New York City
Interscholastic
Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER ONE** | **FALL 1997**

PART I: 10 Minutes | **NYCIML Contest One** | **Fall 1997**

F97J1. The sum of the first 1000 positive even integers is A. The sum of the first 1000 positive odd integers is B. Compute A–B.

F97J2. Compute the product of all x values such that $x! + \frac{48}{x!} = 26$. (Note that x! stands for x factorial.)

PART II: 10 Minutes | **NYCIML Contest One** | **Fall 1997**

F97J3. Compute the units digit of $(1997)^{1997}$.

F97J4. In the game of "TETRIS," polygons drop down to the floor, one after the other. Each polygon is made up of four squares, each with at least one edge pasted to an edge of another square. Two polygons are considered to be duplicates if one of them can be rotated and/or translated to coincide with the other. How many polygons must drop before a duplicate polygon *must* drop?

PART III: 10 Minutes | **NYCIML Contest One** | **Fall 1997**

F97J5. In an isosoceles triangle, a leg has length $3x - 2$ and the base has length $4x + 10$. Compute the set of all possible x values.

F97J6. An old fashioned bicycle was constructed so that the radius of the front wheel is 25 inches and the radius of the back wheel is 10 inches. If the common internal tangent is perpendicular to the ground, find the distance between the centers of the two wheels.

Answers

1. 1000	**3.** 7	**5.** $\{x \mid x>7\}$ or $x > 7$
2. 8	**4.** 7	**6.** $\sqrt{1450}$ or equivalent

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JUNIOR DIVISION | **CONTEST NUMBER TWO** | **FALL 1997**

PART I: 10 Minutes | **NYCIML Contest Two** | **Fall 1997**

F97J7. If n! represents n factorial, compute all values of x such that $x^2 = (4! + 4! + 4! + 4! + 4!)^2$.

F97J8. Compute the number of rings that can be formed using the areas between 1997 concentric circles.

PART II: 10 Minutes | **NYCIML Contest Two** | **Fall 1997**

F97J9. Points A and B lie 90° apart on Circle O with radius 6. If D is a point on minor arc $\overset{\frown}{AB}$ equidistant from point E on $\overline{OA}$ and point C on $\overline{OB}$, find the area of OCDE

F97J10. Two sides of an isosceles triangle have lengths 2x + 3 and 3x − 2. If the perimeter of the triangle is 25, compute the sum of all possible x values.

PART III: 10 Minutes | **NYCIML Contest Two** | **Fall 1997**

F97J11. A line passes through the point (a,b) and has slope m and x-intercept $\frac{c}{m}$. Compute c in terms of a, b, and m.

F97J12. In the game of "PENTRIS," polygons drop down to the floor, one after the other. Each polygon is made up of *five* squares, each with at least one edge pasted to an edge of another square. Two polygons are considered to be duplicates if one of them can be rotated and/or translated to coincide with the other. How many polygons must drop before a duplicate polygon *must* drop?

Answers

7. −120, +120	**9.** 18	**11.** ma−b
8. 1993006	**10.** $6\frac{1}{4}$	**12.** 18

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER THREE** **FALL 1997**

PART I: 10 Minutes **NYCIML Contest Three** **Fall 1997**

F97J13. Points P and Q lie 90° apart on Circle O with radius 6. Tangents drawn at P and Q meet at A. Compute the area of ΔAPQ.

F97J14. The length of a side of a square is 2. The corners are cut off so that the resulting figure is a regular octagon. Compute the area of the octagon.

PART II: 10 Minutes **NYCIML Contest Three** **Fall 1997**

F97J15. Compute the value of $v - z$ if

$$v + w = 10$$
$$w + x = 9$$
$$x + y = 8$$
$$y + z = 7$$

F97J16. Four couples go to the movies. In how many ways can they be seated in eight seats if each couple must sit together?

PART III: 10 Minutes **NYCIML Contest Three** **Fall 1997**

F97J17. A room in the shape of a cube has an edge 6 feet long. A fly starts from the midpoint of a bottom edge and flies to an opposite upper corner. Compute the number of feet it travels.

F97J18. The length of an edge of a cube is 10 inches. Compute the volume of the octahedron formed by connecting the centers of the faces of the cube.

Answers

13. 18 **15.** 2 **17.** 9

14. $8\sqrt{2} - 8$ (or equivalent) **16.** 384 **18.** $\frac{500}{3}$ (or equivalent)

New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER ONE** | **FALL 1997**

Solutions

F97J1. A wonderful problem solving technique is to change the problem to a *simpler* one and then try to generalize. Let N = the amount of numbers added. If N = 2, A = 2+4 = 6, B = 1 + 3, and A–B = 2. If N = 3, A = 2+4 + 6 = 12, B = 1 + 3 + 5 = 9, and A–B = 3. If N = 4, A = 20, B = 16, so A–B =4. If N = 5, A = 30, B = 25, so A–B = 5. In general, A – B = N.

Answer: 1000

F97J2. $x! + \frac{48}{x!} = 26$ ☞ $(x!)^2 + 48 = 26x!$ ☞ $(x!)^2 - 26x! + 48 = 0$

$(x!-2)(x!-24) = 0$ ☞ $x! = 2$ or $x! = 24$ ☞ $x = 2$ or $x = 4$ **Answer:** 8

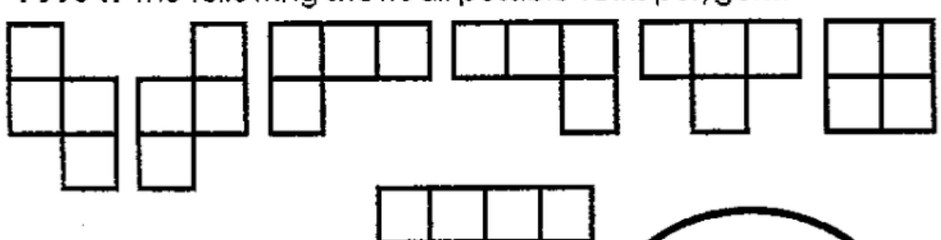
F97J3. A number, x, ending in "7" has the following units digit:

Power of x	Units Digit
x^0	1
x^1	7
x^2	9
x^3	3
x^4	1
x^5	7

Power of x	Units Digit
x^6	9
x^7	3
x^8	1
x^9	7
x^{10}	9
• • •	

The pattern indicates that one should reduce the exponent modulo 4 and use this table. Thus $(1997)^{1997}$ has the same units digit as x^1 or 7. **Answer:** 7

F97J4. The following shows all possible Tetris polygons:

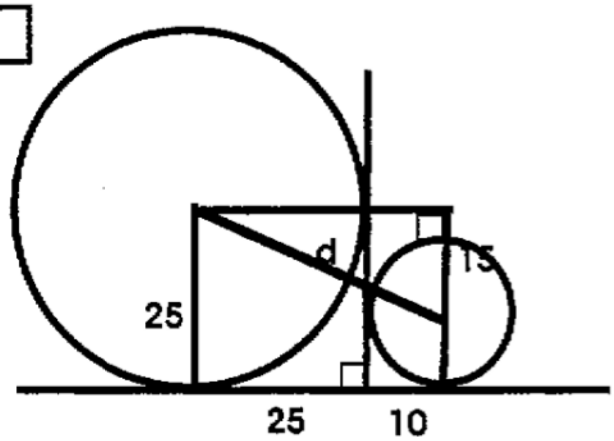


Answer: 7

F97J5. The triangle inequality yields two inequalities: $6x-4 > 4x + 10$ which gives $x > 7$. Also, $7x + 8 > 3x-2$ or $x > -\frac{5}{2}$ **Answer:** $\{x \mid x>7\}$

F97J6. $35^2 + 15^2 = d^2$, so $d^2 = 1450$

Thus $d = \sqrt{1450}$ **Answer:** $\sqrt{1450}$



Please note: Concepts used today will be repeated later this year.

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER TWO** **FALL 1997**

Solutions

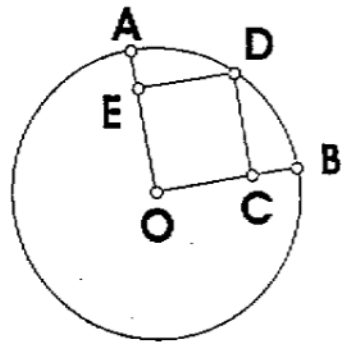
F97J7. $x^2 = (4! + 4! + 4! + 4! + 4!)^2$ ☞ $x^2 = (5 \cdot 4!)^2$ ☞ $x^2 = (5!)^2$

Thus, $x = \pm 120$. **Answer:** ± 120

F97J8. Since two concentric circles create a ring, there can be ${}_{1997}C_2 = \frac{1997 \cdot 1996}{2} = 1993006$ such rings. **Answer:** 1993006

F97J9. OCDE must be a square, its diagonal has length 6 so that its area is $\frac{1}{2} \cdot 6^2 = 18$

Answer: 18



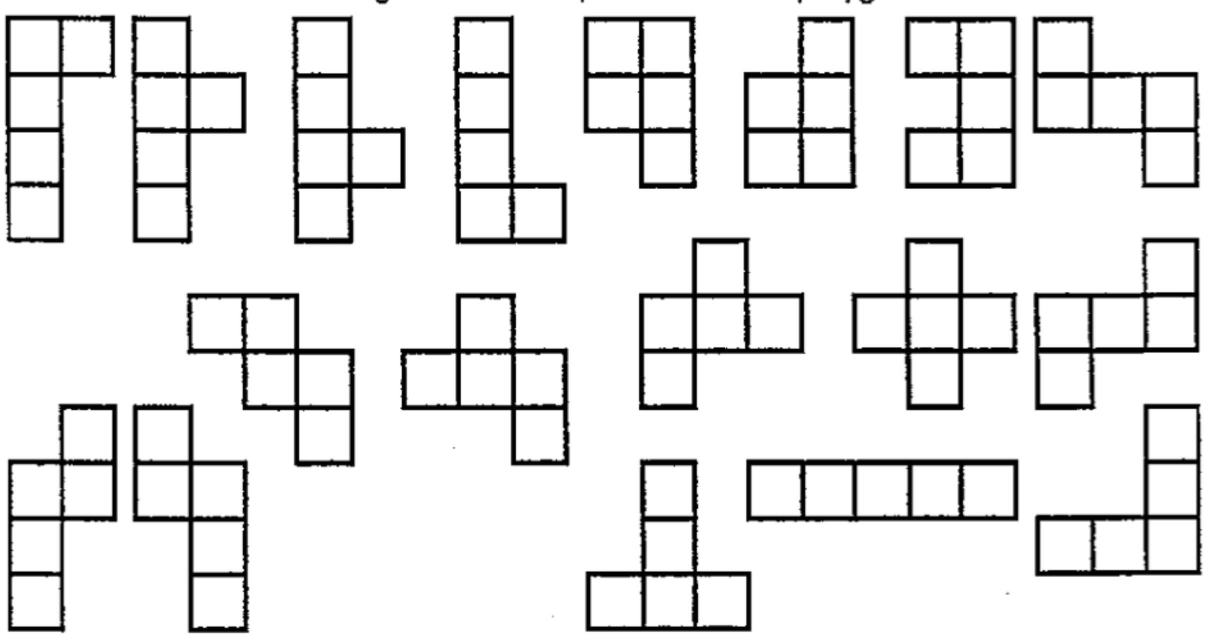
F97J10. The sides of the triangle may have lengths $3x-2$, $3x-2$, and $2x+3$ which gives a perimeter of $8x-1 = 25$ ☞ $x = 3\frac{1}{4}$.

The three sides can also have lengths $3x-2$, $2x+3$, and $2x+3$ which gives a perimeter of $7x+4=25$ ☞ $x = 3$. A third possibility is that $2x + 3 = 3x - 2$ ☞ $x = 5$. (reject)

The sum of all possible x values is therefore $6\frac{1}{4}$. **Answer:** $6\frac{1}{4}$.

F97J11. The equation of the line is $y - b = m(x - a)$ ☞ $y = mx - ma + b$. If $y = 0$, $mx = ma - b$ ☞ $x = \frac{ma-b}{m}$ ☞ $c = ma - b$ **Answer:** $ma - b$

F97J12. The following shows all possible Pentris polygons:



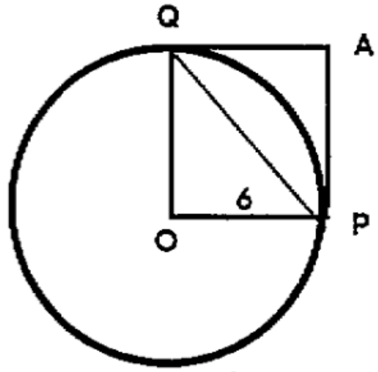
Answer: 18

New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER THREE** | **FALL 1997**

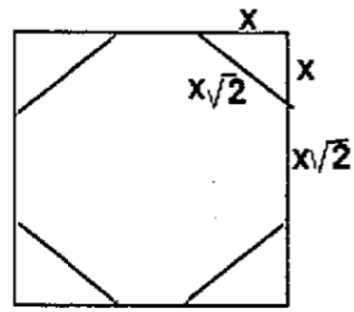
Solutions

F97J13. Connect the center to Q and to P. OPAQ is a square with area 36. The area of ΔAPQ is therefore 18.



Answer: 18

F97J14. Let x= the length cut off from each side. $2x + x\sqrt{2} = 2$ ☞ $x = \frac{2}{2+\sqrt{2}}$ ☞ $x^2 = \frac{4}{6+4\sqrt{2}} = \frac{2}{3+2\sqrt{2}}$. The total area cut off is $2x^2 = \frac{4}{3+2\sqrt{2}} = 12 - 8\sqrt{2}$. The area of the square is 4. The area of the octagon is $8\sqrt{2} - 8$.



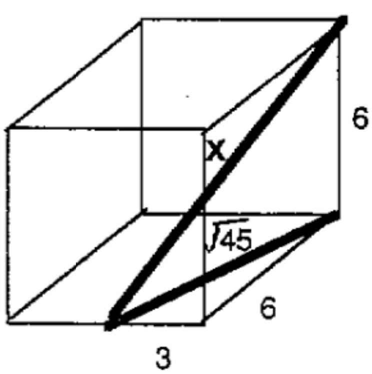
Answer: $8\sqrt{2} - 8$.

F97J15. Subtracting the first two equations, $v-x = 1$. This equation, and the third equation, $x+y = 8$, gives $v+y = 9$. Since $y+z=7$, $v-z = 2$. **Answer:** 2

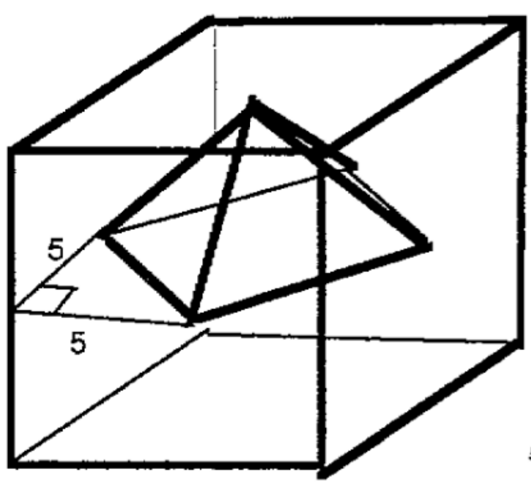
F97J16. There are 4! ways of permuting four couples. Each couple can be permuted in 2 ways. Thus the answer is $4!\cdot2\cdot2\cdot2\cdot2 = 384$ **Answer:** 384

F97J17. Let x = the distance flown. $6^2+45 = x^2$. so $x = 9$.

Answer: 9



F97J18. In the diagram on the right, only the top half of the octahedron is shown. It is a pyramid a square base, each of whose side has length $5\sqrt{2}$. The height of the pyramid is 5 and its tvolume is $\frac{1}{3}\cdot(5\sqrt{2})^2\cdot5 = \frac{250}{3}$. The volume of the octahedron is double this or $\frac{500}{3}$.



Answer: $\frac{500}{3}$.

January 25, 1998

Dear Math Team Coach,

Enclosed is your copy of the Fall, 1997 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Junior	F97J9 was eliminated	It was poorly worded.

Have a great Spring term!

Sincerely yours,

Richard Geller

Secretary, NYCIML