NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER ONE

PART I TIME: 10 MINUTES SPRING 1996

S96S01 Let a be the square root of the sum of the first 100 consecutive odd positive integers. Let b be the square root of the sum of the first 100 consecutive even positive integers. Compute the value of b - a.

S96S02 Compute all the integer solutions to $3x^4 - 9x^3 + 5x^2 + 3x - 2 = 0$.

PART II TIME: 10 MINUTES SPRING 1996

S96S03 For positive integers n, let f(n) = 2n-1 if n is even and 2n+1 if n is odd. Compute the sum of the values of f for n = 1,2,3,...,100.

S96S04 Compute all the ordered pairs of positive integers (a,b) which satisfy $76a^b + 3b^a = 1996$.

PART III TIME: 10 MINUTES SPRING 1996

S96S05 The average of five numbers is $1/3$ and the average of three of these numbers is $1/5$. Compute the average of the other two numbers.

S96S06 A storekeeper wants to clear his inventory of the software package Mathzhard because he hadn't sold a single copy. He originally hoped to have a gross revenue of $100,000 if all the software had sold at the retail price. He knows that if he creates a four week plan in which during week w he sells exactly 10w% of the inventory present at the beginning of week w at a 10w% discount off the retail price, he will recover his cost for the software. Compute the profit (in dollars) he originally expected to make.

ANSWERS

S96S01 $10\sqrt{101} - 100$
S96S02 x = 1 and x = 2
S96S03 10,100
S96S04 (1,640) and (5,2).
S96S05 $8/15$
S96S06 49,384

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER TWO

PART I TIME: 10 MINUTES SPRING 1996

S96S07 If $\log_6 3 = x$, express $\log_6 72$ in terms of x.

S96S08 When fully expanded, $(x^3 + 9)^{96} = c_0 + c_1x^3 + c_2x^6 + \ldots + c_{96}x^{192}$.

If $2(c_0 + c_2 + c_4 + \ldots + c_{96}) = a^{96} + b^{96}$ where $a - b = 2$, compute the ordered pair of positive integers (a,b).

PART II TIME: 10 MINUTES SPRING 1996

S96S09 Compute the positive acute angle θ such that $2\csc\theta = \tan 72° + \tan 18°$

S96S10 Compute the remainder when $3^{3^{3^{3}}}$ is divided by 7.

PART III TIME: 10 MINUTES SPRING 1996

S96S11 Circles of radii $\sqrt{2}$ and $\sqrt{3}$ are concentric with center O. Point A lies on the larger circle, point B lies on the smaller circle. Compute the distance from A to B when ∠OAB is a maximum.

S96S12 If all but one of five not necessarily distinct positive integers are added, the possible sums are 33, 35, 36 and 40 with one of the sums repeated. List all possible sets of five such integers.

ANSWERS

S96S07	3 - x
S96S08	(10,8)
S96S09	36°
S96S10	6
S96S11	1
S96S12	{5, 9, 9, 10, 12} and {6, 6, 10, 11, 13}

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER THREE

PART I TIME: 10 MINUTES SPRING 1996

S96S13 A figure G is drawn in the Cartesian plane consisting of a circle and a square inscribed within. A transformation of the plane is performed under which the image of (x,y) is (ax,by). Write, in simplest form, the ratio of the area of the image of the square to the area of the original square in terms of a and b.

S96S14 The roots of $x^2 + 19x + 96 = 0$ are r_1 and r_2. Compute $r_1^2 + r_2^2$.

PART II TIME: 10 MINUTES SPRING 1996

S96S15 In ΔABC, $AB = AC$, altitudes BD and CE intersect at F, and $EF:FC = 1:\sqrt{2}$. Compute the measure of $\angle A$.

S96S16 Not including $x = 0$ and $x = 1$, compute the number of real solutions of $\sqrt[84]{x} + \sqrt[182]{x} = \sqrt[91]{x} + \sqrt[156]{x}$.

PART III TIME: 10 MINUTES SPRING 1996

S96S17 The perimeter of ΔABC, with integer sides, is 11. Compute the maximum perimeter of a triangle whose sides are the reciprocals of the sides of ΔABC.

S96S18 Let $f(n) = n^2 - n$. Compute the sum of the reciprocals of f(n) for all positive integers $n > 1$.

ANSWERS

S96S13	ab:1 or ab
S96S14	169
S96S15	45°
S96S16	0
S96S17	13/15
S96S18	1

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FOUR

PART I TIME: 10 MINUTES SPRING 1996

S96S19 Four line segments are each tangent to a circle of radius 2 and meet at their endpoints to form an isosceles trapezoid. If the area of the trapezoid is 36, compute its perimeter.

S96S20 The pair of simultaneous nonlinear equations
$2x^2 + 5y^2 + 3z^2 + 6p = 0$ and $3x^2 + 8y^2 + 4z^2 + p = 0$, where p is an odd prime, has solutions where y and z are positive integers. Compute the number of ordered pairs (y,z) of these integers.

PART II TIME: 10 MINUTES SPRING 1996

S96S21 A function f(x) has the property that $f(1 + x) + kf(1 - x) = x^3$. If $k > 0$, write an expression in simplest form for f(3) in terms of k.

S96S22 For a positive integer n, let R_n denote the nonempty set of all integers between 1 and n that are relatively prime to n. Compute the largest two digit integer, n, such that all the integers in R_n are prime.

PART III TIME: 10 MINUTES SPRING 1996

S96S23 The Fibonnaci Sequence is defined as $u_1 = u_2 = 1$ and $u_{n+2} = u_{n+1} + u_n$. Compute the remainder when u_{60} is divided by 4.

S96S24 In the following base 10 addition problem, 3 is the middle digit of the first addend and the places marked by ● are filled by each of the nine other distinct digits. If the sum of the digits of the sum is a multiple of 9 and in each column the digit of the upper addend is less than the digit of the second addend, compute all possible sums. (The leftmost digit of any of the three integers is not 0.)

```
  ●3●
  ●●●
 ----
 ●●●●
```

ANSWERS

S96S19	36
S96S20	3
S96S21	$8/(1 - k)$
S96S22	30
S96S23	0
S96S24	1026 and 1089

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FIVE

PART I TIME: 10 MINUTES SPRING 1996

S96S25 In triangle ABC, altitude $\overline{CD}$ creates the ratio AD:DB = 2:3. If $\tan A = 3/2$, compute the ratio AC:BC.

S96S26 A line tangent to the parabolas $y = x^2$ and $y = -x^2 + 8x - 16$ passes through the point (5,24). Compute, in simplest radical form, the distance between the points of tangency.

PART II TIME: 10 MINUTES SPRING 1996

S96S27 Compute the three digit positive integer that is equal to the sum of the factorials of its digits.

S96S28 At Mathzfun High School, there is a make-up test policy that is unusual. The student begins with 64 points and is asked 10 questions. For each correct response the student receives ¼ of his or her current point score and for each incorrect response the student loses ½ of his or her current point score. Compute the least number of questions that must be answered correctly in order to pass the test, if passing requires 65 or more points.

PART III TIME: 10 MINUTES SPRING 1996

S96S29 Compute $\dfrac{1996}{1997^2 - 1996 \cdot 1998}$.

S96S30 In right triangle ABC, hypotenuse $AB = 2\sqrt{3}$ and $m\angle A = 30°$. The angle bisectors, AE and BF with E on BC and F on AC, meet at D. If DF = x and $CD^2 = ax^2 + bx + c$, compute the ordered triplet (a,b,c).

ANSWERS
S96S25 $\sqrt{26}/6$
S96S26 $4\sqrt{65}$
S96S27 145
S96S28 8
S96S29 1996
S96S30 (1,-1,1)

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER ONE
SOLUTIONS - SPRING 1996

S96S01 $a^2 = 1 + 3 + 5 + \ldots + 199 = 100^2$ and $b^2 = 2(1 + 2 + 3 + \ldots + 100) = 100 \cdot 101$. Therefore, $a = 100$ and $b = 10\sqrt{101}$ and $b - a = 10\sqrt{101} - 100$.

S96S02 $3x^4 - 9x^3 + 5x^2 + 3x - 2$
$= 3x^4 - 9x^3 + 6x^2 - x^2 + 3x - 2$
$= 3x^2(x^2 - 3x + 2) - (x^2 - 3x + 2)$
$= (3x^2 - 1)(x^2 - 3x + 2) = (3x^2 - 1)(x - 1)(x - 2) = 0$.
Therefore, the integer solutions are $x = 1$ and $x = 2$.

S96S03 $f(1) + f(2) + f(3) + f(4) + f(5) + f(6) + \ldots + f(97) + f(98) + f(99) + f(100)$
$= 3 + 3 + 7 + 7 + 11 + 11 + \ldots + 195 + 195 + 199 + 199$
$= 2(3 + 7 + 11 + \ldots + 99 + 103 + \ldots + 191 + 195 + 199)$
$= 2 \cdot 202 \cdot 25 = 10{,}100$.

S96S04 If $a = 1$, then $76 + 3b = 1996 \rightarrow b = 640$.
If $b = 1$, then $76a + 3 = 1996 \rightarrow 76a = 1993$ which has no integer solutions since $76a$ is even. Along the same line of thinking, b^a must be even forcing b to be even. a^b and b^a cannot be equal since 79 is not a divisor of 1996. (This also precludes $a = b$.) Also, $76a^b < 1996 \rightarrow a^b < 27$. Hence, for $a,b \neq 1$, the only possible values for a^b are 3^2 and 5^2. The values of b^a would therefore be 2^3 and 2^5, respectively. Checking these, we find that $a = 5$ and $b = 2$ is The only other solution.

A much quicker way to find the second pair: $1996 = 1900 + 96 = 19 \cdot 4 \cdot 25 + 3 \cdot 32 = 76 \cdot 5^2 + 3 \cdot 2^5$.

S96S05 The sum of the five numbers is $5/3$. The sum of the three whose average is $1/5$ is $3/5$. Therefore, the remaining two numbers have a sum of $5/3 - 3/5 = 16/15$ and their average is $8/15$.

S96S06 Let T be the total inventory and P be the original price. The prediscount value of the entire stock is $TP = 10^5$.
During week 1 10% of T are sold at 90% of P
During week 2 20% of 90% of T are sold at 80% of P
During week 3 30% of 80% of 90% of T are sold at 70% of P
During week 4 40% of 70% of 80% of 90% of T are sold at 60% of P

The revenue for these four weeks is $\left(\frac{1 \cdot 9}{10^2} + \frac{2 \cdot 9 \cdot 8}{10^3} + \frac{3 \cdot 9 \cdot 8 \cdot 7}{10^4} + \frac{4 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{10^5}\right) \cdot TP$

$$\left(\frac{1 \cdot 9}{10^2} + \frac{2 \cdot 9 \cdot 8}{10^3} + \frac{3 \cdot 9 \cdot 8 \cdot 7}{10^4} + \frac{4 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{10^5}\right) \cdot 10^5$$

$= 9{,}000 + 14{,}400 + 15{,}120 + 12{,}096 = 50{,}616$. The expected profit was \$49,384.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER TWO
SOLUTIONS - SPRING 1996

S96S07 $\log_6 72 = \log_6(216/3)$
$= \log_6 216 - \log_6 3 = 3 - x$.

S96S08 When x = 1, we have $10^{96} = c_0 + c_1 + c_2 + \dots + c_{96}$.
When x = -1, we have $8^{96} = c_0 - c_1 + c_2 - \dots + c_{96}$.
Therefore, $10^{96} + 8^{96} = 2(c_0 + c_2 + c_4 + \dots + c_{96})$ and (a,b) = (10,8).
This answer must be unique since $a^{96} + b^{96} = (b+2)^{96} + b^{96}$ is increasing for increasing values of b and $2(c_0 + c_2 + c_4 + \dots + c_{96})$ is constant.

S96S09

$$2\csc\theta = \tan 72° + \tan 18° = \frac{\sin 72°}{\cos 72°} + \frac{\sin 18°}{\cos 18°} = \frac{\sin 72° \cos 18° + \cos 72° \sin 18°}{½(2\sin 18° \cos 18°)} = \frac{2\sin 90°}{\sin 36°} = 2\csc 36°$$

Therefore, θ = 36°.

S96S10 Let a ≡b (mod c) mean that b is the remainder when a is divided by c. The remainders when dividing whole number powers of 3 by 7 are cyclic with

$3^{0 (\text{mod } 6)} = 1 \pmod 7$
$3^{1 (\text{mod } 6)} = 3 \pmod 7$
$3^{2 (\text{mod } 6)} = 2 \pmod 7$
$3^{3 (\text{mod } 6)} = 6 \pmod 7$
$3^{4 (\text{mod } 6)} = 4 \pmod 7$
$3^{5 (\text{mod } 6)} = 5 \pmod 7$

Also, $3^n = 3 \pmod 6$ for all integers n ≥ 1.
Starting with the upper most exponentiation,

$$3^{3^{3^3}} = 3^{\left(3^{\left(3^3\right)}\right)} = 3^{\left(3^{27}\right)} = 3^{3(\text{mod}6)} = 6(\text{mod}7)$$

S96S11 The maximum value of ∠OAB occurs when AB is tangent to the smaller circle. Therefore, ΔOBA is a right triangle and $AB = \sqrt{3 - 2} = 1$.

S96S12 Since there are only four distinct sums, one of the sums must result twice. Therefore, the possible sums of sums are 144 + 33, 144 + 35, 144 + 36 and 144 + 40 or 177, 179, 180 or 184. The sum of the sums of four of the five integers is four times the sum of the five nondistinct integers. Therefore, the sum of the sums must be divisible by 4 leaving us with only 180 and 184 as possibilities or with 36 and 40 as the repeated sums. Also, these possible sums of sums forces the sum of the five nondistinct integers to be either 45 or 46.
The numbers are found as the differences between S and the possible sums of four integers. The following table clarifies the situation:

Repeated sum	Sum of sums	Sum of the five integers *	* - 33	* - 35	* - 36	* - 40	* - repeat
33	177						
35	179						
36	180	45	12	10	9	5	9
40	184	46	13	11	10	6	6

Therefore, the only possibilities are 5,9,9,10 and 12 or 6,6,10,11 and 13.
Checking to see that the given sums are attainable, we find:
6 + 6 + 10 + 11 = 33 = 5 + 9 + 9 + 10
6 + 6 + 10 + 13 = 35 = 5 + 9 + 9 + 12
6 + 6 + 11 + 13 = 36 = 5 + 9 + 10 + 12
6 + 10 + 11 + 13 = 40 = 9 + 9 + 10 + 12.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER THREE
SOLUTIONS - SPRING 1996

S96S13 Assume without loss of generality that the circle is centered at the origin with radius $\sqrt{2}$ and the square is inscribed so that a vertex lies on the circle at (1,1). The image of the circle is an ellipse and the image of the square is a rectangle inscribed within the ellipse with a vertex on the ellipse at the point (a,b). The area of the rectangle is (2a)(2b) = 4ab and the area of the square is 4. Therefore, the ratio we seek is ab:1 or ab.

S96S14 $(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1r_2$ --> $(-19)^2 = r_1^2 + r_2^2 + 2 \cdot 96$ --> $r_1^2 + r_2^2 = 361 - 192 = 169$.

S96S15 Since F lies on the altitude from A to BC and this altitude is a line of symmetry for ΔABC, we have $DF:FB = EF:FC = 1 : \sqrt{2} = EF:FB$. Therefore, right triangle BEF is isosceles and $m\angle BFE = 45°$. By similar reasoning, we also have $m\angle DFC = 45°$. Also, ΔBFC is isosceles with $m\angle BFC = 135°$.
Hence, $m\angle B + m\angle C = 45° + 45° + (180 - 135)° = 135°$ and $m\angle A = 45°$.

S96S16 The given equation indicates that $x \geq 0$. The prime factorizations of the indices are the key to determining if there are solutions besides x = 0 and x = 1. $84 = 2^2 \cdot 3 \cdot 7$, $182 = 2 \cdot 7 \cdot 13$, $91 = 7 \cdot 13$ and $156 = 2^2 \cdot 3 \cdot 13$

Let $x = y^{2^2 \cdot 3 \cdot 7 \cdot 13}$. The given equation now becomes $y^{13} + y^6 = y^{12} + y^7$.
Moving terms to one side and factoring, we have
$y^{12}(y - 1) - y^6(y - 1) = y^6(y^6 - 1)(y - 1) = 0$. The solutions to this equation are $y = 0$, $y^6 = 1$ and $y = 1$.

Since $x = \left(y^6\right)^{182}$, we clearly see that the only solutions to the original equation are x = 0 and x = 1.
Therefore, the number of solutions other than x = 0 and x = 1 is 0.

S96S17 The only possible triples of sides for ΔABC are {3,3,5}, {3,4,4}, {2,4,5} and {1,5,5}. Therefore, the possible triples for sides of the triangle we seek are {1/3, 1/3, 1/5}, {1/3, 1/4, 1/4}, {1/2, 1/4, 1/5} and {1, 1/5, 1/5}. The last two triples do not constitute sides of a triangle. The first two triples produce perimeters of 13/15 and 5/6. The maximum perimeter is 13/15.

S96S18 The sum we seek is $S = \sum_{n=2}^{\infty} \frac{1}{n(n-1)} = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \ldots$

Since $f(n) = \frac{1}{n^2 - n} = \frac{1}{n(n-1)} = \frac{1}{n-1} - \frac{1}{n}$,

S is equivalent to $\left(1 - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \ldots$. Therefore, S = 1.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FOUR
SOLUTIONS - SPRING 1996

S96S19 The diameter of the circle is the height of the trapezoid. Let the bases of the trapezoid be 2a and 2b. The legs would then each be a + b by the congruence of segments of tangents from a common external point. The area of the trapezoid is therefore ½·4(2a+2b) = 4(a+b) which is also the perimeter of the trapezoid! Hence, the perimeter is 36.

S96S20 Multiplying the first equation by 3 and the second by 2 and subtracting, we have the equation $-y^2 + z^2 + 16p = 0$ or $y^2 - z^2 = (y - z)(y + z) = 16p$. The possible values for these factors are given in the table below:

y-z	y+z	2y	y	z
1	16p	16p+1	impossible	
2	8p	8p+2	4p+1	4p-1
4	4p	4p+4	2p+2	2p-2
8	2p	2p+8	p+4	p-4
16	p	p+16	impossible	

Therefore, there are 3 such ordered pairs of integers.

S96S21 If x = 2, we have f(3) + kf(-1) = 8.
If x = -2, we have f(-1) + kf(3) = -8.
Adding these two equations gives us (f(3) + f(-1))(1 + k) = 0. Since k > 0, it must be the case that f(-1) = -f(3). Using this in the first equation, we have f(3) = 8/(1 - k).

S96S22 Since multiples of 2, 3 and 5 are "dense" among the set of integers, it is reasonable to have n being a multiple of 30. The only multiples of primes less than 30 are those of 7, 11, and 13. These, however, are also multiples of 2, 3 or 5 and, therefore, excluded from R_{30}. If n = 60, R_{60} would include 7^2 which is not prime. To exclude it we would have to have n = 210 which is too large. Therefore, the largest two digit n is 30 with
R_{30} = {7,11,13,17,19,23,29}.

S96S23 Let a = b (mod c) mean that b is the remainder when a is divided by c. It is easy to demonstrate that if a = b (mod c) and d = e (mod c), then a + d = (b + e) (mod c). Therefore, we can construct the Fibonacci Sequence (mod 4) as follows: 1, 1, 2, 3, 1, 0, 1, 1, 2, 3, 1, 0, ... and every sixth term will be a multiple of 4. Therefore, u_{60} leaves a remainder of 0 when divided by 4.

S96S24
Let S be the sum of the digits of the sum. Since the sum is a multiple of 9, S must also be a multiple of 9.
Since the leftmost digit of the sum has to be a 1 and the other digits are at least 0, 2 and 4, and at most 7,8 and 9, we have $7 \le S \le 25$.
Hence, the only possibilities are S = 9 or S = 18.

If S = 9, then the other three digits have to be 0, 2 and 6. The unused digits are 4,5,7,8 and 9.
Looking at the possible pairwise sums of these remaining digits, we see that it is not possible for 0 to be the units digit of the sum.

If 2 were the units digit of the sum, the units digits of the addends could be 4 and 8 or 5 and 7.
In either case, a 1 would carry to the tens column and we would not be able to produce a 0 or a 6 in the tens column of the sum with the remaining digits. Therefore, 2 is also not the units digit of the sum.

Using 6 as the units digit of the sum, we find the forced solution

$$\begin{array}{r} 437 \\ +589 \\ \hline 1026 \end{array}$$

for S = 9.

If S = 18, then, since the left digit of the sum is 1, the possible triplets for the other three digits are:
0,8,9 ; 2,6,9 ; 2,7,8 ; 4,5,8 ; 4,6,7

For 0,8,9 , there is a solution

$$\begin{array}{r} 432 \\ +657 \\ \hline 1089 \end{array}$$

All other cases are easily ruled out by checking the possibility of any of the digits being the units digit of the sum.

Therefore, the only possible sums are 1026 and 1089.

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FIVE
SOLUTIONS - SPRING 1996

S96S25 Let AD = 2x and DB = 3x. From right triangle CAD and tan A = 3/2, we have CD = 3x and AC = $x\sqrt{13}$. From right triangle CBD, we have BC = $x\sqrt{18}$. Therefore, AC:BC = $x\sqrt{13}/x\sqrt{18} = \sqrt{26}/6$.

S96S26 $y = -x^2 + 8x - 16$ is a reflection of $y = x^2$ in the point (2,0). The line tangent to both parabolas would necessarily contain this point. Using (2,0) and (5,24), we compute the slope of the line to be 8. The equation of the line is $y - 0 = 8(x - 2)$ or $y = 8x - 16$. On $y = x^2$, the x-coordinate of the point of tangency would satisfy $x^2 - 8x + 16 = 0$ resulting in x = 4 and the point of tangency is (4,16). The point of tangency on the other parabola is its image by the point reflection, (0,-16). The distance between the points is $4\sqrt{65}$.

S96S27 Since the sum of the 3 factorials is between 100 and 999 inclusive, we easily see that 7,8, and 9 cannot be digits of our number. Since 6! = 720, we can rule out 6 as well because any factorial added to this would give us 7,8 or 9 as a digit. We can also rule out 555, since 3(5!) = 360 which clearly doesn't satisfy the conditions. We can also rule out any arrangement of 55x, since 2(5!) = 240 and no value of x for 0,1,...,4 would create a sum containing two 5's. Since there is only one 5, then the number is in the list 105,115,125,135,145,150,151,152,153,154. However, the largest possible sum of factorials in this list is 1! + 4! + 5! = 145. We have stumbled upon our number. (We were told that there is only one.)
To complete the thinking process:
For 1X5 = 1! + 5! + X! = 121 + X!, X! must have a 4 as its last digit and only X=4 has this property and our number is 145.

S96S28 Regardless of the sequence of correct and incorrect responses, for exactly x questions answered correctly, the final score will be $64\left(\frac{5}{4}\right)^x\left(\frac{1}{2}\right)^{10-x} = \frac{5^x}{2^{x+4}}$. Since we need a minimum score of $64 = 2^6$, x must satisfy $5^x > 2^{x+10}$ or

$x\log 5 > (x + 10)\log 2$. Since $10 = 5 \cdot 2$, we have $1 = \log 5 + \log 2$. Since $2^{10} \approx 10^3$ yielding $10\log 2 \approx 3$, we can approximate $\log 2 \approx .3$ and $\log 5 \approx .7$. (These approximations will be accurate enough to produce a minimum x due to the rapid increasing behavior of exponential functions.) Therefore, $.7x > .3(x + 10)$ which requires $x > 7.5$ and the minimum number of correct responses to insure a passing grade is 8. (It would be interesting to compute this passing grade and compare it to the maximum failing grade!)

S96S29 Let x = 1996 and the problem is of the form $\dfrac{x}{(x+1)^2 - x(x+2)} = \dfrac{x}{x^2 + 2x + 1 - x^2 - 2x} = x$.

S96S30 The sides of ΔABC are AB = $2\sqrt{3}$ (given), BC = $\sqrt{3}$ and AC = 3. ΔBCF is also a 30-60-90 right triangle giving us CF = 1 and BF = 2. Letting DF = x and BD = y, we can apply Stewart's Theorem on ΔBCF:
$CF^2 \cdot BD + BC^2 \cdot DF = BF(CD^2 + BD \cdot DF)$
$1y + 3x = 2(CD^2 + xy)$
$y + 3x = 2CD^2 + 2xy$
$2 - x + 3x = 2CD^2 + 2x(2-x)$
$2 + 2x = 2CD^2 + 4x - 2x^2$
$2x^2 - 2x + 2 = 2CD^2$
$x^2 - x + 1 = CD^2$
Therefore, the ordered triplet of the coefficients of the quadratic is (1,-1,1).

May 10, 1996

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1996 NYCIML contests that you requested on the application form.

The following questions had different answers than the given one or were eliminated from the competitions.

	Question	Correct answer
Senior A	S96S07	add "with no logarithmic functions"
	S96S30	any ordered triple of the form ((b+3)/2, b, -b)
Senior B	S96B14	add "with no logarithmic functions"
Junior	S96J07	15

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML