New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER ONE** | **SPRING 1998**

PART I: 10 Minutes | **NYCIML Contest One** | **Spring 1998**

S98J1. In ΔABC, altitudes $\overline{BE}$ and $\overline{CD}$ are drawn. If $AD = 5$, $DB = 2$ and $AE = 4$, compute the length of $\overline{EC}$.

S98J2. Compute the value of $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$

PART II: 10 Minutes | **NYCIML Contest One** | **Spring 1998**

S98J3. Compute $x + y$ where $\frac{(5!)!}{(4!)!} = {}_xP_y$. <u>Note:</u> $n!$ represents n factorial and ${}_xP_y$ represents the number of permuations of x things y at a time.

S98J4. The points (1,3), (−1,2), and (2,5) lie on the graph of $y = ax^2 + bx + c$. Compute the product abc.

PART III: 10 Minutes | **NYCIML Contest One** | **Spring 1998**

S98J5. During her vacation, Roz drove from home to the shore along a straight highway and averaged 50 mph. On the way home, she had traffic and only averaged 40 mph along the same route. If she took an hour longer to drive home than to go, compute the distance from her home to the shore.

S98J6. An isosceles triangle has base of length $3x + 6$ and legs of length $x^2 - 2x + 1$. Find the set of all possible values of x.

<u>**Answers**</u>

1. $4\frac{3}{4}$ or equivalent	**3.** 216	**5.** 200 miles
2. 2	**4.** $\frac{1}{2}$	**6.** $\{x: (-2 < x < -\frac{1}{2}) \cup (x > 4)\}$ or equivalent

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER TWO** **SPRING 1998**

PART I: 10 Minutes **NYCIML Contest Two** **Spring 1998**

S98J7. A model train set is in the shape of a circle with twelve equally spaced stations along the tracks. A train moves at a constant speed from the first station to the third station (without stopping) in 12 seconds, Compute how many seconds it will take the train to make a complete loop around the track without stopping.

S98J8. Circles O and P having radii 4 and 5 respectively are concentric. If circle P is inscribed in square ABCD and circle O is circumscribed about square EFGH, compute the area between the two squares.

PART II: 10 Minutes **NYCIML Contest Two** **Spring 1998**

S98J9. Pablo had a 95 average on eight tests. Compute the lowest mark he may have received. (Assume all tests are marked between 0 and 100 points inclusively.)

S98J10. Compute the area enclosed by the graph of $|x| + |y| = 2$.

PART III: 10 Minutes **NYCIML Contest Two** **Spring 1998**

S98J11. Compute $(x-y)!$ where $\frac{((25!)!)!}{((3!)!)!} = {}_xP_y$. Note: n! represents n factorial and ${}_xP_y$ represents the number of permuations of x things y at a time.

S98J12. Benjamin is flying a kite he constructed. One pair of adjacent sides are congruent and enclose a 90° angle. The other two sides are also congruent and enclose a 60° angle. If the shorter diagonal has length 8, compute the ratio of the area of the kite to the length of its longer diagonal.

Answers

7. 72	**9.** 60	**11.** 720
8. 68	**10.** 8	**12.** 4

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER THREE** **SPRING 1998**

PART I: 10 Minutes **NYCIML Contest Three** **Spring 1998**

S98J13. In square ABCD, the midpoints of $\overline{DC}$ and $\overline{BC}$ are connected to one another and to point A, creating four non–overlapping triangles. How many times larger is the area of the largest triangle than the smallest?

S98J14. Compute the units digit of $1997^{1998} + 1998^{1999} + 1999^{2000}$.

PART II: 10 Minutes **NYCIML Contest Three** **Spring 1998**

S98J15. A prism is constructed so that the congruent bases are decagons. Compute the number of internal diagonals in the prism. (Internal diagonals do not include diagonals of faces.)

S98J16. If $\begin{cases} a^3 + b^3 = 175 \\ a^2 - ab + b^2 = 25 \end{cases}$, compute the value of a+b.

PART III: 10 Minutes **NYCIML Contest Three** **Spring 1998**

S98J17. Two circles with radii 4 and 6. are externally tangent. A quadrilateral is formed by their line of centers, a common external tangent and two radii drawn to this common external tangent. Compute the area of this quadrilateral in simplest form.

S98J18. If $x = \dfrac{\sqrt{\sqrt{5}+2} - \sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}-1}} - \sqrt{3-2\sqrt{2}}$, rewrite x in simplest form.

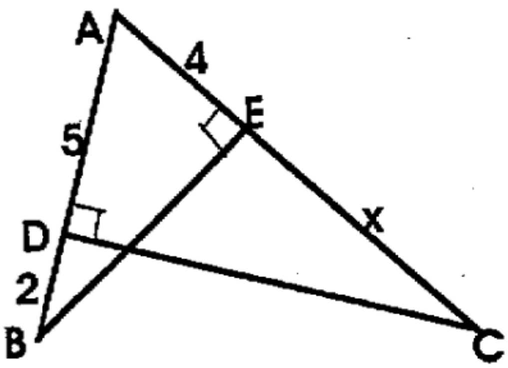
Answers

13. 3	**15.** 70	**17.** $20\sqrt{6}$
14. 2	**16.** 7	**18.** 1

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER ONE** **SPRING 1998**

Solutions



S98J1. Using similar triangles, we get $\frac{5}{x+4} = \frac{4}{7}$ which means that $x = 4\frac{3}{4}$. **Answer:** $4\frac{3}{4}$.

S98J2. Let $x = \sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$. This means that $x^2 = 2 + x$ so that $(x-2)(x+1) = 0$. Since x is obviously positive, we take only the positive result, $x = 2$

Answer: 2

S98J3. $\frac{(5!)!}{(4!)!} = \frac{120!}{24!} = {}_{120}P_{96}$. This means that x = 120 and y = 96. Thus $x+y = 216$. **Answer:** 216

S98J4. Substituting the three points into the given equation, we get the following:

$$\begin{cases} 3 = a + b + c \\ 2 = a - b + c \\ 5 = 4a + 2b + c \end{cases}$$

Solving gives $a = b = \frac{1}{2}$ and $c = 2$. Thus the product $abc = \frac{1}{2}$.

Answer: $\frac{1}{2}$

S98J5. Let d = the distance from Roz's home to the shore. Her time going was $\frac{d}{50}$. Her time returning was $\frac{d}{40}$. This means that $\frac{d}{40} = \frac{d}{50} + 1$. This gives $5d = 4d + 200$, so that $d = 200$ **Answer:** 200 miles

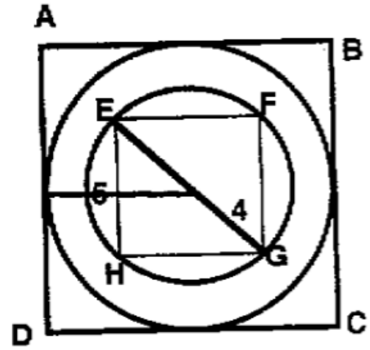
S98J6. First check that the lengths are positive: $3x + 6 > 0 \rightarrow x > -2$. Likewise, $x^2 - 2x + 1 > 0 \rightarrow x \neq 1$. Thus all answers must be larger than –2, but not equal to 1. Now check the triangle inequality: $2(x^2 - 2x + 1) > 3x + 6 \rightarrow x > 4$ OR $x < -\frac{1}{2}$. Also: $(x^2 - 2x + 1) + 3x + 6 > x^2 - 2x + 1 \rightarrow$ which also gives $x > -2$. Combining the above gives $\{x: (-2<x<-\frac{1}{2}) \cup (x>4)\}$

Answer: $\{x: (-2<x<-\frac{1}{2}) \cup (x>4)\}$

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER TWO** **SPRING 1998**

Solutions



S98J7. It takes the train 6 seconds to go from one station to the next. Thus to do the entire circuit, the train must take $6 \cdot 12 = 72$ seconds.

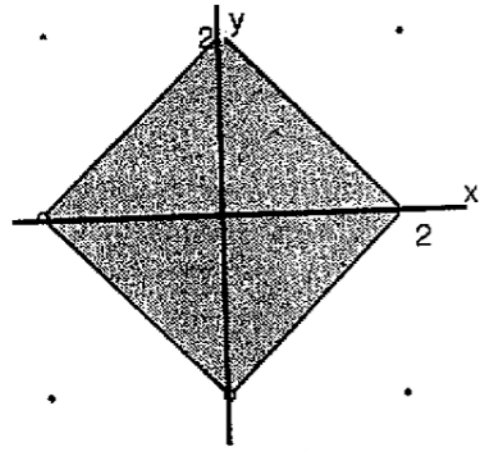
Answer: 72

S98J8. The bigger square has area 100 and the smaller one has area 32 (half the square of the length of the diagonal). The area between the two is 68.

Answer: 68

S98J9. Eight tests averaging 95 must total 760. If Pablo scored 100 on seven of the tests, he would have had to score 60 to reach this total. Thus the lowest score on any test would be 60.

Answer: 60

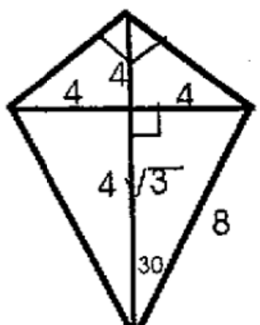


S98J10. The graph is a square with diagonal four units long. Thus the area is $\frac{1}{2} \cdot (4)^2 = 8$

Answer: 8

S98J11. Note that the denominator is the same as 720!. We can disregard the numerator since ${}_nP_r = \frac{n!}{(n-r)!}$ tells us that $(n-r)! = 720!$

Answer: 720



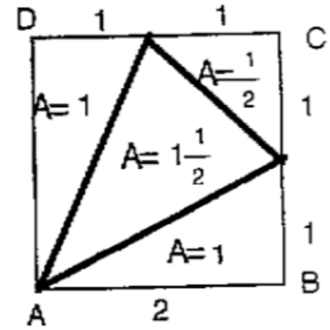
S98J12. The area of the top right triangle is $\frac{1}{2} \cdot 8 \cdot 4 = 16$. The area of the bottom triangle is $\frac{1}{2} \cdot 8 \cdot 4\sqrt{3} = 16\sqrt{3}$. The total area of the kite is $16(1+\sqrt{3})$. The longer diagonal has length $4 + 4\sqrt{3} = 4(1+\sqrt{3})$. Thus the desired ratio is 4.

Answer: 4

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER THREE** **SPRING 1998**

Solutions



S98J13. Since no lengths were given, the easiest approach is to let a side of the given square be any length, say 2, as in the diagram on the side. This leads to the smallest area being $\frac{1}{2}$ and the largest area being $1\frac{1}{2}$. The largest area is therefore three times as large as the smallest area. **Answer:** 3

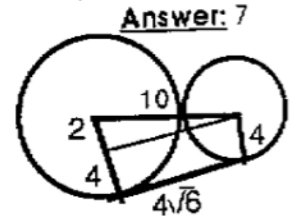
S98J14. The table below shows: The last digit of 1997^n repeats with a cycle of 4. Thus 1997^{1998} has the same last digit as 1997^2, or 9. The last digit of

Exponent	Last digit of 1997^n	Last digit of 1998^n	Last digit of 1999^n
1	7	8	9
2	9	4	1
3	3	2	9
4	1	6	1
5	7	8	9
6	9	4	1
7	3	2	9

1998^n also has a cycle of 4, so 1998^{1999} has the same last digit as 1998^3, or 2. The last digit of 1999^n fluctuates between 9 and 1. Thus 1999^{2000} has the same last digit as 1999^2 or 1. Adding 9+2+1 gives the desired units digit 2. **Answer:** 2

S98J15. The easiest approach is to realize that each of the vertices on the top base must be connected to vertices on the bottom base. To avoid forming edges or surface diagonals, each vertex can be connected to seven bottom vertices. This gives a total of $10 \cdot 7 = 70$ internal diagonals. **Answer:** 70

S98J16. The easiest approach is to use $(a+b)^3 = (a+b)(a^2-ab+b^2)$. This means that $175 = (a+b)(25)$ so that $a+b = 7$. **Answer:** 7



S98J17. The quadrilateral is composed of a rectangle with area $16\sqrt{6}$ and a right triangle with area $4\sqrt{6}$. (Its hypotenuse has length 10 and a leg has length 2. The other leg has length $\sqrt{96} = 4\sqrt{6}$.) The total area is $20\sqrt{6}$.

Answer: $20\sqrt{6}$.

S98J18. To simplify x, look at the two numbers being

subtracted: $\left(\frac{\sqrt{\sqrt{5}+2}-\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}-1}}\right)^2 = 2$. So the first term is $\sqrt{2}$. The second term: $\sqrt{3-2\sqrt{2}}$

$= \sqrt{2-2\sqrt{2}+1}$, $= \sqrt{2}-1$. Thus $x = \sqrt{2} - (\sqrt{2}-1) = 1$ **Answer:** 1

June 3, 1998

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1998 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Senior A	S98S1	$n^2 - n$ or any number of that form > 0 was accepted
Junior	S98J11	was eliminated. The problem should have read "Compute x - y"

Have a great summer!
MATH IS # 1

Sincerely yours,

Richard Geller

Secretary, NYCIML