NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER ONE

PART I: TIME: 10 MINUTES SPRING, 1995

S95B1 Express S in simplest form if
$S = i + i^2 + i^3 + i^4 + \ldots + i^{50}$ where $i = \sqrt{-1}$.

S95B2 Find the smallest positive number which leaves a remainder of 9 when divided by 10, a remainder of 8 when divided by 9, ... etc. down to a remainder of 1 when divided by 2.

PART II: TIME: 10 MINUTES SPRING, 1995

S95B3 Find the remainder when 10^{109} is divided by 7.

S95B4 Compute the numerical value of
$\sin(\text{Arcsin}(4/5) + \text{Arcsin}(5/13))$.

PART III: TIME: 10 MINUTES SPRING, 1995

S95B5 Lisa drives to a distant town averaging 30 mph and returns over the same route averaging 50 mph. What was her average rate for the trip?

S95B6 If $x + y = a$, $x^3 + y^3 = b$, and $x^2 + y^2 = c$, express c in terms of a and b.

ANSWERS

1. $i-1$	3. 3	5. 37.5
2. 2519	4. 63/65	6. $(a^3 + 2b)/3a$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER TWO

PART I: TIME: 10 MINUTES SPRING, 1995

S95B7 Solve for x: $8^x = 1/32$

S95B8 Find the area of a regular 12 sided polygon which is inscribed in a circle of radius 12.

PART II: TIME: 10 MINUTES SPRING, 1995

S95B9 Working alone, John can paint a room in 8 hours. His son can paint the same room in 10 hours. How long would it take them if they work together?

S95B10 Solve for x: $x^{x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}} = 3$.

PART III: TIME: 10 MINUTES SPRING, 1995

S95B11 If $\log(x + y) = \log x + \log y$, express y in terms of x.

S95B12 If $(x + 1/x)^2 = 3$, find the value of $x^3 + 1/x^3$.

ANSWERS

7. $-5/3$
8. 432
9. $40/9$
10. $\sqrt[3]{3}$
11. $x/(x - 1)$
12. 0

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER THREE

PART I: TIME: 10 MINUTES SPRING, 1995

S95B13 Find the product of the first 5 terms of a geometric progression if the first term is 1 and the fifth term is 9.

S95B14 Solve for all values of x: $(x + 3)^{x+6} = 1$.

PART II: TIME: 10 MINUTES SPRING, 1995

S95B15 Find the volume of a box (rectangular solid) if 3 of its faces have areas of 15, 30, and 18.

S95B16 Find the coordinates of the point of intersection of the graphs $y = 2\log x$ and $y = \log 2x$.

PART III: TIME: 10 MINUTES SPRING, 1995

S95B17 Square ABCD has sides of length 10. A circle is drawn through points A and B and is tangent to CD. Find the radius of the circle.

S95B18 Find the sum of the infinite series
$1/10 + 2/10^2 + 3/10^3 + 4/10^4 \ldots$.

ANSWERS

13. 243
14. $\{-2,-4,-6\}$
15. 90
16. (2,log4)
17. 6.25
18. 10/81

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FOUR

PART I: TIME: 10 MINUTES SPRING, 1995

S95B19 Solve for all values of x: $x + \sqrt{8 - x} = 6$.

S95B20 A circle is inscribed in an equilateral triangle with sides of length 4. Find the sum of the areas of the three regions which are outside the circle and inside the triangle.

PART II: TIME: 10 MINUTES SPRING, 1995

S95B21 Given the product $10^{1/3} \cdot 10^{2/3} \cdot 10^{3/3} \cdots 10^{N/3}$. Find the smallest value of N such that the product exceeds one billion.

S95B22 Find the sum of the coefficients of the expansion of $(x - 3)^6$.

PART III: TIME: 10 MINUTES SPRING, 1995

S95B23 Compute the sum

$$100^2 - 99^2 + 98^2 - 97^2 + 96^2 - 95^2 + \ldots + 2^2 - 1^2$$

S95B24 The interior angles of a regular polygon with M sides each measure 2/3 the interior angles of a regular polygon with N sides. Express in ordered pair form (M,N) all possible values of M and N.

ANSWERS

19. 4
20. $4\sqrt{3} - 4\pi/3$
21. 7
22. 64
23. 5050
24. (3,4), (4,8), (5,20)

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER FIVE

PART I: TIME: 10 MINUTES SPRING, 1995

S95B25 How many points with integral coordinates (x,y) lie on the graph of $x^2 + y^2 = 169$?

S95B26 Two poles are 20 feet and 80 feet tall respectively. Their bases are 100 feet from each other. If cables join the top of each pole to the base of the other, how high is the point of intersection of the cables?

PART II: TIME: 10 MINUTES SPRING, 1995

S95B27 Solve for all values of x: $[2x/3] = 9$. ([x] represents the greatest integer less than or equal to x.)

S95B28 Three fair dice are rolled simultaneously. Find the probability that the three numbers obtained can be arranged in an arithmetic progression.

PART III: TIME: 10 MINUTES SPRING, 1995

S95B29 Express, in terms of N, the average of the first N positive integers.

S95B30 An isosceles triangle has sides 8, 10, and 10. Find the length of the radius of the circumscribed circle.

ANSWERS

25. 12
26. 16
27. $27/2 \leq x < 15$
28. 1/6
29. (N+1)/2
30. $25\sqrt{21}/21$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS CONTEST NUMBER ONE SPRING, 1995

S95B1 The powers of i are in cycles: $i^2 = -1$, $i^3 = -i$, and $i^4 = 1$. The sum of any 4 consecutive powers is 0. Thus, $S = i + i^2 + i^3 + i^4 + \ldots + i^{50} = i^{49} + i^{50} = i - 1$.

S95B2 If one is added to this number, it becomes the least common denominator for the first 10 integers, which is 2520. N = 2519.

S95B3
$10^2 \equiv 100 \equiv 2 \pmod 7$
$10^4 \equiv 4 \pmod 7$
$10^6 \equiv 4 \cdot 2 \pmod 7 \equiv 1 \pmod 7$
$10^{108} = (10^6)^{18} \equiv (1)^{18} \pmod 7 \equiv 1 \pmod 7$
$10^{109} = 10^{108} \cdot 10 \equiv 10 \pmod 7 \equiv 3 \pmod 7$

S95B4 Using the sine of the sum formula,
$\sin(\text{Arcsin}(4/5) + \text{Arcsin}(5/13))$
$= (\sin\text{Arcsin}(4/5))(\cos\text{Arcsin}(5/13)) + (\sin\text{Arcsin}(5/13))(\cos\text{Arcsin}(4/5))$
$= (4/5)(12/13) + (5/13)(3/5) = 48/65 + 15/65 = 63/65$.

S95B5 Let x be the distance between the towns.
$R = D/T = 2x/(x/30 + x/50) = 300x/8x = 37.5$

S95B6
$(x + y)^3 = x^3 + y^3 + 3x^2y + 3xy^2$
$a^3 = b + 3axy$ or $xy = (a^3 - b)/3a$
$c = (x + y)^2 - 2xy = a^2 - 2(a^3 - b)/3a = (a^3 + 2b)/3a$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS CONTEST NUMBER TWO SPRING, 1995

S95B7 $2^{3x} = 2^{-5}$. $3x = -5$. $x = -5/3$.

S95B8 The central angle is 30°. Each triangle has area $\frac{1}{2} \cdot 12 \cdot 12 \cdot \sin 30^\circ = 36$. Area $= 36 \cdot 12 = 432$.

S95B9 $x/10 + x/8 = 1$. $4x + 5x = 40$. $x = 40/9$

S95B10 $x^3 = 3$. $x = \sqrt[3]{3}$

S95B11 $\log(x + y) = \log xy$, $x + y = xy$, $x = xy - y$, $x = y(x - 1)$, $y = x/(x - 1)$.

S95B12 $(x + 1/x)^2 = 3$, $x^2 + 2 + 1/x^2 = 3$.
$x^2 + 1/x^2 = 1$, $x + 1/x = \sqrt{3}$.
$(x^2 + 1/x^2)(x + 1/x) = \sqrt{3}$.
$x^3 + x + 1/x + 1/x^3 = \sqrt{3}$.
$x^3 + \sqrt{3} + 1/x^3 = \sqrt{3}$.
$x^3 + 1/x^3 = 0$.
(Note: Solution is the same if $x + 1/x = -\sqrt{3}$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS CONTEST NUMBER THREE SPRING, 1995

S95B13 The first term is 3^0, the fifth term is 3^2. Therefore, the common ratio is $3^{1/2}$ and the product is $3^0 \cdot 3^{1/2} \cdot 3^1 \cdot 3^{3/2} \cdot 3^2 = 3^5 = 243$.

S95B14 Either $x + 6 = 0$, $x + 3 = 1$ or $x + 3 = -1$ and $x + 6$ is even. The solution is $\{-6, -2, -4\}$.

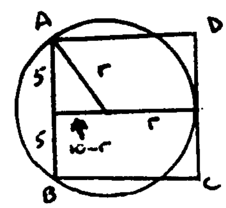
S95B15 Let x, y and z be the dimensions. $xy = 15$, $yz = 30$, and $xz = 18$. $x^2y^2z^2 = 15 \cdot 30 \cdot 18 = 3^2 \cdot 5^2 \cdot 6^2$. $V = xyz = 3 \cdot 5 \cdot 6 = 90$.

S95B16 $\log 2x = 2\log x = \log x^2$. $x^2 = 2x$. $x = 2$ (since x cannot be 0). Point of intersection is $(2, \log 4)$ or $(2, 2\log 2)$.

S95B17 $5^2 + (10 - r)^2 = r^2$

$5^2 + 100 - 20r + r^2 = r^2$

$r = 125/20 = 6.25$



S95B18 The series is the sum of other series: $(1/10 + 1/10^2 + 1/10^3 + \dots) + (1/10^2 + 1/10^3 + 1/10^4 + \dots) + (1/10^3 + 1/10^4 + 1/10^5 + \dots) +$ etc. Using Sum $= a_1/(1 - r)$, these series have sums of $1/9 + 1/90 + 1/900 + \dots = (1/9)/(1 - 1/10) = 10/81$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

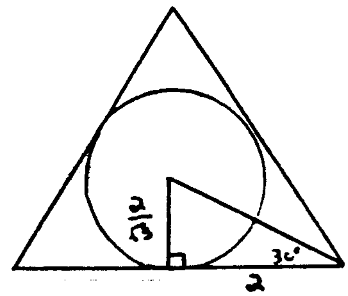
SENIOR B DIVISION SOLUTIONS CONTEST NUMBER FOUR SPRING, 1995

S95B19 $\sqrt{8 - x} = 6 - x$

$8 - x = 36 - 12x + x^2$

$x^2 - 11x + 28 = 0$. Thus, $x = 7, 4$. However, 7 does not check. Therefore, 4 is the only solution.

S95B20 The area of the triangle is $4^2\sqrt{3}/4 = 4\sqrt{3}$. The area of the circle is $4\pi/3$. The desired area is $4\sqrt{3} - 4\pi/3$.



S95B21 One billion = 10^9. We therefore need the smallest N such that $1 + 2 + 3 + \ldots + N > 27$. $N = 7$

S95B22 This can be done most easily by letting $x = 1$. $(1 - 3)^6 = (-2)^6 = 64$.

S95B23 Grouping the numbers in pairs and factoring, $(100 + 99)(100 - 99) + (98 + 97)(98 - 97)$, we have an arithmetic progression $199 + 195 + 191 + \ldots + 3$. $S = (50/2)(199 + 3) = 5050$.

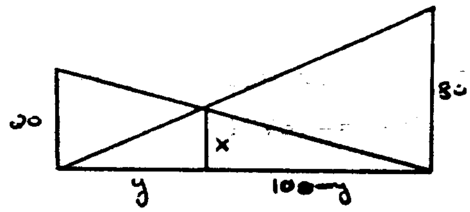
S95B24 $(N - 2)\cdot 180/N = (3/2)\cdot(M - 2)\cdot 180/M$. $N = 4M/(6 - M)$. The only M's that will produce positive integers for N are 3, 4, and 5. Thus, (3,4), (4,8) and (5,20) are all the solutions required.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS CONTEST NUMBER FIVE SPRING, 1995

S95B25 Considering the circle which is the graph of $x^2 + y^2 = 169$, there are 4 points on the axes plus 2 points in each quadrant, with x and y absolute values 5 and 12. $4 + 2 \cdot 4 = 12$.

S95B26 $x/80 = y/100$ or $100x = 80y$.
$x/20 = (100 - y)/100$ or
$100x = 2000 - 20y = 80y$
$y = 20$ and $x = 16$.

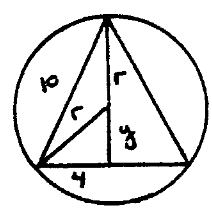


S95B27 $9 \leq 2x/3 < 10$, $27 \leq 2x < 30$, $27/2 \leq x < 15$.

S95B28 There are 6^3 ways the dice can land. There are 6 different arithmetic progressions possible: 1,2,3; 2,3,4; 3,4,5; 4,5,6; 1,3,5; and 2,4,6. For each of these, there are 3! or 6 ways they can turn up on the three dice. $6 \cdot 6/6^3 = 1/6$.

S95B29 The sum is $N(N+1)/2$. The average is $N(N+1)/2N = (N+1)/2$.

S95B30 The height $h = \sqrt{84}$.
$r^2 = 4^2 + y^2 = 4^2 + (h-r)^2$
$= 16 + h^2 - 2hr + r^2 = r^2$
$r = (16 + h^2)/2h = (16 + 84)/2\sqrt{84}$
$= 50/\sqrt{84} = 25\sqrt{21}/21$.



May 20, 1995

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1995 NYCIML contests that you requested on the application form.

The following are the corrected or alternative answers for the enclosed contests.. The original answers were inaccurate.

	Question	Correct answer
Senior A	S95S16	$3\sqrt{3}$ or $\frac{3\sqrt{3}}{2}$ if C=P or $\frac{3\sqrt{3}}{4}$ if D= P
	S95S25	1/6 or 7/36 (if d = 0)
	S95S27	16
Senior B	S95B28	1/6 or 7/36 (if d = 0)
Junior	S95J12	$\frac{\sqrt{3}}{4}$ or $\frac{\sqrt{3}}{16}$ (if the process was repeated an extra time)

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML

~~only one~~