NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION

CONTEST NUMBER ONE

PART I *FALL, 1997* *CONTEST 1* *TIME: 10 MINUTES*

F97S1 The arithmetic mean (average) of k consecutive integers is 37. If the first integer is also k, compute k.

F97S2 Compute all real values of x which satisfy $|x-6| + |2x-6| + |3x-6| > 12$.

PART II *FALL, 1997* *CONTEST 1* *TIME: 10 MINUTES*

F97S3 If $x^2 - xy = 75$, $xy - y^2 = 50$, and $x > y$, compute the ordered pair (x,y).

F97S4 In circle O, chord AC = 13 and chord AD = 7. Segments CM and DN are drawn perpendicular to diameter AB, where M and N are points in $\overline{AB}$. If AB = 20, then compute MN.

PART III *FALL, 1997* *CONTEST 1* *TIME: 10 MINUTES*

F97S5 Compute all values of x such that ${}_{x^2}C_{5x-14} = {}_{x^2}C_{4x-6}$

F97S6 Given 5 consecutive positive integers such that the first integer is divisible by 7, the second integer is divisible by 6, the third is divisible by 5, the fourth is divisible by 4, and the fifth is divisible by 3. Compute the smallest possible value of the first integer.

ANSWERS:

ANSWERS:	F97S1	25
	F97S2	$(x<1) \vee (x>4.5)$
	F97S3	(15,10)
	F97S4	6
	F97S5	4, 5, 8
	F97S6	413

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION **CONTEST NUMBER TWO**

PART I *FALL, 1997* *CONTEST 2* *TIME: 10 MINUTES*

F97S7 Compute all real values of m such that $\sqrt{m} - \sqrt{m-10} > 1$.

F97S8 The circle which passes through the points $(^{-}5, 12)$, $(9,14)$, and $(3, {}^{-}4)$ has center (h,k) and radius r. Compute the ordered triple (h,k,r).

PART II *FALL, 1997* *CONTEST 2* *TIME: 10 MINUTES*

F97S9 Compute the smallest prime factor of 8,000,027.

F97S10 The bases of a right prism are congruent polygons of n sides. Express the number of all diagonals that can be drawn in simplest form in terms of n. Include both interior and face diagonals.

PART III *FALL, 1997* *CONTEST 2* *TIME: 10 MINUTES*

F97S11 If $\log 2 = a$, express $\log {}^5/_8$ in simplest form, in terms of a, with no logarithms.

F97S12 The roots of $x^3 - 5x^2 + 7x - 9 = 0$ are a, b, and c. The roots of $x^3 - px^2 + qx - r = 0$ are $a+b$, $b+c$, and $c+a$. Compute r.

ANSWERS:

F97S7	$10 \le m < 30.25$
F97S8	(3,6,10)
F97S9	7
F97S10	, or $2n^2 - 4n$
F97S11	$1 - 4a$
F97S12	26

New York City Interscholastic Math League

Senior A Division — **Contest Number Three**

Part I — *Fall, 1997* — *Contest 3* — *Time: 10 Minutes*

F97S13 In a certain golf tournament, each match groups four people together, so that one person wins and goes on to another match in the next round, while the other three lose and are each eliminated. The tournament continues until only one person remains undefeated. If 256 players enter the tournament, how many matches must be played?

F97S14 If $\cos\theta = -n$ for $\frac{\pi}{2} < \theta < \pi$, express $\cot(\theta - \frac{\pi}{2})$ in simplest form in terms of n.

Part II — *Fall, 1997* — *Contest 3* — *Time: 10 Minutes*

F97S15 The roots of $mx^2 - 3x - 2 = 0$ differ by 1. Compute both values of m.

F97S16 The *n*th term of the sequence $\sqrt[3]{1997}, \sqrt[6]{1997}, \sqrt[12]{1997}, \sqrt[24]{1997}, \ldots,$ is given by the formula $\sqrt[3\cdot 2^{n-1}]{1997}$. The product of this infinite sequence is given by $\sqrt[a]{1997^b}$. Compute the ordered pair (a,b) if the highest common factor of a and b is 1.

Part III — *Fall, 1997* — *Contest 3* — *Time: 10 Minutes*

F97S17 Two circles are concentric. In the larger circle two distinct chords are drawn so that each chord is tangent to the smaller circle. If the length of one chord is $x^2 - 7$ and the length of the other chord is $5 - 4x$, compute the value of x.

F97S18 The lengths of the two smaller sides of a triangle are 5 and 8, and the area of the triangle is $4\sqrt{21}$. Compute the length of the third side of the triangle.

Answers:

F97S13	85
F97S14	$\sqrt{1-n^2}/n$.
F97S15	$-1, 9$
F97S16	(3,2)
F97S17	-6
F97S18	11

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER FOUR**

PART I *FALL, 1997* *CONTEST 4* *TIME: 10 MINUTES*

F97S19 A circle is inscribed in a rhombus whose diagonals have lengths of 10 and 24. Compute the length of the radius of the circle.

F97S20 Express $x^4 + 64$ as the product of two trinomials with integral coefficients.

PART II *FALL, 1997* *CONTEST 4* *TIME: 10 MINUTES*

F97S21 If $\tan\theta + \cot\theta = {}^{a}/_{b}$, express $\sin 2\theta$ in simplest form in terms of a and b.

F97S22 Two trains, 1000 miles apart, start directly for each other at the same time. During successive hours, one travels 30 miles, 31 miles, 32 miles, and so on, increasing by one mile each hour. During those same hours, the other train travels 30 miles, 29 miles, 28 miles, and so on, decreasing by one mile each hour. In each hour, each train travels at a constant rate. How many miles apart are they 23 minutes before they pass each other?

PART III *FALL, 1997* *CONTEST 4* *TIME: 10 MINUTES*

F97S23 A deck of cards consists of 1 one, 2 twos, 3 threes, and so on, up to 10 tens, with no other cards. Two cards are picked at random without replacement. Compute the probability that they are a pair (two cards of the same value).

F97S24 Compute all values of x that satisfy: $(\log_4 x^2 - 4)^3 + (\log_8 x^3 - 8)^3 = (\log_2 x^2 - 12)^3$

ANSWERS:

F97S19	${}^{60}/_{13}$
F97S20	$(x^2 - 4x + 8)(x^2 + 4x + 8)$
F97S21	${}^{2b}/_{a}$
F97S22	23
F97S23	${}^{1}/_{9}$
F97S24	16, 64, 256

New York City Interscholastic Math League

Senior A Division | **Contest Number Five**

Part I *Fall, 1997* *Contest 5* *Time: 10 Minutes*

F97S25 The lengths of the sides of a triangle are represented by $x+30$, $2x-6$, and $5x$. Compute all values of x.

F97S26 Given three different positive integers, a, b, and c, that satisfy both $2a+3b+4c=30$ and $4a+3b+2c=42$. Compute the ordered triple (a,b,c).

Part II *Fall, 1997* *Contest 5* *Time: 10 Minutes*

F97S27 Compute the sum of $\sin\frac{\pi}{7} + \sin\frac{4\pi}{7} + \sin\frac{7\pi}{7} + \sin\frac{10\pi}{7} + \sin\frac{13\pi}{7}$.

F97S28 Two circles whose radii measure 5 and 7 are externally tangent. Both common external tangents are drawn. Their common internal tangent intersects the external tangents at points M and N. Compute MN.

Part III *Fall, 1997* *Contest 5* *Time: 10 Minutes*

F97S29 If the eight digit integer AB,123,456 is divisible by 99, compute the ordered pair (A,B).

F97S30 Quadrilateral ABCD is inscribed in circle O and points A, O, and C are collinear. If the four sides of the quadrilateral have different integral lengths and AD = 1, compute the smallest perimeter possible for ABCD.

Answers:

	F97S25	$6 < x < 12$
	F97S26	(7,4,1)
	F97S27	0
	F97S28	$2\sqrt{35}$
	F97S29	(9,6)
	F97S30	20

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER ONE**

SOLUTIONS

F97S1 **Answer: 25** Use the formula $S = \frac{1}{2}(n)(a_1 + a_n)$, where $n = a_1 = k$ and $a_n = k + (k-1)$. Then $37 = S/k = \frac{1}{2}(3k-1)$ and $k = 25$.

F97S2 **Answer: $(x < 1) \vee (x > 4.5)$** If $a - b \geq 0$, then $|a-b| = a - b$, but if $a - b \leq 0$, then $|a-b| = b - a$. Separate the left side into four intervals by setting each absolute value expression equal to zero. Thus, the "break-points" are at 6, 3, and 2.

[1] Let $x \geq 6 \Rightarrow (x-6) + (2x-6) + (3x-6) > 12 \Rightarrow x > 5$. In this interval, $x \geq 6$.

[2] Let $6 > x \geq 3 \Rightarrow (6-x) + (2x-6) + (3x-6) > 12 \Rightarrow x > 4.5$. In this interval, $4.5 < x < 6$.

[3] Let $3 > x \geq 2 \Rightarrow (6-x) + (6-2x) + (3x-6) > 12 \Rightarrow 6 > 12$. In this interval, there are no solutions.

[4] Let $x < 2 \Rightarrow (6-x) + (6-2x) + (6-3x) > 12 \Rightarrow x < 1$. In this interval, $x < 1$.

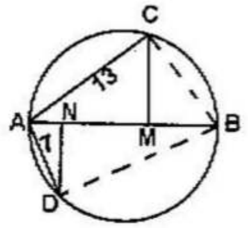
Therefore, $(x < 1) \vee (x > 4.5)$.

F97S3 **Answer: (15,10)** **METHOD 1:** Subtract equations: $x^2 - 2xy + y^2 = 25 \Rightarrow x - y = 5$. Then $x^2 - xy = 75 \Rightarrow x(x-y) = x(5) = 75$. Thus, $(x,y) = (15,10)$.

METHOD 2: Divide equations to get $x/y = 3/2 \Rightarrow y = 2x/3$. Substitute back: $x - y = 5 \Rightarrow x - 2x/3 = 5 \Rightarrow x = 15 \Rightarrow y = 10$.

METHOD 3: Add equations: $x^2 - y^2 = 125$. Then $(x-y)(x+y) = 1\times125$ or 5×25, if x and y are integers or half-integers. The system $x + y = 125$ and $x - y = 1$ produces (63,62) [extraneous], while the system $x + y = 25$ and $x - y = 5$ produces (15,10) [good].

F97S4 **Answer: 6** **METHOD 1:** Complete right triangles ACB and ADB. By the mean proportional theorem, $13^2 = (AM)(20)$ and $7^2 = (AN)20$. Subtracting equations, $13^2 - 7^2 = (MN)20 \Rightarrow MN = 6$. *Note: Placing C and D on the same side of the diameter does not affect the solution or the answer.*



METHOD 2: Let O = the center of the circle. Draw radius OD $= 10$. Let $NO = x$, $AN = 10-x$, and $ND = y$. The system $x^2 + y^2 = 10^2$ and $(10-x)^2 + y^2 = 7^2$ yields $x = 7.55 = NO$. Similarly, $MO = 1.55$. Since $13 < 10\sqrt{2}$, points M and N are on the same side of point O. Thus, $MN = NO - MO = 6$.

F97S5 **Answer: 4, 5, 8** If ${}_nC_a = {}_nC_b$, then either $a = b$ or $a + b = n$.

[Case 1] $5x - 14 = 4x - 6 \Rightarrow x = 8$.

[Case 2] $x^2 = (5x-14) + (4x-6) \Rightarrow x = 4 \vee x = 5$.

$\therefore x \in \{4, 5, 8\}$.

F97S6 **Answer: 413** Add 7 to the first number, 6 to the second number, 5 to the third number, 4 to the fourth number and 3 to the fifth number to get an integer which is a multiple of 7, 6, 5, 4, and 3. The smallest common multiple is 420. Hence, the first of the consecutive integers must be $420 - 7 = 413$.

SOLUTIONS

F97S7 **Answer: $10 \le m < 30.25$** Rearrange terms: $\sqrt{m} - 1 > \sqrt{m-10}$, and square. This yields $m < 30.25$. But $\sqrt{m-10} \ge 0$, so $m \ge 10$. Thus, $10 \le m < 30.25$.

F97S8 **Answer: (3,6,10)** The midpoint of A(−5, 12) and B(9,14) is (2,13). The slope of chord AB is 1/7. The diameter of the circle is the perpendicular bisector of chord AB. Thus the diameter's slope is −7 and its equation is $y - 13 = -7(x - 2)$, or $y = -7x + 27$. Similarly, the equation of the perpendicular bisector (diameter) of the chord determined by B(9,14) and C(3,−4) is $y = -\frac{1}{3}x + 7$. They intersect at the center, so equate: $-7x + 27 = -\frac{1}{3}x + 7$. Thus the center is (3,6) and the radius is 10.

Another way is to equate in pairs the three distances from the center to the given points.

F97S9 **Answer: 7** $8{,}000{,}027 = (200)^3 + (3)^3 = (200 + 3)(200^2 - 200\cdot 3 + 3^2) = 203\times 39{,}409$. Then $203 = 210 - 7 = 7\times 30 - 7\times 1 = 7\times(30 - 1) = 7\times 29$.

F97S10 **Answer: $2n^2 - 4n$ or $2n(n-2)$** Handle as 3 cases: top, bottom, and top-to-bottom. In the top surface of the prism: from each of n vertices $n - 3$ diagonals can be drawn. But since AB and BA are the same segment, the top surface actually contains $\frac{1}{2}n(n-3)$ diagonals. Likewise, the bottom surface of the prism also contains $\frac{1}{2}n(n-3)$ diagonals. From top-to-bottom: from each of n vertices there are $n - 1$ diagonals with no repetitions. So both the lateral faces and the interior of the prism contain $n(n-1)$ diagonals. The total number of diagonals are $2n^2 - 4n$.

Note: This procedure and formula are unaffected by congruence of the bases or by concave vs. convex. Only the bases agreeing in the number of sides is necessary.

F97S11 **Answer: $1 - 4a$** $\log 5/8 = \log 5 - \log 8 = (\log {}^{10}/_{2}) - \log 2^3 = (\log 10 - \log 2) - 3\log 2 = (1 - a) - 3a = 1 - 4a$.

Variation: $\log 5/8 = \log 10/16 = \log 10 - \log 16 = 1 - 4\log 2 = 1 - 4a$.

F97S12 **Answer: 26** First note that $a + b + c = 5$, $ab + bc + ca = 7$ and $abc = 9$.

$$
\begin{aligned}
r &= (a + b)(b + c)(c + a) \\
&= a^2b + ab^2 + b^2c + bc^2 + c^2a + ca^2 + 2abc \\
&= (a^2b + ab^2 + abc) + (abc + b^2c + bc^2) + (ca^2 + abc + c^2a) - abc \\
&= ab(a + b + c) + bc(a + b + c) + ca(a + b + c) - abc \\
&= (ab + bc + ca)(a + b + c) - abc \\
&= (7)(5) - 9 \\
&= 26
\end{aligned}
$$

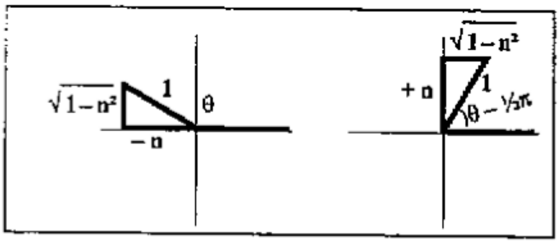
NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION | **CONTEST NUMBER THREE**

SOLUTIONS

F97S13 **Answer: 85** **METHOD I:** To eliminate 255 players three at a time, $255 \div 3 = 85$ matches are needed.
METHOD II: $256 \to 64$ foursomes with 64 winners; $64 \to 16$ foursomes with 16 winners; $16 \to 4$ foursomes with 4 winners; $4 \to 1$ foursomes with 1 winners; Then $64 + 16 + 4 + 1 = 85$ matches.

F97S14 **Answer: $\sqrt{1-n^2}/n$** $\cot(\theta - \frac{1}{2}\pi) = \cos(\theta - \frac{1}{2}\pi) \div \sin(\theta - \frac{1}{2}\pi) = \sin\theta \div -(\cos\theta) = (\sqrt{1-n^2}) \div -(-n) = \sqrt{1-n^2}/n$.



This problem can also be solved by rotating the triangle on the left ¼ turn clockwise about the origin, as shown.

F97S15 **Answer: −1, 9** Using the quadratic formula, the difference of the roots of $mx^2-3x-2=0$ is

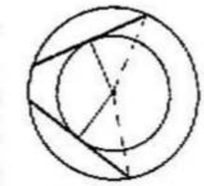
$$\frac{3+\sqrt{9+8m}}{2m} - \frac{3-\sqrt{9+8m}}{2m} = 1$$

This yields $m^2 - 8m - 9 = 0$, so that $m = -1, 9$.

Another approach is to use the formulas for sum and product of the roots of a quadratic equation and then solve the resulting system of equations in *m*.

F97S16 **Answer: (3, 2)** The product $= 1997^{1/3} \cdot 1997^{1/6} \cdot 1997^{1/12} \cdot 1997^{1/24} \cdot \ldots = 1997^{1/3+1/6+1/12+1/24+\cdots} = 1997^{2/3} = \sqrt[3]{1997^2}$. Thus, $(a,b) = \mathbf{(3,2)}$.

F97S17 **Answer: −6** Since both chords are the same distance from the center, the chords are congruent. Thus $x^2 - 7 = 5 - 4x \Rightarrow x = -6$ or $x = 2$. Suppose $x = -6$: since $(-6)^2 - 7$ and $5 - 4(-6)$ are positive (29), the chords are possible. *Accept −6.* Suppose $x = 2$: since $(2)^2 - 7$ and $5 - 4(2)$ are negative (-3), the chords are not possible. *Reject 2.* The only answer is −6. *Note: Without loss of generality, make the chords parallel. Seeing relationships becomes easier. In fact, thinking of the chords as rotating about the center allows us to regard them as the same chord.*



F97S18 **Answer: 11** Let C = the measure of the included angle and let c = the length of the desired side. Using the SAS area formula, $4\sqrt{21} = \frac{1}{2}\cdot 8\cdot 5 \sin C$, so that $\sin C = \frac{1}{5}\sqrt{21}$. Then using $\sin^2 C + \cos^2 C = 1$, $\cos C = \pm \frac{2}{5}$. Finally, $c^2 = 5^2 + 8^2 - 2\cdot 5\cdot 8\cdot \cos C = 89 \pm 32 = 121$ or 57. Since c is the largest side of the triangle, $c = 11$. *Another method is possible using Hero's Formula.*

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE

SENIOR A DIVISION

CONTEST NUMBER FOUR

SOLUTIONS

F97S19 **Answer: 60/13** The diagonals divide the rhombus into four right triangles whose sides measure 5, 12, and 13. The altitude to the hypotenuse of the triangle is also a radius *r* of the circle. Then the area of the triangle = $\frac{1}{2} \cdot 5 \cdot 12 = \frac{1}{2} \cdot 13 \cdot r$. Thus $r = {}^{60}/_{13}$.

F97S20 **Answer: $(x^2 - 4x + 8)(x^2 + 4x + 8)$** **METHOD 1:** To factor, rewrite $x^4 + 64$ as the difference of two squares, where the first trinomial is a perfect square: $(x^4 + 16x^2 + 64) - (16x^2)$. Then $(x^2 + 8)^2 - (4x)^2$ factors into $[(x^2 + 8) - (4x)][(x^2 + 8) + (4x)] = (x^2 - 4x + 8)(x^2 + 4x + 8)$.

METHOD 2: Assume $x^4 + 64 = (x^2 + ax + b)(x^2 + cx + d)$, where b and d have the same sign. Because in $x^4 + 64$ the coefficient of x^3 is 0, $a = -c$. Because in $x^4 + 64$ the coefficient of x is 0, $b = d = 8$. Then $(x^2 + ax + 8)(x^2 - ax + 8)$ produces $a = \pm 4$.

F97S21 **Answer: 2b/a** $a/b = \tan\theta + \cot\theta = \sin\theta / \cos\theta + \cos\theta / \sin\theta = (\sin^2\theta + \cos^2\theta) / \sin\theta\cos\theta = 1 / \sin\theta\cos\theta$. $\therefore \sin\theta\cos\theta = b/a$, and $\sin 2\theta = 2b/a$.

F97S22 **Answer: 23** Together they travel 60 miles every hour. Thus they travel 1 mile every minute. Twenty-three minutes before they meet, they will be 23 miles apart.

F97S23 **Answer: 1/9** $({}_2C_2 + {}_3C_2 + {}_4C_2 + {}_5C_2 + {}_6C_2 + {}_7C_2 + {}_8C_2 + {}_9C_2 + {}_{10}C_2) \div {}_{55}C_2 = (1+3+6+10+15+21+28+36+45) \div (55 \cdot 27) = 165 \div (55 \cdot 27) = 3 \div 27 = 1/9$.

F97S24 **Answer: 16, 64, 256** By the Change-of-Base Law, $\log_4 x^2 = (2 \log x)/(2 \log 2) = \log_2 x$. Similarly, $\log_8 x^3 = (3 \log x) / (3 \log 2) = \log_2 x$. Since the sum of $\log_2 x - 4$ and $\log_2 x - 8$ is $\log_2 x^2 - 12$, use the theorem, *If $A^3 + B^3 = (A + B)^3$, then $A = 0$, $B = 0$, or $A + B = 0$.* If $\log_4 x^2 - 4 = 0$, then $x = 16$. If $\log_8 x^3 - 8 = 0$, then $x = 256$. If $\log_2 x^2 - 12 = 0$, then $x = 64$. The roots are 16, 64, and 256.

SOLUTIONS

F97S25 **Answer: $6 < x < 12$** Since $5x > 0$, x is positive. $\therefore$ $5x > 2x - 6$, and $2x - 6$ cannot be the largest side. Then use the Triangle Inequality for the other two cases:

I. Assume $(5x)$ is the largest side: $(x + 30) + (2x - 6) > 5x$. Thus $x < 12$.

II. Assume $(x + 30)$ is the largest side: $(5x) + (2x - 6) > x + 30$. Thus, $x > 6$.

Therefore, $6 < x < 12$.

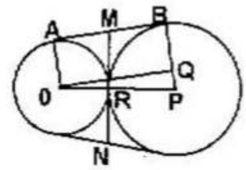
F97S26 **Answer: (7, 4, 1)** Add both equations to get $6a + 6b + 6c = 72$, or $a + b + c = 12$. Subtract both equations to get $2a - 2c = 12$, or $a = c + 6$. Substitute the second result into the first result to get $b = -2c + 6$. Since each variable is a positive integer, c can only be 1 or 2. Then the two triples are (7,4,1) and (8,2,2). Since the variables are distinct, the only triple is (7,4,1).

F97S27 **Answer: 0** **METHOD 1:** In the unit circle, the points $\pi/7$ and $13\pi/7$ are reflections of each other over the x-axis. Their sines are opposites so the sum of their sines is zero. Likewise, the points $4\pi/7$ and $10\pi/7$ also are reflections of each other over the x-axis and the sum of their sines is zero. The point $7\pi/7$ lies on the x-axis, so its sine is zero. The sum of zeros is zero.

METHOD 2: Graph the sine curve. Note that $\sin x = -\sin(2\pi - x)$ for every point between 0 and 2π. The sum of the given sines is zero.

METHOD 3: Note that $\pi/7 = \pi - 6\pi/7$ while $13\pi/7 = \pi + 6\pi/7$. Then $\sin(\pi + 6\pi/7) + \sin(\pi - 6\pi/7) = 2\sin\pi\cos 6\pi/7 = 0$. Etcetera.

F97S28 **Answer: $2\sqrt{35}$** $OP = 7 + 5 = 12$. $QP = 7 - 5 = 2$. $\therefore$ by Pythagoras $\sqrt{140} = 2\sqrt{35} = OQ = AB$. Since tangents from a point to a circle are congruent, $MA = MR = MB$. $\therefore$ $MN = AB = 2\sqrt{35}$.



F97S29 **Answer: (9,6)** Any number divisible by 99 is divisible by 11 and 9 (and 3). Since 9 divides AB123456, 9 divides the sum of the digits, $21 + A + B$. So $A + B = 6$ or 15. Since 11 divides AB123456, then 11 divides $(9 + A) - (12 + B) = A - B - 3$. A is a single digit so $A = B + 3$. Substituting, $2B + 3 = 6$ or $2B + 3 = 15$. Thus, $B = 6$ and $A = 9$.

F97S30 **Answer: 20** AC is a diameter, so ΔABC and ΔADC are right triangles. Let $AB = a$, $BC = b$, $CD = c$, and $DA = 1$. Then $a^2 + b^2 = (AC)^2 = c^2 + 1^2 \Rightarrow a^2 - 1 = c^2 - b^2 \Rightarrow (a-1)(a+1) = (c-b)(c+b)$. $a-1$ and $a+1$ differ by 2 while $c-b$ and $c+b$ represent another pair of factors of the same product.

Suppose $a - 1 = 1$. Then $a + 1 = 3$ and $c^2 - b^2 = 3$. This yields $c - b = 1$ and $c + b = 3$. Reject, because at least two sides will have the same length..

Suppose $a - 1 = 2$. Then $a + 1 = 4$, and $c^2 - b^2 = 8$. For c and b to be integers, $c - b$ and $c + b$ must have the same parity. Reject, because at least two sides will have the same length..

Suppose $a - 1 = 3$. Then $c^2 - b^2 = 15$. To obtain unique lengths, $c - b$ must equal 1 and $c + b$ must equal 15. Thus $(a, b, c, 1) = (4, 7, 8, 1)$, and the least perimeter is 20.