NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER ONE

PART I | TIME: 10 MINUTES | SPRING 1997

S97S1 The length, width and height of a rectangular solid are in the ratio of 4:6:9 respectively. If the volume of the solid is 6,000, compute the volume of the sphere inscribed in a cube whose edge is equal to the width of the rectangular solid.

S97S2 Two numbers, x and y, are inserted between $\sqrt{5}$ and $\sqrt{180}$ such that the first three numbers form a geometric progression while the last three numbers form an arithmetic progression. Compute the sum of all possible values of x.

PART II | TIME: 10 MINUTES | SPRING 1997

S97S3 Legend has it that if Newton and Liebnitz had worked together, they would have discovered all the calculus completely in 30 years. However, if they had only worked together for 18 years, it would have taken Newton an additional 20 years to finish. Compute the number of years this legend suggests that it would have taken Liebnitz to discover all the calculus by himself?

S97S4 A square is constructed on one side of a regular hexagon and a regular octagon is constructed on the parallel opposite side of the hexagon. The perimeter of the figure containing the square, hexagon and octagon is $\sqrt{392}$. If the distance from the center of the square to the center of the octagon can be expressed as $a+b\sqrt{2}+c\sqrt{3}$, write a+b+c in simplest form.

PART III | TIME: 10 MINUTES | SPRING 1997

S97S5 If $\log_a b^3 = c$ and $\log_b a^2 = d$, compute, in simplest form with no logarithms, the value of cd.

S97S6 Compute the number of distinct positive integers such that the sum of its not necessarily distinct prime factors is 14.

ANSWERS

S97S1 1000π

S97S2 $\sqrt{5}/2$

S97S3 75

S97S4 $2+\sqrt{2}$

S97S5 6

S97S6 10

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER TWO

PART I — TIME: 10 MINUTES — SPRING 1997

S97S7 Compute N, if N is a 3 digit base ten number whose middle digit is 0 and when N is divided by 11, the quotient is equal to the sum of the squares of the digits of N and the remainder is 0.

S97S8 A right regular hexagonal prism (2 congruent regular hexagonal bases and 6 congruent rectangular lateral sides perpendicular to the bases) is such that a side of the base is $\sqrt{1997}$. Compute the difference of the squares of the two different length internal space diagonals.

PART II — TIME: 10 MINUTES — SPRING 1997

S97S9 In quadrilateral ABCD, M and N are the midpoints of $\overline{AD}$ and $\overline{BC}$, repsectively. If $\overline{MN}$ and $\overline{BD}$ bisect each other at point P and AP = 3, compute AP+PC.

S97S10 The sum of two of the roots of $4x^3 + 12x^2 - 67x + K = 0$ is 3. Compute the value of K.

PART III — TIME: 10 MINUTES — SPRING 1997

S97S11 If one simply guesses, the product of the probabilities of getting this question right and of getting this question wrong is $6/25$. If the probability that 3 or more of the five team members will get the problem right when all guess is $K/3125$, compute the maximum value of K.

S97S12 Let $\hat{n}$ be the repeating decimal (less than 1) formed by the digits of n.

For example, $\widehat{17}$ = .17171717...Compute the positive integer n for which $\dfrac{n! - n}{\hat{n}} - \sqrt{n} = 43$

ANSWERS

S97S7	803
S97S8	1997
S97S9	6
S97S10	30
S97S11	2133
S97S12	4

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER THREE

PART I | TIME: 10 MINUTES | SPRING 1997

S97S13 Compute the sum of the products of the coordinates of all ordered pairs (x,y) formed from the elements of {a,b,c} where $a + b + c = 1$.

S97S14 Within regular hexagon ABCDEF, there is a point P exacty 1 unit from A, $\sqrt{13}$ units from B and F, $\sqrt{37}$ units from C and E, and 7 units from D. Compute the perimeter of the hexagon.

PART II | TIME: 10 MINUTES | SPRING 1997

S97S15 In base b, the first term of an arithmetic progression is 121 and the second term is 232. Compute the value of b, if the 100^{th} term of the progression is 44410.

S97S16 Four sequences $\{a_n\}$, $\{b_n\}$, $\{c_n\}$, and $\{d_n\}$ are defined as follows for $n \geq 3$:

$$a_n = n^3 + 4n^2 + n - 6$$
$$b_n = n^3 + 2n^2 - 5n - 6$$
$$c_n = n^3 + 6n^2 + 11n + 6$$
$$d_n = n^3 - 7n + 6$$

Compute $\dfrac{a_{1997}\, b_{1997}}{c_{1997}\, d_{1997}}$

PART III | TIME: 10 MINUTES | SPRING 1997

S97S17 The final exams in the Math Department at Mathzfun High School always start between 8am and 9am and exactly at the moment when the minute and hour hands on the clock are coincident. Students have at most four hours to complete the exam, but can only hand in the exam when the hands are again coincident. Compute the exact maximum number of minutes the students actually have for the exam. Express your answer as a mixed number in simplest form.

S97S18 Compute the following for $x = \pi/9$:

$$\frac{\log\cos 2x - \log(1 + \sin 2x)}{\log(\cot x + 1) - \log(\cot x - 1)}$$

ANSWERS

S97S13 1
S97S14 24
S97S15 5
S97S16 1
S97S17 196 4/11
S97S18 -1

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FOUR

PART I | TIME: 10 MINUTES | SPRING 1997

S97S19 A square of side 3 is within right triangle ABC such that 2 sides of the square are coincident with the legs of the triangle and the hypotenuse of the triangle contains a vertex of the square. Compute the sum of the lengths of the legs of the triangle, if the length of hypotenuse AB is $\sqrt{135}$.

S97S20 Let x be the non-zero x-coordinate of a point of intersection of $y = x^4 + 1$ and $y = x^2 + x + 1$. Compute the exact numerical value of
$[x]^{11} - [x]^8 - [x]^3 + 1$ where [x] is the greatest integer less than or equal to x.

PART II | TIME: 10 MINUTES | SPRING 1997

S97S21 If xy = 7 and $\frac{1}{x^2}+\frac{1}{y^2}=9$, compute the value of $(x+y)^2$.

S97S22 Write the smallest ordered pair of consecutive positive integers (a,b) such that a is divisible by 25 and b is divisible by 36.

PART III | TIME: 10 MINUTES | SPRING 1997

S97S23 The sum of the roots of $2x + \frac{k}{3x} = 3$ is three times the difference of the roots.

Compute the value of k.

S97S24 One side of one of a pair of ordinary dice is altered so that two sides of the die are identical. The other die is unchanged. After this alteration, only the probability of rolling a sum of ten is reduced. Compute the number of dots on one of the identical sides of the altered die.

ANSWERS
S97S19 15
S97S20 0
S97S21 455
S97S22 (575,576)
S97S23 3
S97S24 3

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FIVE

PART I | TIME: 10 MINUTES | SPRING 1997

S97S25 In ΔABC altitude CD is drawn to AB. Circle O with area $3\pi^3$ is inscribed in ΔADC and Circle P with area $5\pi^3$ is inscribed in ΔBDC. Compute the length of the line of centers, OP.

S97S26 Compute the value of $\tan(5°)\tan(15°)\tan(25°)\tan(35°)...\tan(335°)\tan(345°)\tan(355°)$

PART II | TIME: 10 MINUTES | SPRING 1997

S97S27 A triangle whose perimeter is $\pi^2/2$ cm is in the interior of a closed region R. The boundary of R is exactly π cm from any point on the triangle. Write, in terms of π and in simplest form, the area of the portion of R that is exterior to the triangle.

S97S28 The quadratic equations $x^2 - 11x + p = 0$ and $x^2 + 11x - q = 0$ have exactly one integer root in common. If p and q are positive integers between 10 and 100 and the digits of p are in the reverse order as the digits of q, compute the numerical value of p.

PART III | TIME: 10 MINUTES | SPRING 1997

S97S29 Compute the probability of picking, at random, an integer between 100 and 1000 that is a multiple of 2, 3 and 11 and having no perfect cube divisors.

S97S30 Let N be the smallest integer such that $N/4 = Q$ where Q can found by removing the leftmost digit of N and placing it after its rightmost digit. Compute the number of digits of N.

ANSWERS
S97S25 4π
S97S26 1
S97S27 $3\pi^3/2$
S97S28 24
S97S29 $10/899$
S97S30 6

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SPRING 1997 SENIOR A DIVISION CONTEST NUMBER ONE
SOLUTIONS

S97S1

Let the length, width and height of the solid respectively be 4x, 6x and 9x. The sphere inscribed in a cube with edgelength 6x has a radius of 3x and its volume is $(4/3)\pi(3x)^3 = 36\pi x^3$. The volume of the rectangular solid is $6 \cdot 36x^3 = 6000$. Therefore, the volume of the sphere is 1000π.

S97S2

The first three terms must satisfy $x^2 = y\sqrt{5}$ and the last three terms must satisy $y = (x + \sqrt{180})/2$. Substituting for y into the first equation and simplifying gives the quadratic equation $2x^2 - x\sqrt{5} - 30 = 0$ whose roots are the two possible values for x. The sum is $\sqrt{5}/2$.

S97S3

Let N and L be the number of years that Newton and Liebnitz take to complete the job, respectively. The two situations give the following:

$$\frac{1}{N} + \frac{1}{L} = \frac{1}{30} \text{ and } 18\left(\frac{1}{N} + \frac{1}{L}\right) + \frac{20}{N} = 1$$

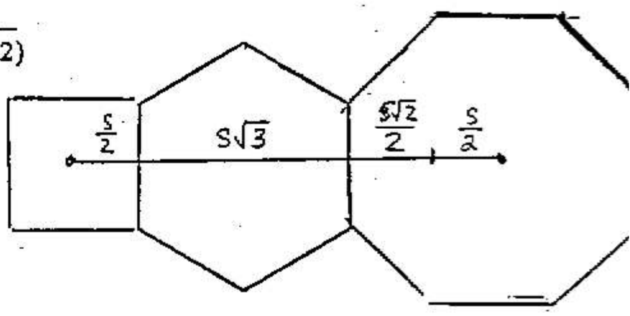
Substituting into the second equation gives $\frac{18}{30} + \frac{20}{N} = 1$. N = 50 and L = 75.

S97S4

There are 14 congruent exterior sides to the figure. Let s be the length of a side. Therefore, $14s = \sqrt{392}$ and $s = \sqrt{2}$. The portion of the line of centers within the square is $s/2$, the portion within the hexagon is $s\sqrt{3}$,
and the portion within the octagon is $s(\sqrt{2} + 1)/2$.

The sum is $(\sqrt{2}/2)(1 + 2\sqrt{3} + \sqrt{2} + 1) = (\sqrt{2}/2)(2 + 2\sqrt{3} + \sqrt{2}) = 1 + \sqrt{2} + \sqrt{2}\sqrt{3}$.

Therefore, $a + b + c = 1 + 1 + \sqrt{2} = 2 + \sqrt{2}$



S97S5

$\log_a b^3 = c$ and $\log_b a^2 = d$

implies that $c = 3\frac{\log b}{\log a}$ and $d = 2\frac{\log a}{\log b}$. Therefore, cd = 6.

S97S6

Let N be an integer satisfying the condition. Then, $N = 2^a 3^b 5^c 7^d 11^e$ (N cannot have a factor of 13 or higher if the sum of the prime factors must be 14.) The condition is that $2a + 3b + 5c + 7d + 11e = 14$ for a,b,c,d and $e \geq 0$ and the problem is the same as finding the number of ways 14 can be expressed as a sum of primes less than or equal to 11.
Listing the possibilities starting with 11 makes the situation clear:
With a summand of 11, there is only 1 result, 11 + 3;
Without 11, but with a summand of 7, there are 3 results, 7+7, 7+2+5 and 7+2+2+3;
Without 11 and 7, but with a summand of 5, there are 3 results, 5+5+2+2, 5+3+2+2+2 and 5+3+3+3;
Without 11,7 and 5, but with a summand of 3, there are 2 results, 3+3+3+3+2 and 3+3+2+2+2+2;
and with only 2, there is 1 result, 2+2+2+2+2+2+2. Hence, there are 10 possibilities.

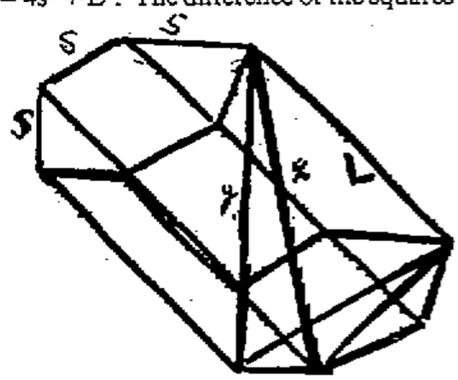
NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SPRING 1997 SENIOR A DIVISION CONTEST NUMBER TWO
SOLUTIONS

S97S7

Let N be the numeral a0b. Divisibility by 11 requires that $a - 0 + b$ be divisible by 11. Therefore, N is among the following: 209, 308, 407, 506, 605, 704, 803 and 902.
Dividing each by 11, we have the respective quotients:
19, 28, 37, 46, 55, 64, 73 and 82.
803, giving $8^2 + 3^2 = 73$ is the only number satisfying the condition.

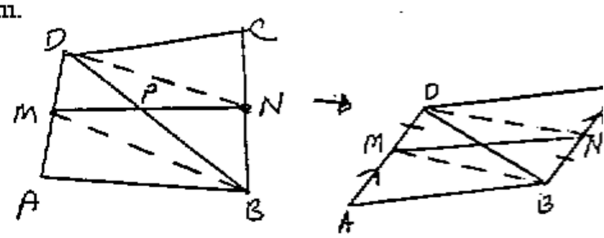
S97S8

Let the shorter space diagonal have length x and the longer have length y. Also let the lateral side have length L. The shorter space diagonal is the hypotenuse of the right triangle whose legs are the short diagonal of the base and the lateral side. Therefore, $x^2 = (s\sqrt{3})^2 + L^2 = 3s^2 + L^2$ where s is a side of the base. The longer space diagonal is the hypotenuse of the right triangle whose legs are the long diagonal of the base and a lateral side. Therefore, $y^2 = (2s)^2 + L^2 = 4s^2 + L^2$. The difference of the squares of the lengths of the space diagonals is $y^2 - x^2 = s^2 = 1997$.



S97S9

Since MN and BD bisect each other, quadrilateral MBND is a parallelogram.
Therefore, MD // BN and MD ≅ BN.
Furthermore, since M and N are midpoints of AD and BC,
we have AD // BC and AD ≅ BC giving us that ABCD is a parallelogram.
Therefore, AP+PC is a diagonal of the parallelogram with midpoint P and its length must be 6.



S97S10

Let the roots be a, b and c with $a + b = 3$. The sum of the roots $a + b + c = -12/4 = -3$, giving $c = -6$.
$ab + ac + bc = ab + c(a+b) = -67/4$, giving $ab - 18 = -67/4$ or $ab = 5/4$.
$-K/4$ is the product of the roots $abc = -30/4$. Therefore, $K = 30$.

S97S11

Let p be the probability of guessing correctly and $q = 1 - p$ be the probability of guessing incorrectly. $pq = p(1 - p) = p - p^2 = 6/25$ or $25p^2 - 25p + 6 = 0$. Therefore, $p = 2/5$ or $3/5$. The situation is a Bernoulli experiment and the answer is

${}_5C_3p^3q^2 + {}_5C_4p^4q + {}_5C_5p^5$. The maximum will occur for $p = 3/5$ producing $\dfrac{10\bullet 27\bullet 4 + 5\bullet 81\bullet 2 + 1\bullet 243}{3125} = \dfrac{2133}{3125}$. Thus, $K = 2133$.

S97S12

Clearly, n must be small since n! is very large compared to 43 for integers greater than 5. For single digits .nnnn... = n/9. Therefore, the equation becomes $\dfrac{9(n! - n)}{n} - \sqrt{n} = 43$ and n must be a perfect square. $n = 4$ is easily seen to satisfy the equation.

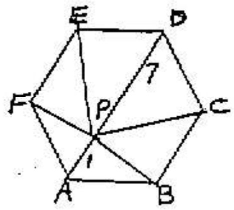
NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SPRING 1997 SENIOR A DIVISION CONTEST NUMBER THREE
SOLUTIONS

S97S13

Since the pairs are ordered, the products are not necessarily unique in form. The possible products are a^2, b^2, c^2, ab, ba, ac, ca, bc and cb. The sum $a^2 + b^2 + c^2 + 2ab + 2ac + 2bc = (a + b + c)^2 = 1$.

S97S14

Clearly, there are three pairs of congruent triangles formed with common vertex P. The sum of the vertex angles around P is 360° and, therefore, the sum of the three distinct angles is 180°. Therefore, points A, P and D are collinear and APD is a long diagonal of the hexagon whose length, 8, is twice that of the side. Hence, a side of the hexagon is 4 and the perimeter is 24.
(An alternative solution involves using triangles APC and DPC with the Law of Cosines and realizing that the short diagonal is $s\sqrt{3}$ where s is the side of the regular hexagon.)



S97S15

The constant difference of the arithmetic progression is 111 in base b. Converting to base ten, the first term is $b^2 + 2b + 1$, the difference is $b^2 + b + 1$, and the 100^{th} term is $4b^4 + 4b^3 + 4b^2 + b$. Thus, $4b^4 + 4b^3 + 4b^2 + b = b^2 + 2b + 1 + 99(b^2 + b + 1)$ or $b^4 + b^3 - 24b^2 - 25b - 25 = 0$. Since we are looking for a positive integer b>1, we can turn to the Rational Root Theorem that tells us that the only possible such positive integer b is 5. Checking 5 in the equation verifies that this is the value of b.

S97S16

By inspection we find that 1 is a root of the cubic a_n and, therefore, a_n has a factor n-1. Manipulating the terms of a_n we can factor a_n as follows:

$$\begin{aligned} a_n &= n^3 + 4n^2 + n - 6 \\ &= n^3 - n^2 + 5n^2 - 5n + 6n - 6 \\ &= n^2(n - 1) + 5n(n - 1) + 6(n - 1) \\ &= (n^2 + 5n + 6)(n - 1) \\ &= (n + 2)(n + 3)(n - 1) \end{aligned}$$

Similarly, by inspection we find that −1 is a root of the cubic b_n and, therefore, b has a factor n+1. Manipulating the terms of b_n we have:

$$\begin{aligned} b_n &= n^3 + 2n^2 - 5n - 6 \\ &= n^3 + n^2 + n^2 + n - 6n - 6 \\ &= n^2(n + 1) + n(n + 1) - 6(n + 1) \\ &= (n^2 + n - 6)(n + 1) \\ &= (n + 3)(n - 2)(n + 1) \end{aligned}$$

In the same fashion, we find that n + 1 is a factor of c_n and n − 1 is a factor of d_n giving
$c_n = n^3 + 6n^2 + 11n + 6 = (n + 1)(n + 2)(n + 3)$ and
$d_n = n^3 - 7n + 6 = (n + 3)(n - 2)(n - 1)$
The fraction a_nb_n / c_nd_n exists and simplifies to 1 for all $n \geq 3$.

S97S17

Since there are 11 points at which the hands are coincident, they must be so at intervals of 60/11 minutes around the clock. The actual times are, therefore, 1:05 5/11, 2:10 10/11, 3:15 15/11, 4:20 20/11, etc.. The starting time for the final is 8:40 40/11 or 8:43 7/11 and the actual ending time is before 12:43 7/11. The last possible time to hand in the exam is, therefore, 12 Noon. The students have 3 hours and 16 4/11 minutes or 196 4/11 minutes to complete the exam.

S97S18

Using the laws of logarithms, the expression simplifies to $\dfrac{\log\left(\dfrac{\cos 2x}{1+\sin 2x}\right)}{\log\left(\dfrac{\cot x+1}{\cot x-1}\right)}$

It is easily proven that $\dfrac{\cot x+1}{\cot x-1}$ is the reciprocal of $\dfrac{\cos 2x}{1+\sin 2x}$

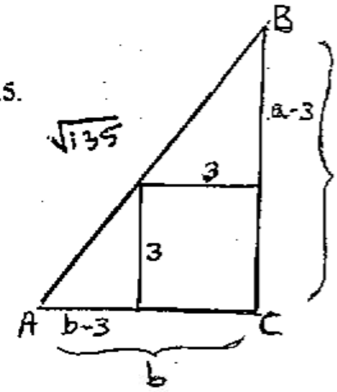
Therefore, the log expression is equal to −1 for all x for which the expressions are defined and the denominator is not zero and certainly for $x = \pi/9$.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SPRING 1997 SENIOR A DIVISION CONTEST NUMBER FOUR
SOLUTIONS

S97S19

The square forms smaller triangles similar to ΔABC. Therefore, $\frac{3}{b-3} = \frac{a}{b}$ and $3(a+b) = ab$.

Let $x = a+b$, we have $x^2 = a^2 + b^2 + 2ab = 135 + 6x$ or $x^2 - 6x - 135 = 0$ which has a positive root of 15.



S97S20

$x^4 + 1 = x^2 + x + 1$ gives the equation $x(x^3 - x - 1)=0$. Therefore, the non-zero root satisfies $x^3 = x + 1$. Since both sides of this equation represent increasing functions, we easily see that the root must be between 1 and 2 since, for $x=1$, $x^3 < x + 1$ while for $x=2$, $x^3 > x + 1$. Hence, $[x] = 1$ and the expression we wish to evaluate is clearly 0.

S97S21

$$(x+y)^2 = x^2 + 2xy + y^2. \quad \frac{1}{x^2} + \frac{1}{y^2} = 9. \quad \frac{x^2+y^2}{x^2y^2} = 9. \quad \frac{x^2+y^2}{49} = 9.$$

$$x^2 + y^2 = 441. (x + y)^2 = 441 + 2(7) = 455$$

S97S22

Since the last digits of multiples of 25 are either 0 or 5 and multiples of 36 do not have last digits of 1, the integers must be such that the last digit of a is 5 and the last digit of b is 6. Therefore, b is of the form $(5K + 1) \cdot 36 = 180K + 36$ for positive integers K. The first such integer that is 1 more than a multiple of 25 occurs for K=3 yielding b=576. The ordered pair we seek is (575,576).

S97S23

Simplifying the quadratic yields $6x^2 - 9x + k = 0$. Let the roots be r_1 and r_2. The sum of the roots $r_1 + r_2$ is $3/2$ and the condition of the problem dictates that $r_1 - r_2 = 1/2$. Therefore, $r_1 = 1$ and $r_2 = 1/2$. k is 6 times the product of the roots. Hence, $k = 3$.

S97S24

Let (a,b) represent the roll of the two dice where "a" is the number from the unaltered die and "b" is the number from the altered die. Since rolling a 12 can happen in only one way, (6,6), if the six was changed, the new probability of rolling a 12 would be zero which would be a reduction, but ten is the only sum reduced. Therefore, the 6 is left alone.

If the 5 were changed to a 6, then the probability of rolling an 11 would be reduced, since this can only occur by rolling (5,6) or (6,5). Therefore, the 5 is left alone.

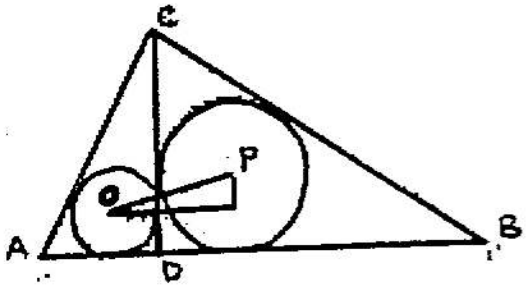
Since rolling a sum of 10 involves only 4,5 or 6 on each die, it must be that the 4 was altered. In the absence of 4, the number of ways of rolling a 5,6,7,8 or 9 are also each reduced by 1, thus reducing their respective probabilities.

Suppose 4 is replaced by x, the rolls involving x would be (1,x), (2,x), (3,x), (4,x), (5,x) and (6,x). If $x<3$, then the probability of rolling a 9 would remain reduced. Therefore, the 4 is replaced by a 3.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SPRING 1997 SENIOR A DIVISION CONTEST NUMBER FIVE
SOLUTIONS

S97S25

Let x be the radius of the smaller circle and y be the radius of the larger circle. Therefore, $x = \pi\sqrt{3}$ and $y = \pi\sqrt{5}$. The right triangle whose hypotenuse is the line of centers of the circles and legs parallel to AB and CD, has leg lengths of y - x and y + x. Therefore, letting d be the distance between the centers, we have $d^2 = (y-x)^2 + (y+x)^2 = 2(x^2 + y^2) = 16\pi^2$. Thus, $d = 4\pi$.



S97S26

The first nine terms of the product can be regrouped to

tan(5°)tan(85°)tan(15°)tan(75°)tan(25°)tan(65°)tan(35°)tan(55°)tan(45°)

= tan(5°)cot(5°)tan(15°)cot(15°)tan(25°)cot(25°)tan(35°)cot(35°)tan(45°) = 1

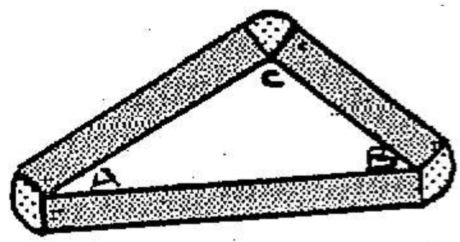
The second nine terms can be regrouped similarly to

tan(95°)tan(175°)tan(105°)tan(165°)tan(115°)tan(155°)tan(125°)tan(145°)tan(135°)

Reducing to equivalent functions with acute angles less than or equal to 45°, this part of the product is equal to -1.
The next nine terms involve angles in quadrant III and would have a product of +1 when regrouped in a similar fashion and the last nine terms involving angles in quadrant IV would have a product of -1. Therefore, the entire product is +1.

S97S27

The region within R and exterior to the triangle consists of three rectangles each having width π, but with lengths equal to the side of the triangle to which it is adjacent. Their combined area is $\pi \cdot \text{perimeter} = \pi^3/2$. The remaining portion of this region consists of 3 sectors with radius π, but with angles equal to the supplements of the angles of the triangle.
Their combined area is $(\pi^2/2)(3\pi-(A+B+C))=(\pi^2/2)(2\pi)=\pi^3$. The total area is $3\pi^3/2$.



S97S28

Adding the two equations gives $x^2 = (q - p)/2$ and subtracting the equations gives $x = (q + p)/22$.

Let $p = 10a + b$ and $q = 10b + a$ producing $q - p = 9(b - a)$ and $q + p = 11(b + a)$.

Therefore, $x = (b + a)/2$ and $x^2 = 9(b - a)/2$. Using these we find that

$(b + a)^2 = 18(b - a)$. Since a and b are single digits with a and b at most 9, the only possibilities for which $(b + a)^2$ is a multiple of 18 are 36 and 144. The first possibility gives $b + a = 6$ and $b - a=2$ giving $b = 4$ and $a = 2$ forcing $p = 24$. The second possibility gives $b + a = 12$ and $b - a = 8$, but this gives $b = 10$. Thus, $p = 24$ is the only solution.

S97S29

Let N be an integer satisfying the condition. Therefore, $N=2\cdot3\cdot11\cdot K$ where K is an integer whose prime factorization does not include 2^2, 3^2, 11^2 or p^3 for any prime p different from 2, 3 or 11. Also, $K \le 15$ since $66\cdot15 = 990$ is the largest multiple of 66 less than 1000 and we have that p could be 5, 7, 11 or 13. The possibilities for K are easily seen to be 2, 3, 5, 7, 11, 13, $2\cdot3$, $2\cdot5$, $2\cdot7$, and $3\cdot5$. Since there are exactly 899 integers between 10 and 1000, the probability is 10/899.

S97S30

Suppose N has x digits. Let $N = 10^{x-1}a+b$ where a is the first digit of N and b is the number represented by the digits of N when a is removed.

$10^{x-1}a+b = 4(10b + a)$ according to the conditions of the problem and, therefore, $b = (10^{x-1} - 4)a/39$.

$10^{x-1} - 4$ are numbers of the form 99...96 which are divisible by 3. Hence, we seek the smallest $10^{x-1} - 4$ which is divisible by 13 or the smallest 10^{x-1} which is congruent to 4 mod 13. The first such power of 10 is 10^5 and, therefore, $x = 6$.

May 10, 1997

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1997 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Senior A	S97S2	$\frac{27\sqrt{5}}{4}$ was also accepted
	S97S4	was eliminated. There were no restrictions on a, b, c.
	S96S22	(325, 324) was also accepted. It should have included "a < b"
Junior	S96J7	was eliminated. The wording led to misinterpretation of the problem.

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML