NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER ONE**

PART I: TIME 10 MINUTES SPRING 1997

S97B1 Find the absolute value of the difference between the roots of $x^2+8x+1=0$.

S97B2 Alone, a man can do a job in 8 minutes. Together, the man and his son can do the job in 2 minutes. How long, in minutes, would it take the son to do the job alone?

PART II: TIME 10 MINUTES SPRING 1997

S97B3 Solve for all values of x: $x+\sqrt{x+1}=5$.

S97B4 Two circles have radii 2 and 3, and the distance between their centers is 15. Compute the length of their common internal tangent.

PART III: TIME 10 MINUTES SPRING 1997

S97B5 In terms of x, compute the area of an isosceles right triangle with perimeter x.

S97B6 If $\left(a+\frac{1}{a}\right)^2=5$ and a is positive, compute the value of $a^3+\frac{1}{a^3}$.

ANSWERS

1. $\sqrt{60}$ or $2\sqrt{15}$
2. $\frac{8}{3}$
3. 3
4. $\sqrt{200}$ or $10\sqrt{2}$
5. $\frac{x^2(3-2\sqrt{2})}{4}$
6. $2\sqrt{5}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER TWO**

PART I: TIME 10 MINUTES SPRING 1997

S97B7 Compute the area of the largest triangle which can be inscribed in a semicircle with radius 5?

S97B8 Compute the sum of the coefficients of the expansion of $(x-3)^{10}$.

PART II: TIME 10 MINUTES SPRING 1997

S97B9 A man drove a distance of 144 miles. If he had driven 6 miles per hour faster, he could have made the trip in 20 minutes less time. How fast did he drive?

S97B10 In how many different ways can 6 charms be arranged on a circular bracelet?

PART III: TIME 10 MINUTES SPRING 1997

S97B11 A 25 foot ladder is placed against a vertical wall. The foot of the ladder is 15 feet from the wall. If the foot is pulled 9 more feet away from the wall, how far down the wall does the top of the ladder slip?

S97B12 $\frac{x}{y}+\frac{x}{y^2}+\frac{x}{y^3}+\ldots$ is an infinite series with a sum of $\frac{1}{3}$. If x and y are one digit positive integers, find all possible ordered pairs (x,y).

ANSWERS

7. 25	9. 48	11. 13
8. 1024	10. 60	12. (1,4), (2,7)

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER THREE**

PART I: TIME 10 MINUTES **SPRING 1997**

S97B13 At a party, 66 handshakes are exchanged. If everyone shakes hands with everyone else, how many people were at the party?

S97B14 Compute the value of $\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$.

PART II: TIME 10 MINUTES **SPRING 1997**

S97B15 If $4x^2+4x+y^2+6y=-10$, find the ordered pair (x,y).

S97B16 The denominator of $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$ is rationalized, and the new denominator is 6. Find the numerator of the new fraction.

PART III: TIME 10 MINUTES **SPRING 1997**

S97B17 The diagonals of a rectangle intersect at a point which is 5 inches closer to the length than to the width. If the perimeter of the rectangle is 76, find the area of the rectangle.

S97B18 How many integral solutions exist for $\left(x^2-x-1\right)^{\left(x^2-10x+24\right)}=1$?

ANSWERS

13. 12

14. $\dfrac{1+\sqrt{5}}{2}$

15. $\left(\dfrac{-1}{2},-3\right)$

16. $3+\sqrt{6}+\sqrt{15}$

17. 336

18. 5

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER FOUR**

PART I: TIME 10 MINUTES **SPRING 1997**

S97B19 Bob is three times as old as Bill. Twelve years ago, Bob was six times as old as Bill. How old is Bob now?

S97B20 Nine square tiles are placed on a 3x3 larger square. Three of these tiles are chosen at random. Compute the probability that they are in a horizontal, vertical, or diagonal row.

PART II: TIME 10 MINUTES **SPRING 1997**

S97B21 The points (-3,1), (5,-2) and (7,k) lie on a straight line. Compute the value of k.

S97B22 If xy = 7 and $\frac{1}{x^2}+\frac{1}{y^2}=9$, compute the value of $(x+y)^2$.

PART III: TIME 10 MINUTES **SPRING 1997**

S97B23 An equilateral triangle with area 18 is inscribed in a circle. Compute the area of the circle.

S97B24 Written in base b, $(34)^2_{\text{base b}} = 1277_{\text{base b}}$. Compute b.

ANSWERS

19. 60

21. $\frac{-11}{4}$

23. $\frac{24\pi}{\sqrt{3}}$ or $8\pi\sqrt{3}$

20. $\frac{2}{21}$

22. 455

24. 9

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER FIVE**

PART I: TIME 10 MINUTES **SPRING 1997**

S97B25 If $(3!)(5!)(7!) = (b!)$, compute b.

S97B26 Compute the average of the first 100 odd integers.

PART II: TIME 10 MINUTES **SPRING 1997**

S97B27 If $4^{x+y} = 32$ and $32^{y-x} = 4$, compute the product xy.

S97B28 How many ounces of water must be evaporated from 20 ounces of a 4% acid solution to make a 6% acid solution?

PART III: TIME 10 MINUTES **SPRING 1997**

S97B29 Compute the product $\left(1-\frac{1}{4}\right)\left(1-\frac{1}{5}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{7}\right)\ldots\left(1-\frac{1}{100}\right)$.

S97B30 The medians to the legs of a right triangle are 4 and 7 respectively. Compute the length of the hypotenuse.

ANSWERS

25. 10

26. 100

27. $\frac{609}{400}$

28. $6\frac{2}{3}$

29. $\frac{3}{100}$

30. $\sqrt{52}$ or $2\sqrt{13}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

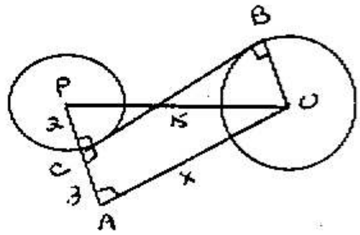
SENIOR B SOLUTIONS SPRING, 1997 CONTEST ONE

S97B1 $\frac{-8+\sqrt{60}}{2}-\left(\frac{-8-\sqrt{60}}{2}\right)=\sqrt{60}$ or $2\sqrt{15}$.

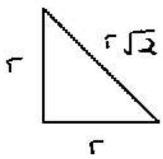
S97B2 $\frac{2}{8}+\frac{2}{x}=1.\ x=\frac{16}{6}=\frac{8}{3}$

S97B3 $\sqrt{x+1}=5-x.\ x+1=25-10x+x^2.\ x=8,3.$ But 8 does not check. Therefore, x = 3.

S97B4 $x^2+5^2=15^2.$
$x=\sqrt{200}.$
$BC=OA=\sqrt{200}.$



S97B5
$$\begin{cases} 2r+r\sqrt{2}=x.\ r=\frac{x}{2+\sqrt{2}}. \\ A=\frac{r^2}{2}=\frac{x^2}{12+8\sqrt{2}}=\frac{x^2\left(3-2\sqrt{2}\right)}{4}. \end{cases}$$



S97B6 $a+\frac{1}{a}=\sqrt{5}.\ a^2+2+\frac{1}{a^2}=5.\ a^2+\frac{1}{a^2}=3.$

$$\left(a+\frac{1}{a}\right)\left(a^2+\frac{1}{a^2}\right)=3\sqrt{5}=a^3+a+\frac{1}{a}+\frac{1}{a^3}=a^3+\sqrt{5}+\frac{1}{a^3}.$$

$$a^3+\frac{1}{a^3}=3\sqrt{5}-\sqrt{5}=2\sqrt{5}.$$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1997 CONTEST TWO

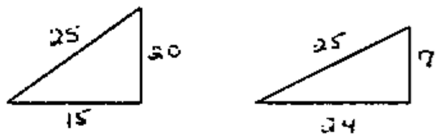
S97B7 The largest triangle will have the diameter as its base and the perpendicular radius as its height. $A=\frac{1}{2}\cdot 5\cdot 10=25$.

S97B8 Using x = 1, the sum of the coefficients is $(1-3)^{10}=(-2)^{10}=1024$.

S97B9 Let x = rate. Since $\frac{D}{R}=T$, $\frac{144}{x}=\frac{144}{x+6}+\frac{1}{3}$. Clearing the fractions, $x^2+6x-2592=0$. $x=48$.

S97B10 The number of ways N objects can be arranged in a circle is (N - 1)! However, since a charm bracelet can be turned over, the number is $\frac{5!}{2}=60$.

S97B11 The top slips 13 feet.



S97B12 Using the sum of an infinite series, $\frac{1}{3}=\frac{\frac{x}{y}}{1-\frac{1}{y}}=\frac{x}{y-1}$. Since x and y are one digit integers, $\frac{1}{4}$ *and* $\frac{2}{7}$ are the only possibilities. (1,4) and (2,7).

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1997 CONTEST THREE

S97B13 $66=\dfrac{N(N-1)}{2}$. N = 12.

S97B14 $x=\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$

$x=\sqrt{1+x}$. $x^2=1+x$. $x^2-x-1=0$. $x=\dfrac{1+\sqrt{5}}{2}$. (negative answer is rejected)

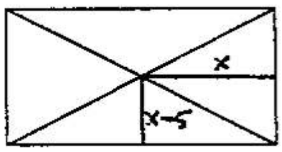
S97B15 $4x^2+4x+y^2+6y=-10$, $4x^2+4x+1+y^2+6y+9=0$,

$(2x+1)^2+(y+3)^2=0$. $\left(\dfrac{-1}{2},-3\right)$.

S97B16 $\dfrac{\sqrt{2}}{\sqrt{2}+\sqrt{3}-\sqrt{5}}\cdot\dfrac{\sqrt{2}+\sqrt{3}+\sqrt{5}}{\sqrt{2}+\sqrt{3}+\sqrt{5}}=$

$\dfrac{2+\sqrt{6}+\sqrt{10}}{2\sqrt{6}}\cdot\dfrac{\sqrt{6}}{\sqrt{6}}=\dfrac{2\sqrt{6}+6+2\sqrt{15}}{12}=\dfrac{3+\sqrt{6}+\sqrt{15}}{6}$.

S97B17 P = 2(2x) + 2(2[x - 5]). 4x + 4x - 20 = 76.
8x = 96. x = 12.
A = 24·14 = 336.



S97B18 If $x^2 - x - 1 = 1$, $x^2 - x - 2 = 0$, x = 2, -1.
If $x^2 - 10x + 24 = 0$, x = 6, 4.
If $x^2 - x - 1 = -1$, $x^2 - x = 0$, x = 0, 1.
But, the exponent is only even if x = 0, not if x = 1. Therefore, the solutions are 0, -1, 2, 4, 6.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1997 CONTEST FOUR

S97B19 3x - 12 = 6(x - 12). x = 20. 3x = 60.

S97B20 There are 8 ways that the 3 squares will be in a row.

$$P=\frac{8}{{}_9C_3}=\frac{8}{84}=\frac{2}{21}.$$

S97B21 The slope of AB = the slope of BC.

$$\frac{3}{-8}=\frac{k+2}{2}.\quad k=\frac{-11}{4}.$$

S97B22 $(x+y)^2=x^2+2xy+y^2$. $\frac{1}{x^2}+\frac{1}{y^2}=9$. $\frac{x^2+y^2}{x^2y^2}=9$. $\frac{x^2+y^2}{49}=9$.

$x^2+y^2=441. (x\ \ +y\ )^2=441+2(7)=455$

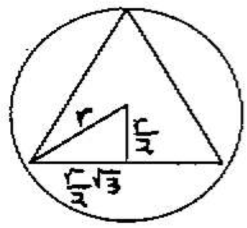
S97B23 It is easiest to find the ratio of the areas of a circle and its inscribed

$$s=r\sqrt{3}.$$

equilateral triangle.

$$A=\frac{\left(r\sqrt{3}\right)^2}{4}\cdot\sqrt{3}=\frac{3}{4}\cdot r^2\sqrt{3}.$$

$$\frac{Area_{triangle}}{Area_{circle}}=\frac{\frac{3}{4}\cdot r^2\sqrt{3}}{\pi r^2}=\frac{3\sqrt{3}}{4\pi}.$$



$$\frac{18}{x}=\frac{3\sqrt{3}}{4\pi}.\quad x=\frac{72\pi}{3\sqrt{3}}=\frac{24\pi}{\sqrt{3}}.$$

S97B24 $(3b + 4)^2 = b^3 + 2b^2 + 7b + 7$.

$9b^2 + 24b + 16 = b^3 + 2b^2 + 7b + 7$

$b^3 - 7b^2 - 17b - 9 = 0$.

$(b + 1)(b + 1)(b - 9) = 0$. b = 9 is the only possible answer.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1997 CONTEST FIVE

S97B25 $(3!)(5!)(7!) = 3 \cdot 2 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 10!$

S97B26 The sum = $\frac{100}{2}(1+199)=50 \cdot 200=10{,}000.$ $\frac{10{,}000}{100}=100.$

S97B27

$2^{2(x+y)}=2^5 \quad 2x+2y=5 \quad 10x+10y=25$

$2^{5(y-x)}=2^2 \quad 5y-5x=2 \quad -10x+10y=4$

Solving for x and y, we obtain, $x=\frac{21}{20}$ *and* $y=\frac{29}{20}$, *thus* $xy=\frac{609}{400}$.

S97B28 Let x = amount of water to be evaporated.

$.04(20) = .06(20 - x).$ $x = 6\frac{2}{3}$.

S97B29 $\frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6} \cdot \frac{6}{7} \cdots \frac{99}{100} = \frac{3}{100}$.

S97B30

$x^2 + (2y)^2 = 4^2$

$(2x)^2 + y^2 = 7^2$

$5x^2 + 5y^2 = 65$

$4x^2 + 4y^2 = c^2 = 52$

Thus, $c=\sqrt{52}$.