{2}(5 \pm \sqrt{13})$
S98S24	0

New York City Interscholastic Math League

Senior A Division | **Contest Number Five**

Part I *Spring, 1998* *Contest 5* *Time: 10 Minutes*

S98S25 Almond Joyner needs 24 seconds to run a complete circuit of a circular track. Kelly Greene runs in the opposite direction and passes her in 15 seconds. Each girl maintains a constant pace, and both girls start at the same time and place. How many seconds does Kelly need to run one complete circuit of the track?

S98S26 If $c = (a + bi)^3 - 47i$ where a, b, and c are each positive integers and $i^2 = -1$, compute the value of c.

Part II *Spring, 1998* *Contest 5* *Time: 10 Minutes*

S98S27 Each of five natural numbers less than 100 have exactly 12 factors. Three of them are 60, 72, and 96. What are the other two natural numbers?

S98S28 A straight line has a y-intercept of **a** and an x-intercept of **b**. The origin is reflected over this line. Express the coordinates of the image of the origin in simplest form, in terms of **a** and **b**.

Part III *Spring, 1998* *Contest 5* *Time: 10 Minutes*

S98S29 If $\log(a - {}^1\!/_a) = \log a + \log {}^1\!/_a$ where $a \neq 0$, compute a.

S98S30 The legs of a right triangle are 3 and 4. Two circles each are tangent to one leg, the hypotenuse and each other. If their radii are in a ratio of 1:4 and the smaller circle is tangent to the smallest side of the triangle, compute the radius of the smaller circle.

Answers:

S98S25	40
S98S26	52
S98S27	90, 84
S98S28	$\left(\frac{2a^2b}{a^2+b^2}, \frac{2ab^2}{a^2+b^2}\right)$
S98S29	$\frac{1}{2}(1+\sqrt{5})$
S98S30	$^5/_{18}$

SOLUTIONS

S98S1 ***Answer:* 1980** The only perfect squares near the year 1998 are 1936, 2025, and 2116. Reject 1936 (not a future year). Then the birth years are 2025 – 45 = 1980 and 2116 – 46 = 2070. Reject 2070: my daughter already was born.

S98S2 ***Answer:* $mn/(m^2+n^2)^{1/2}$ or equivalent** The hypotenuse is $(m^2+n^2)^{1/2}$. If h is the length of the altitude to the hypotenuse, then the area of the triangle = $\frac{1}{2}h(m^2+n^2)^{1/2} = \frac{1}{2}mn$. Then solve for h.

S98S3 ***Answer:* 10** First note that a + b + c = 5, ab + bc + ca = 7 and abc = 9.

p = (a + b) + (b + c) + (c + a)
= 2(a + b + c)
= 2(5)
= 10.

S98S4 ***Answer:* 4:10:55 or 4:32:44** First note that the hour hand leads the minute hand by exactly 60° 11 times in 12 hours. Therefore this position occurs every 1 hour and 60 ÷ 11 minutes = every $1{:}05^5/_{11}$ = $1{:}05{:}27^3/_{11}$. Starting at the simplest time, 2:00, the next such times are 3:05:27 and 4:10:55.

When the minute hand leads the hour hand by 60°, the same reasoning holds. The simplest time is 10:00, and 6 intervals of $1{:}05{:}27^3/_{11} \Rightarrow 10{:}00 + 6{:}30{:}162^{18}/_{11} \Rightarrow 4{:}32{:}43^7/_{11}$.

S98S5 ***Answer:* 213** If the first COI = a, then 3|a ("3 divides a") implies that 3|(a – 3). The second COI = a + 2, so 5|(a + 2) implies that 5|(a + 2 – 5), or 5|(a – 3). The third COI = a + 4, so 7|(a + 4) implies that 7|(a + 4 – 7), or 7|(a – 3). Since a – 3 is divisible by 3, 5, and 7, it is a multiple of 105, their LCM. If a – 3 = 105, then the COI are 108, 110, 112, which are not odd. Thus a – 3 = 2(105) = 210, and the COI are 213, 215, 217.

S98S6 ***Answer:* 37:100** **METHOD 1:** Let AB = c, AC = b, AP = 0.3c, and AR = 0.7b. Then the areas of ΔAPR and ΔABC are in the ratio [½(0.3c)(0.7b)sin A] : [½ cb sin A] = 0.21. Similarly, the ratios of the areas of ΔBPQ and ΔCQR to ΔABC are also each 0.21. Thus, the ratio of the area of ΔPQR to ΔABC is 1 – 3(0.21) = 37:100.
METHOD 2: With no loss of generality, let the triangle be equilateral with sides of length 10. Then proceed as in method 1.

SOLUTIONS

S98S7 ***Answer: 222***

$$\begin{aligned}ABC &= 100A + 10B + C\\ ACB &= 100A + 10C + B\\ BAC &= 100B + 10A + C\\ BCA &= 100B + 10C + A\\ CAB &= 100C + 10A + B\\ CBA &= \underline{100C + 10B + A}\\ \text{Sum} &= 200(A + B + C) + 20(A + B + C) + 2(A + B + C)\\ &= 222(A + B + C)\end{aligned}$$

Divide by $(A + B + C)$ to obtain 222 *regardless of the values of A, B, or C.*

S98S8 ***Answer: 96*** The graph is a rhombus whose diagonals are parallel to the axes. To find the length of the horizontal axis, let $|3y - 12| = 0$: then $0 \le |4x - 12| \le 24$. This implies that $-3 \le x \le 9$, so that one diagonal has a length of 12. To find the length of the vertical axis, let $|4x - 12| = 0$: then $0 \le |3y - 12| \le 24$. This implies that $-4 \le y \le 12$, so that the other diagonal has a length of 16. Then the area of the rhombus $= \frac{1}{2}d_1d_2 = \frac{1}{2}(12)(16) = 96$.

S98S9 ***Answer: −1*** Since we are interested only in the coefficients, let $a = b = c = 1$. Then $(11 - 7 - 5)^{8991} = (-1)^{8991} = -1$.

S98S10 ***Answer: 32*** First note that $a + b + c = 5$, $ab + bc + ca = 7$ and $abc = 9$.
Also note that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$$\begin{aligned} q &= (a + b)(b + c) + (b + c)(c + a) + (c + a)(a + b)\\ &= a^2 + b^2 + c^2 + 3(ab + bc + ca)\\ &= (a + b + c)^2 + (ab + bc + ca)\\ &= (5)^2 + 7\\ &= 32.\end{aligned}$$

S98S11 ***Answer: $(\sqrt{3}, -8\sqrt{3})$*** Note that if the upper right hand angle formed by the line and the x-axis is θ, then $\tan\theta = m$, where m is the slope of the line. Then $y - 0 = \sqrt{3}(x - 8)$. Thus $y = x\sqrt{3} - 8\sqrt{3}$.

S98S12 ***Answer: d^4*** There are several clever ways to solve this. One is as follows:

$$\begin{aligned} & (x + a)(x + a + d)(x + a + 2d)(x + a + 3d) + k\\ =\ & [(x + a)(x + a + 3d)]\,[(x + a + d)(x + a + 2d)] + k\\ =\ & [(x + a)^2 + 3d(x + a)]\,[(x + a)^2 + 3d(x + a) + 2d^2] + k\\ =\ & [\qquad y \qquad]\,[\qquad y \qquad + 2d^2] + k\\ =\ & y^2 + 2d^2y + k.\end{aligned}$$

Then $(y + d^2)^2$ requires $k = d^4$. The polynomial is $[(x + a)^2 + 3d(x + a) + d^2]^2$

SOLUTIONS

S98S13 ***Answer: 130*** 7.5 minutes = $^1/_8$ hour and 10 minutes = $^1/_6$ hour. Distance = R×T = $48(t - {}^1/_8) = 42(t + {}^1/_6)$ where t is the required time in hours. $t = {}^{13}/_6$ hours = 130 minutes.

S98S14 ***Answer: (51,49),(27,23),(15,5)*** For the squares x^2 and y^2, $x^2 - y^2 = 200$. Thus $(x-y)(x+y) = 1\cdot200 = 2\cdot100 = 4\cdot50 = 5\cdot40 = 8\cdot25 = 10\cdot20$. Let $x - y$ = the first factor, $x + y$ = the second factor and solve for (x,y). Since factors of opposite parity produce non-integral solutions, only 2•100, 4•50, and 10•20 will produce integral solutions.

S98S15 ***Answer: 9*** Raise each number to the 2nd, 3rd, 4th, etc. power and look for a pattern in the unit's digit: 1^{1998} ends in 1, 9^{1998} ends in 1, and 8^{1998} ends in 4. Then $(1 + 1 + 1 + 4)^{1998} = 7^{1998}$ ends in 9.

S98S16 ***Answer: $y = (x^2 + 2) / x$*** Let A = arctan x and B = arctan y. Then A+B = arc tan (x–y). ∴tan (A+B) = x – y. Expand: (tan A + tan B) / (1 – tan A tan B) = x–y. Now substitute: $(x+y) / (1 - xy) = x - y$. Solving yields $y = (x^2 + 2) / x$.

S98S17 ***Answer: 5, $^{7\pi}/_6$, $^{11\pi}/_6$*** Three cases induce a value of 1:
[1] (Any value)^0; [2] $1^{\text{any value}}$; [3] $(-1)^{\text{even value}}$.
[1] $4 \sin x + 2 = 0$. Thus $x = {}^{7\pi}/_6, {}^{11\pi}/_6$, both of which are between 0 and 2π.
[2] Log (4x – 10) = 1. Thus 4x – 10 = 10, so that x = 5, which is between 0 and 2π.
[3] Log (4x – 10) = – 1. Thus 4x – 10 = 0.1, so that x = 2.525. Reject: 4 sin x + 2 is not even.

S98S18 ***Answer: $\pm\sqrt{13}$*** Let $A = (2x + 5)^{1/3}$ and $B = (2x - 5)^{1/3}$.

	Then $A - B = 1$.
Cube both sides:	$A^3 - 3A^2B + 3AB^2 - B^3 = 1$
Regroup:	$A^3 - B^3 - 3AB(A - B) = 1$
Substitute:	$A^3 - B^3 - 3AB(\ 1\ ) = 1$
Thus	$(2x + 5) - (2x - 5) - 3[(2x + 5)^{1/3} (2x - 5)^{1/3}] = 1$
	$10 - 3(4x^2 - 25)^{1/3} = 1$
	$(4x^2 - 25)^{1/3} = 3$.
Cube both sides:	$x^2 = 13$.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER FOUR**

SOLUTIONS

S98S19 ***Answer:*** $\frac{1}{2}$ $32^{0n} \cdot 16^{1n} \cdot 8^{2n} \cdot 4^{3n} \cdot 2^{4n} \cdot 1^{5n} = 1 \cdot (2^4)^{1n} \cdot (2^3)^{2n} \cdot (2^2)^{3n} \cdot (2^1)^{4n} \cdot 1 = 2^{10}$. $2^{4n} \cdot 2^{6n} \cdot 2^{6n} \cdot 2^{4n} = 2^{4n+6n+6n+4n} = 2^{20n} = 2^{10}$. Then $20n = 10$ so $n = \frac{1}{2}$.

S98S20 ***Answer:* 23** (Euclidean Algorithm) Let n = the required number, and represent 1414 as $an + r$ and 1023 as $bn + r$. Then $1414 - 1023 = (an+r) - (bn+r)$, so $391 = (a - b)n$, which implies that $n|391$. Likewise, $1023 - 724 = 299$, which also is divisible by n. Then: $n|(391 - 299)$ implies $n|92$, and $n|(299 - 92 - 92 - 92)$ implies $n|23$. Since $23|92$, then $n = 23$. [LEMMA: *If* $A \equiv B \pmod m$, *then* $m|(A - B)$.]

Note: $23|92$, $23|299$, and $23|391$; and 23 leaves the same remainder upon dividing 1414, 1023, and 724.

S98S21 ***Answer:* 28** **METHOD 1:** Suppose Janet has selected exactly 11 pairs (22 socks). She can then select 5 single socks, one of each color. The next sock she draws, the 28th, will force the twelfth pair.

METHOD 2: Try simpler cases. If she wants one pair, she needs to select 6 socks to guarantee 1 match. If she wants two pairs, she needs to select 8 socks to guarantee 2 matches. If she wants three pairs, she needs to select 10 socks to guarantee 3 matches. If she wants four pairs, she needs to select 12 socks to guarantee 4 matches. Each new pair requires two more selections. Thus to create $12 - 4$ = eight more pairs, she needs to select 16 more socks. The total is $12 + 16 = 28$ socks.

S98S22 ***Answer:*** $\frac{1}{2}(x + y)$ Draw ΔPQR. If the measures of the angles of this triangle are P, Q, and R respectively, then arc PQ contains 2R degrees, arc QR contains 2P degrees, and arc RP contains 2Q degrees. Then use the angle-arc formula for two tangents:

$$x = \tfrac{1}{2}(2P + 2R - 2Q) = P - Q + R.$$
$$y = \tfrac{1}{2}(2P + 2Q - 2R) = P + Q - R.$$
$$x + y = 2P$$

S98S23 ***Answer:*** $\frac{1}{2}(5 \pm \sqrt{13})$ Let $z = x^2 - 5x + 6$. Then $z^2 - 5z + 6 = 0$, so $z = 3$ or 2.

If $x^2 - 5x + 6 = 2$, then x is rational (1 or 4).

If $x^2 - 5x + 6 = 3$, then x is irrational. Use the quadratic formula.

S98S24 ***Answer:* 0** **METHOD 1:** Graph the tangent function $y = \tan x$. The domain varies from just over $-\frac{1}{2}\pi$ to just under $+\frac{1}{2}\pi$. It is symmetrical about the origin. For every point left of the y-axis there is a corresponding point on the right and conversely. The sum of each of the 49 corresponding pairs of values of y is 0. Also, for $n = 50$, $\tan 0 = 0$. Then the sum of 50 zeroes is zero.

METHOD 2: For $n = 50$, $\tan 0 = 0$. Pair values as follows: For $n = 1$ and 99, $\tan(^-.49\pi) + \tan(.49\pi) = {}^-\tan(.49\pi) + \tan(.49\pi) = 0$. Likewise, for $n = 2$ and $n = 98$, $\tan(^-.48\pi) + \tan .48\pi = 0, \ldots$ Again, the sum of 50 zeroes is zero.

New York City Interscholastic Math League

Senior A Division | **Contest Number Five**

Solutions

S98S25 ***Answer:* 40** Draw a circular track. The GCF of 15 and 24 is 3, so assume Almond covers a circuit in 8 segments of 3 seconds each. She meets Kelly after 15 seconds (5 segments), so Kelly must cover 3 segments in her 15 seconds. Thus one segment takes 5 seconds and all 8 segments require 40 seconds.

S98S26 ***Answer:* 52** $c = a^3 + 3a^2bi - 3ab^2 - b^3i - 47i$. Since c is an integer, $3a^2bi - b^3i - 47i = 0$. Then $3a^2b - b^3 - 47 = 0$, which implies that $b(3a^2 - b^2) = 47$. Since b is an integer, either $b = 1$ or $b = 47$. If $b = 47$, then $3a^2 - 2209 = 1$. Reject — a is not integral. If $b = 1$, then $3a^2 - 1 = 47$, so that $a = \pm 4$. Accept the positive integer 4. Then $c = a^3 - 3ab^2 = 52$.

S98S27 ***Answer:* 84, 90** Let p, q, r, . . . be prime numbers: If $N = p^aq^br^c \ldots$, then N has $(a+1)(b+1)(c+1)\ldots$ different factors. Since $12 = 1\times12 = 2\times6 = 3\times4 = 3\times2\times2$, examine 2^{11}, $2^5\times3^1 = 96$, $2^5\times5^1$, $2^3\times3^2 = 72$, $2^2\times3^3$, $2^3\times5^2$, $2^2\times3^1\times5^1 = 60$, $2^1\times3^2\times5^1 = 90$, $2^2\times3^1\times7^1 = 84$.

S98S28 ***Answer:* $(2a^2b/(a^2 + b^2), 2ab^2/(a^2 + b^2))$** **METHOD 1:** The segment joining the origin to its image is perpendicular to the given graph. Let the line and the segment intersect at (x,y). Then the coordinates of the image are (2x,2y). A y-intercept of a and an x-intercept of **b** determines a line whose equation is $ax + by = ab$. A line perpendicular to the given line that contains the origin has an equation of $bx - ay = 0$. Solve this system of equations to find the (x, y). Then double each coordinate to find (2x,2y).

METHOD 2: In quadrant I the diagram is a right triangle with an altitude to the hypotenuse. The foot of the altitude is (x,y) and the lengths of the legs are a and **b**. Use the Leg Proportional to find the segments of the hypotenuse: $a^2/(a^2+b^2)^{1/2}$ and $b^2/(a^2+b^2)^{1/2}$. Use the Altitude Proportional to find the altitude: $ab/(a^2+b^2)^{1/2}$. Then drop a perpendicular from (x,y) to the x-axis. Repeat the Leg and Altitude Proportionals in this smaller right triangle to find (x,y). Remember to double each coordinate.

S98S29 ***Answer:* $\frac{1}{2}(1+\sqrt{5})$** Since $\log a + \log {}^1/_a = \log(a \cdot {}^1/_a) = \log 1$, $\log(a - {}^1/_a) = \log 1$ $\Rightarrow a - {}^1/_a = 1 \Rightarrow a^2 - a - 1 = 0 \Rightarrow a = \frac{1}{2}(1\pm\sqrt{5})$. Reject $a = \frac{1}{2}(1 - \sqrt{5})$ because $\log a$ exists only for $a > 0$.

S98S30 ***Answer:* $^5/_{18}$** Bisect the larger acute angle. The bisector passes through the center of the smaller circle and also divides side 4 in a 3:5 ratio, that is, 1.5 and 2.5. Then draw the radius of the small circle perpendicular to leg 3 to form a right triangle similar to the triangle whose legs are 1.5 and 3. Thus, for a radius of length r, the segment of leg 3 between the point of tangency and the acute angle has length $2r$. Therefore, the distance along the hypotenuse from that vertex to the other point of tangency is also $2r$.

Likewise, the distance along the hypotenuse from the vertex of the smaller acute angle to the point of tangency is $12r$.

Next, join the centers (length $5r$), draw both radii perpendicular to the hypotenuse, and drop a perpendicular from the center of the smaller circle to the longer radius. The length of this perpendicular is $4r$.

Finally $2r + 12r + 4r = 5$ so that $r = {}^5/_{18}$.

June 3, 1998

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1998 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Senior A	S98S1	$n^2 - n$ or any number of that form > 0 was accepted
Junior	S98J11	was eliminated. The problem should have read "Compute x - y"

Have a great summer!
MATH IS # 1

Sincerely yours,

Richard Geller

Secretary, NYCIML