New York City Interscholastic Mathematics League

JUNIOR DIVISION | CONTEST NUMBER ONE | SPRING 1997

PART I: 10 Minutes | **NYCIML Contest One** | **Spring 1997**

S97J1. A farmer has pigs and chickens. There are sixty animals in all. If there is a total of 172 legs among these animals, and no animal is missing a leg, compute the number of chickens the farmer has.

S97J2. Compute the value of x if $\frac{(x!)!}{x!} = 719!$
(where n! represents n factorial)

PART II: 10 Minutes | **NYCIML Contest One** | **Spring 1997**

S97J3. Mrs. Jones has three children. You see her on the street one day with a little girl. She introduces you to this child and tells you that she is her middle child. Compute the probability that her two other children are also girls.

S97J4. Someone sliced a spherical melon perpendicular to a diameter, 8 cm from the top. The circle resulting from the cut had a diameter of 24 cm. Compute the number of centimeters in the length of the diameter of the melon.

PART III: 10 Minutes | **NYCIML Contest One** | **Spring 1997**

S97J5. Compute $1997^2 - 1996^2 + 1995^2 - 1994^2 + \cdots - 4^2 + 3^2 - 2^2 + 1^2$.

S97J6. Four married couples formed a reading club. Compute the number of committees that contain three people from this club, no two of whom are married.

Answers

1. 34	**3.** $\frac{1}{4}$	**5.** 1995003
2. 6	**4.** 26	**6.** 32

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER TWO** **SPRING 1997**

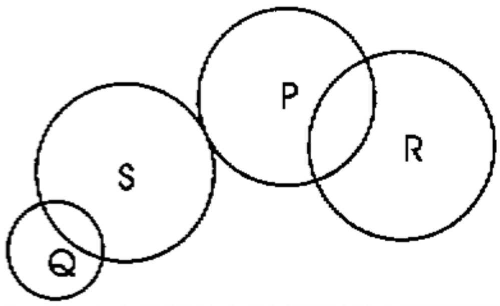
PART I: 10 Minutes **NYCIML Contest Two** **Spring 1997**

S97J7. Mr. Smith has two children. You see him on the street one day and he introduces you to his son, Stephen. Compute the probability that his other child is a boy.

S97J8. Adam rode home from college on vacation using a bus that averaged 40 mph. He returned along the same route with a friend who drove an average of 60 mph. The total time traveling back and forth was 10 hours. Compute the number of miles Adam's college is from his home.

PART II: 10 Minutes **NYCIML Contest Two** **Spring 1997**

S97J9. Compute the product of all real roots of $(x^2-7x+13)^{x^2-25}=1$

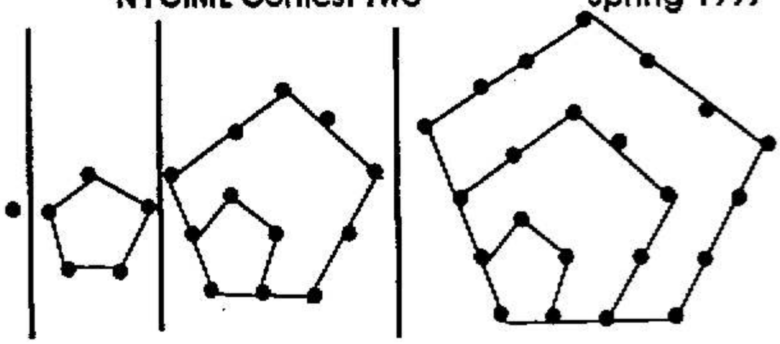


S97J10. For the diagram on the left, compute the total number of common tangents that may be drawn, taken two circles at a time. (Note that circles S and P are tangent to one another.)

PART III: 10 Minutes **NYCIML Contest Two** **Spring 1997**

S97J11. Compute the largest of fifty consecutive integers whose sum is 2075.

S97J12. The first four "pentagonal" numbers, 1,5,12, and 22, are shown geometrically on the right. What is the twentieth pentagonal number?



Answers

7. $\frac{1}{3}$	**9.** -300	**11.** 66
8. 240	**10.** 19	**12.** 590

New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER THREE** | **SPRING 1997**

PART I: 10 Minutes | **NYCIML Contest Three** | **Spring 1997**

S97J13. A town is built on a grid system of two–way streets. North-South blocks are numbered as Streets and East–West blocks are numbered as Avenues. John lives on the corner of 75th Street and 83rd Avenue. Mary lives on the corner of 92nd Street and 65th Avenue. Compute the minimum number of blocks John can drive from home to Mary's house, assuming no illegal maneuvers like driving through backyards.

S97J14. Carol rode her bike to Susan's house, ten miles away. After she arrived, it began to rain, so Susan's mother drove her home, using the same route Carol used coming. The mother's speed was five times as fast as Carol's speed, on average. The total time Carol traveled both ways was 1 hour 12 minutes. How fast did Susan's mother average?

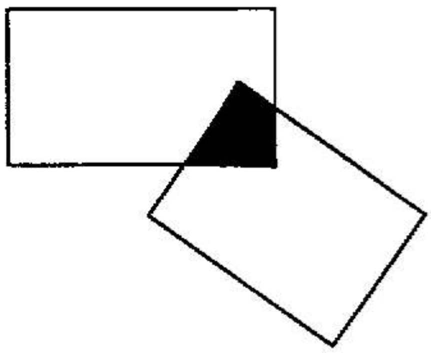
PART II: 10 Minutes | **NYCIML Contest Three** | **Spring 1997**

S97J15. The sum of sixty consecutive odd integers is 9720. Compute the fifth of these odd integers.

S97J16. Compute the sum of the integer roots of $(x^2+2x+1)(x^2+2x-2) = -2$.

PART III: 10 Minutes | **NYCIML Contest Three** | **Spring 1997**

S97J17. A piece of $8\frac{1}{2} \times 14$ inch paper is dropped onto a piece of $8\frac{1}{2} \times 11$ inch paper. The overlapping part (shaded) is cut out from the two original sheets of paper. Compute the difference in the areas remaining on the sheets of paper.



S97J18. What is the maximum number of non-overlapping regions that can be obtained using ten chords in a circle?

Answers

13. 35	**15.** 111	**17.** 25.5 sq. in. or equivalent.
14. 50 mph	**16.** –2	**18.** 56

New York City
Interscholastic
Mathematics League

JUNIOR DIVISION **CONTEST NUMBER ONE** **SPRING 1997**

Solutions

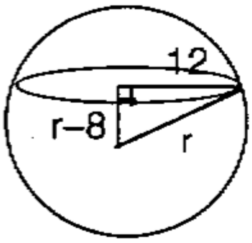
S97J1. Let p = the number of pigs and c = the number of chickens:

$c + p = 60$
$2c + 4p = 172$ This gives p = 26 and c = 34. **Answer:** 34

S97J2. $\frac{(x!)!}{x!} = \frac{(x!)\bullet(x!-1)!}{x!} = (x!-1)!$ So $(x!-1)! = 719!$ and $x! - 1 = 719$.

This means that $x! = 720$ so $x = 6$. **Answer:** 6

S97J3. The sample space for Mrs. Jones' children is {GGG, GGB, BGG, BGB}, where B stands for "boy" and G stands for "girl." Note that the middle child is definitely a girl! Thus $P(GGG) = \frac{1}{4}$. **Answer:** $\frac{1}{4}$



S97J4. Let r = the length of the radius. The Pythagorean Theorem gives $12^2 + (r-8)^2 = r^2$. This is equivalent to $144 - 16r + 64 = 0$, so that r = 13. Thus the diameter has length 26.

Answer: 26

S97J5. Factoring pair-wise, gives:

$1997^2 - 1996^2 + 1995^2 - 1994^2 + \bullet\bullet\bullet - 4^2 + 3^2 - 2^2 + 1^2 =$

$(1997-1996)(1997+1996) + (1995-1994)(1995+1994) + \bullet\bullet\bullet + (3-2)(3+2) + 1$

$= 1997 + 1996 + 1995 + \bullet\bullet\bullet + 1 = \frac{1997}{2}\bullet(1998) = 1995003$ **Answer:** 1995003

S97J6.

Method One:

First choose three of the couples, then choose one person from each couple. This gives ${}_4C_3\bullet2\bullet2\bullet2 = 4\bullet8 = 32$

Method Two:

There are ${}_8C_3$ committees in all (without considering marriage). Subtract off all committees containing a married couple. There are $4\bullet6 = 24$ such committees. Thus the desired number of committees is $56 - 24 = 32$.

Answer: 32

Please note: Concepts used today will be repeated later this term.

New York City Interscholastic Mathematics League

JUNIOR DIVISION **CONTEST NUMBER TWO** **SPRING 1997**

Solutions

S97J7. Normally, the sample space for a two child family is {GG,BG, GB, BB}. The sample space for Mr. Smith, however, is {BG, GB, BB} since he has at least one boy. Thus P(BB)= $\frac{1}{3}$. <u>**Answer:**</u> $\frac{1}{3}$

S97J8. Let d = the distance from Adam's home to college. His two times were $\frac{d}{40}$ coming home and $\frac{d}{60}$ returning to school. Thus we have $\frac{d}{40} + \frac{d}{60} = 10$. This gives d = 240 miles. <u>**Answer:**</u> 240

S97J9. Consider $x^2 - 25 = 0$. This gives x = 5 or x = –5.
Consider $x^2 - 7x + 13 = 1$. This gives x = 3 or x = 4.
Consider $x^2 - 7x + 13 = -1$, this gives complex roots.
Thus the product of the real roots is (5)(–5)(3)(4) =–300 <u>**Answer:**</u> –300

S97J10. Instead of drawing all the tangents, focus on all pairs of circles:

Circles S and P have two external and one internal common tangent.

Circles S and Q have two external common tangent; Likewise for circles P and R.

Circles S and R have two external and two internal common tangent. Likewise for circles P and Q.

Circles Q and R have two external and two internal common tangent.

This gives a total of 19 common tangents. <u>**Answer:**</u> 19

S97J11. Let the consecutive integers be a, a+1, a+2,..a+49. Their sum is $50a + (1 + 2 + 3 + ... + 49) = 50a + \frac{49}{2} \cdot 50 = 50a + 1225 = 2075$ which leads to a = 17. The largest integer is a +49 = 66. <u>**Answer:**</u> 66

S97J12. The sequence 1, 5, 12, 22, 35••• can be studied more closely by forming a new sequence called the "sequence of differences"

Term number (n)	1 2 3 4 5
Original Sequence values (p)	1 5 12 22 35
Its "Sequence of differences"	4 7 10 13
"Sequence of second differences"	3 3 3

Since the sequence of *second* differences is constant, we can find a *second* degree equation for this situation. We have $p = an^2 + bn + c$. Substituting (1,1) (2,5) and (3,12) for (n,p) gives three equations in three variables leading to $a = \frac{3}{2}$, $b = \frac{-1}{2}$ and c = 0. This gives $p = \frac{n}{2} \cdot (3n-1)$. Substituting n = 20 gives p =590. <u>**Answer:**</u> 590

New York City Interscholastic Mathematics League

JUNIOR DIVISION | **CONTEST NUMBER THREE** | **SPRING 1997**

Solutions

S97J13. To find the "taxi–distance" from (x_1,y_1) to (x_2,y_2), we need to calculate the number of horizontal blocks plus the number of vertical blocks. This can be found by computing $|x_1-x_2|+|y_1-y_2|$ which gives $17+18 = 35$. <u>**Answer:**</u> 35

S97J14. Let x = Carol's average speed and 5x = Susan's mother's average speed. Carol's time was $\frac{10}{x}$ and the mother's time was $\frac{10}{5x}$. This means $\frac{10}{x}+\frac{10}{5x}=\frac{6}{5}$. This gives $x= 10$ and $5x = 50$ mph. <u>**Answer:**</u> 50 or 50 mph

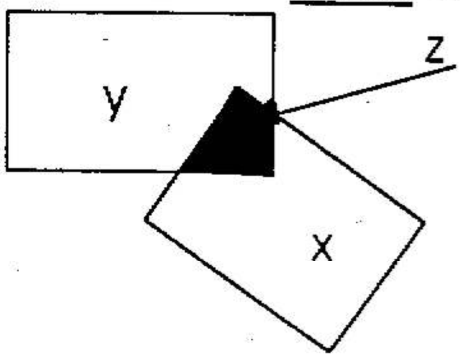
S97J15. Let the consecutive odd integers be a, a+2, a+4,..., a+118. Their sum is $60a + (2 + 4 + \cdots + 118) = 60a + \frac{59}{2}\cdot 120 = 60a + 3540$ which must = 9720, giving a= 103. The fifth consecutive odd integer is 111. <u>**Answer:**</u> 111

S97J16. Let $y=x^2+2x+1$ so that $y-3 = x^2+2x-2$. The original equation becomes $y(y-3) = -2$ or $y^2-3y+2=0$ giving $(y-1)(y-2) = 0$ so that $y = 1$ or $y=2$.

If $y = 1$, we get $x^2 + 2x + 1 = 1$ and $x^2 + 2x = 0$ so that $x=0$ or $x=-2$.

If $y = 2$, we get $x^2 + 2x + 1 = 2$ and $x^2 + 2x - 1 = 0$ which has no integer roots. Thus the sum of the integer roots is -2. <u>**Answer:**</u> -2

S97J17. The difference of the areas of the original sheets is $(y+z) - (x+z) = y-x$. This means that the difference of the resulting areas is the same as the difference of the original areas. This would be true no matter what the shapes of the original papers, an interesting result indeed. Thus the difference is $8\frac{1}{2}(14-11) = 25.5$ square inches. <u>**Answer:**</u> 25.5 sq. in.



S97J18. Build up the following table:

Number of chords (n)	1	2	3	4	5
Maximum # of regions (r)	2	4	7	11	16
Sequence of differences		2	3	4	5
Sequence of second differences		1	1	1	

Since the sequence of *second* differences is constant, we can find a *second* degree equation for this situation. We have $r = an^2 + bn + c$. Substituting (1,2), (2,4) and (3,7) for (n,r) gives three equations in three variables leading to $a=\frac{1}{2}$, $b=\frac{1}{2}$, and $c=1$. This gives $r=\frac{1}{2}(n^2+n+2)$. Substituting n=10 gives r = 56. (Note: This result could have been obtained by extending the table until n = 10, by using the difference pattern.) <u>**Answer:**</u> 56

May 10, 1997

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1997 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Senior A	S97S2	$\frac{27\sqrt{5}}{4}$ was also accepted
	S97S4	was eliminated. There were no restrictions on a, b, c.
	S96S22	(325, 324) was also accepted. It should have included "a < b"
Junior	S96J7	was eliminated. The wording led to misinterpretation of the problem.

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML