NEW YORK CITY
INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER ONE**

PART I: TIME 10 MINUTES **FALL 1995**

F95B1 Find the value of $(3^{-2} + 4^{-1/2})^{-1}$ in simplest form.

F95B2 The digits 1, 3, 5, 7, and 9 are used to form all possible 5 digit numbers with no repetition of digits. If they are listed in ascending numerical order, find the 77th number on the list.

PART II: TIME 10 MINUTES **FALL 1995**

F95B3 If $\sqrt{x\sqrt{x\sqrt{x}}} = x^y$, express y as a fraction in lowest terms.

F95B4 A right circular cone and a right circular cylinder have equal heights and equal volumes. Find the ratio of the radius of the base of the cylinder to the radius of the base of the cone.

PART III: TIME 10 MINUTES **FALL 1995**

F95B5 Find the coefficient of the x^5 term in the expansion of $(x + 3)^7$.

F95B6 Express $\sqrt{(20)(21)(22)(23)+1}$ as an integer.

ANSWERS

1. $\frac{18}{11}$	3. $\frac{7}{8}$	5. 189
2. 71,935	4. $\sqrt{3}/3$	6. 461

NEW YORK CITY
INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER TWO**

PART I: TIME 10 MINUTES **FALL 1995**

F95B7 Each interior angle of a regular polygon measures 170°. How many sides does it have?

F95B8 If the first 3 terms of a geometric progression are 2, $\sqrt[3]{4}$, $\sqrt[6]{4}$, find the fourth term.

PART II: TIME 10 MINUTES **FALL 1995**

F95B9 The diagonal of one square is a side of a second square. Find the ratio of the area of the first square to the area of the second square.

F95B10 A man is 4 miles north and 9 miles west of his cabin. He is also 4 miles south of a river which flows east-west. He wishes to go up to the river, collect water, and then go to his cabin. What is the length of the shortest route that will accomplish this?

PART III: TIME 10 MINUTES **FALL 1995**

F95B11 Mr. Jones gave a math test to his class. The average grade of the boys in the class was 84 and the average grade of the girls was 96. If the average of all the students was 92, find the ratio of boys to girls in the class.

F95B12 The sides of a triangle are 2, 7, and x, and the area is x square units. Compute the value of x.

ANSWERS

7. 36	9. 1:2	11. 1:2
8. 1	10. 15	12. $3\sqrt{5}$

NEW YORK CITY
INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER THREE**

PART I: TIME 10 MINUTES **FALL 1995**

F95B13 What is the volume (in simplest form), in cubic inches, of a cube whose surface area is 48 square inches?

F95B14 If $x^2 + y^2 = 10$ and $xy = 3$, find all possible values of $\frac{1}{x}+\frac{1}{y}$.

PART II: TIME 10 MINUTES **FALL 1995**

F95B15 Compute the area of a regular hexagon which is inscribed in a circle with radius 8.

F95B16 John throws darts at a checkerboard and hits 2 different squares. Compute the probability (in simplest form) that these 2 squares share a common side.

PART III: TIME 10 MINUTES **FALL 1995**

F95B17 What is the prime factorization of $20^3 + 8^3$?

F95B18 Two intersecting circles whose radii are 3 and 7 have their centers 8 units apart. Compute the length of the line segment connecting their points of intersection.

ANSWERS

13. $16\sqrt{2}$ 15. $96\sqrt{3}$ 17. $2^6 \cdot 7 \cdot 19$

14. $\pm\frac{4}{3}$ 16. $\frac{1}{18}$ 18. $3\sqrt{3}$

NEW YORK CITY
INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER FOUR**

PART I: TIME 10 MINUTES **FALL 1995**

F95B19 In triangle ABC, $\angle C$ is a right angle. Find the value of sinAcosB + cosAsinB.

F95B20 A cube whose volume is 1000 cubic units is painted, and then cut into cubes with volume 1 cubic unit. How many of these cubes are painted on exactly 1 face?

PART II: TIME 10 MINUTES **FALL 1995**

F95B21 If $\sqrt{5x^3+5x^3+5x^3+5x^3+5x^3} = 625$ find x.

F95B22 A polygon has 54 diagonals. How many sides does it have?

PART III: TIME 10 MINUTES **FALL 1995**

F95B23 The probability of John passing a test is $\frac{1}{4}$ and the probability of Mary passing a test is $\frac{1}{3}$. Find the probability that exactly one of them passed the test.

F95B24 Find the largest integral N for which 7^N is a factor of 100!.

ANSWERS

19. 1	21. 25	23. $\frac{5}{12}$
20. 384	22. 12	24. 16

NEW YORK CITY
INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION **CONTEST NUMBER FIVE**

PART I: TIME 10 MINUTES **FALL 1995**

F95B25 Solve for all values of x: $\sqrt{4x^2+4x+1} = 2x + 1$

F95B26 John and his grandfather share the same birthday. On their next six birthdays, his grandfather's age will be an integral multiple of John's age. How old will his grandfather be on his next birthday (the first birthday from now)?

PART II: TIME 10 MINUTES **FALL 1995**

F95B27 The product of two prime numbers is 18,881 and one is approximately 3 times the other. Find the larger prime.

F95B28 Jack picked 2 cards from a standard deck of 52 cards. Find the probability, in simplest form, that he picked at least one 2 and at least one club (not necessarily different cards).

PART III: TIME 10 MINUTES **FALL 1995**

F95B29 Find the last two digits (tens and units) of 1! + 2! + 3! + ... + 100!

F95B30 How many different sized right triangles with integral sides have 15 as one of its sides?

ANSWERS

25. $x \geq -1/2$	27. 239	29. 13
26. 61	28. $\frac{29}{442}$	30. 5

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS FALL, 1995 CONTEST ONE

F95B1 $(\frac{1}{9}+\frac{1}{2})^{-1} = \frac{18}{11}$

F95B2 There are 24 which begin with 1, 24 begin with 3, and 24 begin with 5. The 5th number which begins with 7 is 71,935.

F95B3 Squaring both sides 3 times, $x\sqrt{x\sqrt{x}} = x^{2y}$, $x^2\cdot x\cdot\sqrt{x} = x^{4y}$, $x^4\cdot x^2\cdot x = x^{8y}$, $8y = 7$, $y = \frac{7}{8}$.

F95B4 $\frac{1}{3}\cdot\pi\cdot r^2\cdot h = \pi R^2 h$, $\frac{1}{3} = \frac{R^2}{r^2}$, $\frac{R}{r} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

F95B5 The coefficient is ${}_7C_2\cdot 3^2 = 21\cdot 9 = 189$.

F95B6 Using $\sqrt{N(N \div 1)(N+2)(N+3)+1} = N(N+3) + 1$, the integer is 461.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

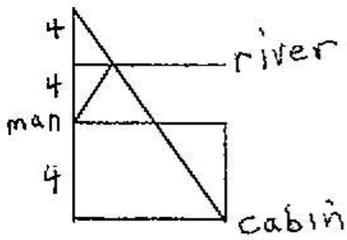
SENIOR B SOLUTIONS FALL, 1995 CONTEST TWO

F95B7 $170 = \dfrac{(N-2)\cdot 180}{N}$, $170N = 180N - 360$, $N = 36$.

F95B8 The first 3 terms are 2, $2^{2/3}$, $2^{1/3}$ and the common ratio is $2^{-1/3}$. The fourth term is $2^0 = 1$.

F95B9 $A_1 = s^2$ and $A_2 = (\sqrt{2}\cdot s)^2 = 2s^2$, $A_1{:}A_2 = 1{:}2$.

F95B10 Imagining his position 4 miles North of the river instead of South, the shortest distance will be the hypotenuse of a right triangle. $9^2 + 12^2 = x^2$ or $x = 15$.



F95B11 $\dfrac{84B + 96G}{B+G} = 92$, $84B + 96G = 92B + 92G$, $4G = 8B$

$\dfrac{1}{2} = \dfrac{B}{G}$.

F95B12 $A = \dfrac{1}{2}ab\sin C = \dfrac{1}{2}\cdot 2\cdot x\cdot \sin C = x$, $x\sin C = x$, $\sin C = 1$, $C = 90°$

$2^2 + x^2 = 7^2$, $x = \sqrt{45}$. $x = 3\sqrt{5}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS FALL, 1995 CONTEST THREE

F95B13 $6e^2 = 48,\ e^2 = 8,\ e = \sqrt{8}$
$V = e^3 = (\sqrt{8})^3 = 8\sqrt{8} = 16\sqrt{2}$

F95B14 $N = \frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy},\ N^2 = \frac{x^2 + 2xy + y^2}{(xy)^2} = \frac{10+6}{3^2}$

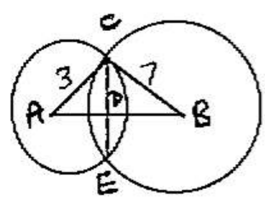
$N = \pm\frac{4}{3}$. (Or, by inspection, one variable is ± 3 and the other variable is ± 1, with both variables having the same sign.
Therefore, $\frac{1}{x} + \frac{1}{y} = \pm\frac{4}{3}$.

F95B15 The hexagon consists of 6 equilateral triangles with side 8.
Therefore, $6 \cdot \frac{8^2}{4} \cdot \sqrt{3} = 96\sqrt{3}$

F95B16 The number of possible hits is ${}_{64}C_2 = 2016$. The number of ways they share a common side is $7 \cdot 8$ horizontally and $7 \cdot 8$ vertically.
$P = \frac{112}{2016} = \frac{1}{18}$.

F95B17 $20^3 + 8^3 = (20 + 8)(20^2 - 20 \cdot 8 + 8^2) = 28(304) = (2^2 \cdot 7)(2^4 \cdot 19) = 2^6 \cdot 7 \cdot 19$.

F95B18 Using Hero's Formula, the area of triangle ABC $= \sqrt{9 \cdot 1 \cdot 2 \cdot 6} = \sqrt{108} = 6\sqrt{3}$



Also, the area is $\frac{1}{2} \cdot CD \cdot AB = \frac{1}{2} \cdot 8 \cdot CD$

$4CD = 6\sqrt{3},\ CD = \frac{3}{2} \cdot \sqrt{3}.\ \ CE = 2CD = 3\sqrt{3}$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS FALL, 1995 CONTEST FOUR

F95B19 $\sin A\cos B + \cos A\sin B = \sin(A + B) = \sin 90^{\circ} = 1$

F95B20 There is an 8x8 square on each of the 6 sides which will be painted on exactly 1 face. 8x8x6 = 384.

F95B21 $\sqrt{5 \cdot 5x^3} = 5\sqrt{x^3} = 625, \sqrt{x^3} = 125$. $x^{\frac{3}{2}} = 125$. x = 25.

F95B22 Since each vertex will have a diagonal from every other vertex except the adjacent ones, the number of diagonals is $\frac{N(N-3)}{2}$.
$N^2 - 3N - 108 = 0$. N = 12.

F95B23 $P = \frac{1}{4} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{3}{4} = \frac{2}{12} + \frac{3}{12} = \frac{5}{12}$.

F95B24 The number of multiples of 7 which are less than 100 is 14. Since 49 and 98 have 2 factors of 7, N = 14 + 2 = 16.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B SOLUTIONS FALL, 1995 CONTEST FIVE

F95B25 This will be true whenever $2x + 1 \geq 0$ or $x \geq -1/2$.

F95B26 In a normal life span, this will only occur if John will be 1 on his next birthday, and his grandfather will be 1 more than the LCM of 2, 3, 4, 5, and 6. This age is 61.

F95B27 $3p{\cdot}p \sim 18881$, $p^2 \sim 6293$. The numbers are 79 and 239.

F95B28 The number of 2 card hands is ${}_{52}C_2 = 1326$. The number with the 2 of clubs is 1x51 = 51. The number with another 2 and another club is 3x12 = 36. $P = \frac{36+51}{1326} = \frac{87}{1326} = \frac{29}{442}$.

F95B29 Since, past 10!, all numbers end in 00, one must only add the tens and units digits of the numbers up to 9!
$1 + 2 + 6 + 24 + 20 + 20 + 40 + 20 + 80 = 213$.

F95B30 $15^2 = 225$ will only be the sum of one pair of squares, $9^2 + 12^2$ so it will be a hypotenuse once. To be a leg, $c^2 - a^2 = 225$, $(c - a)(c + a) = 225$. $225 = 1{\cdot}225 = 3{\cdot}75 = 9{\cdot}25 = 5{\cdot}45$. Since each of these 4 combinations will produce a Pythagorean triple (e.g., $c + a = 75$ and $c - a = 3$ yields $c = 39$ and $a = 36$), there are a total of 5 triangles.