

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION

CONTEST NUMBER ONE

PART I: TIME 10 MINUTES

SPRING 1996

S96B1 The number of subsets of set S is 28 more than the number of subsets of set T. How many elements are in set S?

S96B2 Compute the sum of the series $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \dots$

PART II: TIME 10 MINUTES

SPRING 1996

S96B3 If $[x]$ represents the greatest integer less than or equal to x , solve the following equation for all real x : $\left[\frac{3x}{2} \right] = 10$.

S96B4 Find all values of x such that $x(\sqrt[3]{x^3}) = \frac{x^x}{x}$.

PART III: TIME 10 MINUTES

SPRING 1996

S96B5 The lines $x + 3y = 5$ and $ax - 2y = 7$ intersect at right angles. Compute the value of a .

S96B6 A triangle whose vertices are $(0,0)$, $(0,6)$ and $(x,0)$, where $x > 0$, is divided into two regions of equal area by the line $y = a$. Compute the value of a .

ANSWERS

1. 5

3. $\frac{20}{3} \leq x < \frac{22}{3}$

5. 6

2. $\frac{3}{2}$

4. 3

6. $6 - 3\sqrt{2}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION

CONTEST NUMBER TWO

PART I: TIME 10 MINUTES

SPRING 1996

S96B7 Compute the value of $\log_3[3\log_3 27]$?

S96B8 What is the units digit of 437^{437} ?

PART II: TIME 10 MINUTES

SPRING 1996

S96B9 If the vertex of the parabola $y = 2x^2 - 9x + c$ is on the x-axis, find the value of c.

S96B10 John can do a job alone in 8 hours. When he works together with Bill, they do the job in 2 hours. How many hours would it take Bill to do the job alone?

PART III: TIME 10 MINUTES

SPRING 1996

S96B11 Find all x which satisfy $x + \sqrt{x-2} = 4$.

S96B12 Compute the volume of a tetrahedron (pyramid whose base is a triangle) that has all edges of length one.

ANSWERS

7. 2

9. $\frac{81}{8}$

11. 3

8. 7

10. $\frac{8}{3}$

12. $\frac{\sqrt{2}}{12}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION

CONTEST NUMBER THREE

PART I: TIME 10 MINUTES

SPRING 1996

S96B13 At 2:20 P.M., what is the acute angle made by the hour and the minute hands of a clock?

S96B14 If $\log_{10}2 = x$ and $\log_{10}3 = y$, express $\log_{10}150$ in terms of x and y .

PART II: TIME 10 MINUTES

SPRING 1996

S96B15 Solve for x : $x[x] = 26$. ($[x]$ is the greatest integer of x less than or equal to x .)

S96B16 Compute the sum of the coefficients in the expansion of $(3x - y)^5$.

PART III: TIME 10 MINUTES

SPRING 1996

S96B17 Ten points are drawn on a piece of paper, no three of which are collinear. How many lines do they determine?

S96B18 Three circles with radius 1 are placed in an equilateral triangle so that each is tangent to the other two and to two sides of the triangle. Compute the area of the triangle.

ANSWERS

13. 50°

15. $5\frac{1}{5}$

17. 45

14. $y + 2 - x$

16. 32

18. $4\sqrt{3} + 6$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION

CONTEST NUMBER FOUR

PART I: TIME 10 MINUTES

SPRING 1996

S96B19 In an arithmetic progression, three times the third term equals six times the sixth term. Compute the value of the ninth term.

S96B20 A regular polygon with 12 sides is inscribed in a circle with radius 8. Compute the area of the polygon.

PART II: TIME 10 MINUTES

SPRING 1996

S96B21 Some people were walking their dogs down the street. In the entire group of people and dogs, the number of legs was 14 more than twice the number of heads. How many dogs were there?

S96B22 How many pairs of positive integers (x,y) satisfy the equation $2x + 3y = 1996$?

PART III: TIME 10 MINUTES

SPRING 1996

S96B23 If $27 \cdot 9^{2x} = 4^{y-5}$, then when $y = 5$, compute the value of x .

S96B24 In a circle, a diameter is drawn. One square is inscribed in the circle, and another square is inscribed in the semicircle. Compute the ratio of the area of the smaller square to the area of the larger square.

ANSWERS

19. 0

21. 7

23. $-3/4$

20. 192

22. 332

24. $2/5$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION

CONTEST NUMBER FIVE

PART I: TIME 10 MINUTES

FALL 1996

S96B25 Express $1 - \frac{1}{1 + \frac{3}{1-3}}$ in simplest form.

S96B26 A man invested \$5,000, part at 6% interest per year and part at $6\frac{1}{2}\%$ interest per year. If his annual incomes from the two investments are equal, how much did he invest at 6%?

PART II: TIME 10 MINUTES

FALL 1996

S96B27 If $\log_{10}x = a - \log_{10}y$, express x in terms of a and y with no logarithmic functions.

S96B28 Express the number 1001001001 written in base 2 as a number written in base 3.

PART III: TIME 10 MINUTES

FALL 1996

S96B29 Compute the sum of the reciprocals of the roots of $2x^2 + 3x + 11 = 0$ in simplest form.

S96B30 The medians to the legs of a right triangle measure 5 and $\sqrt{40}$. Compute the length of the hypotenuse.

ANSWERS

25. 3

26. \$2,600

27. $10^a/y$

28. 210200

29. $-3/11$

30. $2\sqrt{13}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1996 CONTEST ONE

S96B1 The number of subsets of a set with N elements is 2^N . The only powers of 2 whose difference is 28 is $2^5 - 2^2$. There are 5 elements in S .

$$S96B2 \quad S = \frac{a}{1-r} = \frac{2}{1-\left(-\frac{1}{3}\right)} = \frac{2}{\frac{4}{3}} = \frac{3}{2}$$

$$S96B3 \quad 10 \leq \frac{3x}{2} < 11, \quad 20 \leq 3x < 22, \quad \frac{20}{3} \leq x < \frac{22}{3}$$

$$S96B4 \quad x \cdot x^{\frac{1}{x}} = x^{x-1}$$

$$\frac{3}{x} + 1 = x - 1$$

$$x^2 - 2x - 3 = 0$$

$$x = 3 \text{ or } x = -1$$

Reject $x = -1$. Therefore, the answer is $x = 3$.

S96B5 The slopes are negative reciprocals. The first has a slope of $-\frac{1}{3}$.

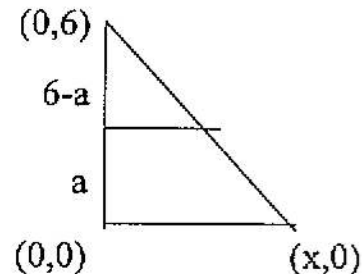
The second must have a slope of 3. $a = 6$.

S96B6 Since the triangles are similar, the ratio

of corresponding sides is $\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$.

$$\frac{6-a}{6} = \frac{1}{\sqrt{2}} \text{ or } 6 = 6\sqrt{2} - \sqrt{2}a$$

$$a = 6 - 3\sqrt{2}$$



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1996 CONTEST TWO

S96B7 $\log_3[3\log_3 27] = \log_3[3 \cdot 3] = \log_3 9 = 2$

S96B8 The powers of a number which ends in 7 has units digits which go in cycles of 7, 9, 3, 1, 7, 9, 3, 1, Since 437 is 1 more than a multiple of 4, the units digit is 7.

S96B9 The axis of symmetry is $x = -\frac{b}{2a} = \frac{9}{4}$. $2\left(\frac{9}{4}\right)^2 - 9\left(\frac{9}{4}\right) + c = 0$. $c = \frac{81}{8}$.

S96B10 John does $\frac{1}{4}$ of the job. Therefore, Bill must do $\frac{3}{4}$ of the job.

$$\frac{2}{x} = \frac{3}{4}. \quad x = \frac{8}{3} \text{ hours.}$$

S96B11 $\sqrt{x-2} = 4-x$ or $x-2 = (4-x)^2$ or $x^2 - 9x + 18 = 0$. Therefore, $x = 6$ and $x = 3$. However, $x = 6$ does not check. 3 is the only solution.

S96B12 $V = \frac{1}{3}Ah$. The area of the base = $\frac{s^2\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$. The height is one leg

of a right triangle whose other leg is $\frac{\sqrt{3}}{3}$ and whose hypotenuse is 1.

$$\left(\frac{\sqrt{3}}{3}\right)^2 + h^2 = 1. \quad h = \frac{\sqrt{2}}{\sqrt{3}}. \quad \text{Therefore, } V = \left(\frac{1}{3}\right)\left(\frac{\sqrt{3}}{4}\right)\left(\frac{\sqrt{2}}{\sqrt{3}}\right) = \frac{\sqrt{2}}{12}.$$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1996 CONTEST THREE

S96B13 The minute hand is at 120° . The hour hand is $1/3$ of the way from 60° to $90^\circ = 70^\circ$. $120^\circ - 70^\circ = 50^\circ$.

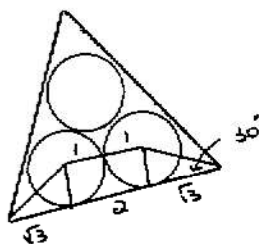
S96B14 $\log_{10}150 = \log_{10}(300/2) = \log_{10}3 + \log_{10}100 - \log_{10}2 = y + 2 - x$.

S96B15 The number is between 5 and 6. $[x] = 5$. $5x = 26$. Thus, $x = 5\frac{1}{5}$.

S96B16 To find the sum of the coefficients, replace x and y with 1.
Therefore, $S = (3 - 1)^5 = 2^5 = 32$.

S96B17 Since 2 points determine a line, the number is ${}_{10}C_2 = 45$.

S96B18 Each side is $2 + 2\sqrt{3}$. $A = s^2\sqrt{3}/4 = (2 + 2\sqrt{3})^2\sqrt{3}/4 = 4\sqrt{3} + 6$.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1996 CONTEST FOUR

S96B19 The terms are $a, a + d, a + 2d, a + 3d$, etc.
 $3(a + 2d) = 6(a + 5d), a = -8d. a + 8d = 0.$

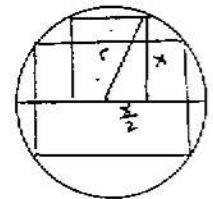
S96B20 The polygon consists of 12 congruent isosceles triangles, each of whose area is $\frac{1}{2} \cdot 8 \cdot 8 \cdot \sin 30^\circ = 16$. Therefore, $16 \cdot 12 = 192$.

S96B21 $2P + 4D = 14 + 2(P + D)$. Therefore, $D = 7$.

S96B22 $x = (1996 - 3y)/2$. y can be any even number, from 2 to 664.
 There are 332 pairs.

S96B23 $3^3 \cdot 3^{4x} = 4^0 = 1. 4x = -3. x = -3/4.$

S96B24 Area of big square = $\frac{1}{2}(2r)^2 = 2r^2$
 Area of small square = x^2
 $x^2 + (x/2)^2 = r^2. 5x^2/4 = r^2. x^2 = 4r^2/5$
 Ratio = $(4r^2/5)/2r^2 = 2/5.$



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS SPRING, 1996 CONTEST FIVE

$$S96B25 \quad 1 - \frac{1}{1 + \frac{3}{1-3}} = 1 - \frac{1}{1 - \frac{3}{2}} = 1 - \frac{1}{-\frac{1}{2}} = 1 + 2 = 3$$

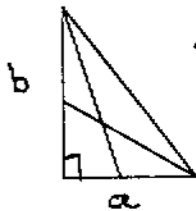
$$S96B26 \quad .06x = .065(5000 - x) \\ 60x = 325,000 - 65x \\ x = 2600$$

$$S96B27 \quad \log_{10}x + \log_{10}y = \log_{10}xy = a. \quad 10^a = xy. \quad x = 10^a/y.$$

$$S96B28 \quad 1001001001_2 = 1 + 8 + 64 + 512 = 585 \\ 585 = 210200_3$$

$$S96B29 \quad \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 + r_2}{r_1 \cdot r_2} = \frac{-3}{\frac{11}{2}} = \frac{-3}{11}$$

$$S96B30 \quad (a/2)^2 + b^2 = 40 \quad \text{and} \quad a^2 + (b/2)^2 = 25. \\ a^2 + 4b^2 = 160 \quad \text{and} \quad 4a^2 + b^2 = 100. \quad \text{Adding these two equations} \\ \text{yields } 5a^2 + 5b^2 = 260 \quad \text{or} \quad a^2 + b^2 = 52. \quad \text{Therefore, } c = \sqrt{52} = 2\sqrt{13}$$



May 10, 1996

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1996 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	<u>Question</u>	<u>Correct answer</u>
Senior A	S96S07	add "with no logarithmic functions"
	S96S30	any ordered triple of the form $((b+3)/2, b, -b)$
Senior B	S96B14	add "with no logarithmic functions"
Junior	S96J07	15

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML