NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER ONE** **SPRING 1995**

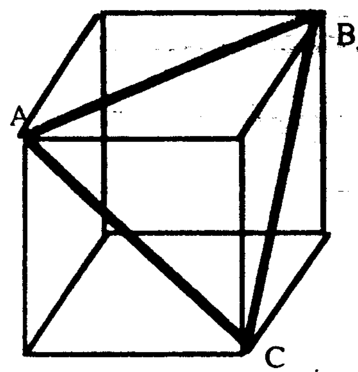
PART I: 10 Minutes **NYCIML Contest One** **Spring 1995**

S95J1. Compute the value of the twelfth term of the geometric sequence: $-1024, 512, -256, 128, \ldots$

S95J2. Four people sit on a park bench. Two of them, Sarah and David, are in love and insist on sitting next to each other. Compute the number of possible seating arrangements with the above condition.

PART II: 10 Minutes **NYCIML Contest One** **Spring 1995**

S95J3. In an election, 12000 people were eligible, but only 75% actually voted. Smith won 30% of the votes. Smythe won 25% of the votes and Schmidt got the rest of the votes. How many more votes did Schmidt receive than Smith?



S95J4. ΔABC is drawn connecting three vertices of the cube as shown. If each side of the cube has length 5, compute the area of ΔABC to the nearest tenth.

PART III: 10 Minutes **NYCIML Contest One** **Spring 1995**

S95J5. Compute the value of $\frac{1}{1995} + \frac{1994 \cdot 1996}{1995} - 1996$.

S95J6. Ruxin added two fractions to get $\frac{9x+1}{3x^2 - x - 2}$. Unfortunately, tea spilled onto her paper and she could not read parts of the original two fractions. If we represent her original sum as $\frac{a}{3x+b} + \frac{c}{x+d}$, where a, b, c, and d are the integers lost by the tea, compute the value of $a-b+c+d$.

Answers

1. $\frac{1}{2}$	**3.** 1350	**5.** -1
2. 12	**4.** 21.7	**6.** 2

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER TWO** **SPRING 1995**

PART I: 10 Minutes **NYCIML Contest Two** **Spring 1995**

S95J7. ΔABC is formed by connecting the x and y intercepts of the graph $y = 9 - x^2$. Another triangle, ΔDEF is formed by connecting the midpoints of $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$. Compute the area of ΔDEF.

S95J8. Square dartboard PQRS has perimeter 20. A dart, randomly thrown, lands at point T in the dartboard. Compute the probability that the area of $\Delta PTQ \le 10$.

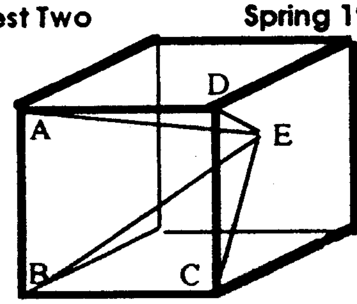
PART II: 10 Minutes **NYCIML Contest Two** **Spring 1995**

S95J9. The least common multiple of two relatively prime positive integers, a and b, is 144. If a ($a \ge 2$) is as *small* as possible, compute the value of b.

S95J10. A circle with radius 3 has its center at the origin. The quadrant I radius lying on the line $y = 2x$ is drawn. Write as an ordered pair, the coordinates of the endpoint of this radius that lies on the circle.

PART III: 10 Minutes **NYCIML Contest Two** **Spring 1995**

S95J11. In the cube shown, each edge has length 5. Point E is the intersection of the diagonals of the face opposite ABCD. A pyramid is formed by connecting points A, B, C and D to E. Compute the volume of E–ABCD.



S95J12. In equilateral ΔABC, AC = 16. The midpoints of sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, D,E, and F, respectively, are connected. ΔDEF is cut out and the three remaining triangles are thrown away. The midpoints of the sides of DEF are connected and this process is repeated a total of four times. Compute the area of the triangle remaining at the end of the process.

Answers

7. $\frac{27}{4}$, 6.75, or equivalent	**9.** 16	**11.** $\frac{125}{3}$ or $41\frac{2}{3}$
8. $\frac{4}{5}$, 0.8, or equivalent	**10.** $(\frac{3}{5}\sqrt{5}, \frac{6}{5}\sqrt{5})$ or equivalent	**12.** $\frac{\sqrt{3}}{4}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER THREE** **SPRING 1995**

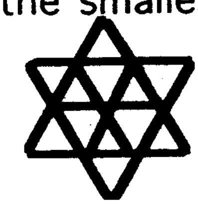
PART I: 10 Minutes **NYCIML Contest Three** **Spring 1995**

S95J13. An equilateral triangle is formed by connecting vertices A(−3,0), B(3,0), and C(0,a), where $a<0$. Compute the value of a.

S95J14. On a recent trip, Amy drove at an average rate of 40 mph. What must her average speed be on the return trip if the average for the entire round trip is 48mph? (Assume she takes the same route both ways.)

PART II: 10 Minutes **NYCIML Contest Three** **Spring 1995**

S95J15. The product of three consecutive *prime* numbers is 7429. Find the product of the smallest and largest of these three integers.



S95J16. How many different triangles are in the accompanying diagram?

PART III: 10 Minutes **NYCIML Contest Three** **Spring 1995**

S95J17. In ΔABC, AC = 7, AB = BC = 25 and altitude $\overline{AD}$ is drawn. Compute to the nearest integer the length of $\overline{DB}$.

S95J18. Secants $\overline{BOCE}$ and $\overline{ADE}$ are drawn to circle O, with points A, B, C, and D lying on the circle. If point D is also on the perpendicular bisector of $\overline{OE}$, and $m\angle BOA = 54$, compute the $m\angle BEA$.

Answers

13. $-3\sqrt{3}$ or equivalent	**15.** 391	**17.** 24
14. 60	**16.** 20	**18.** 18

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER ONE** **SPRING 1995**

Solutions

S95J1. The last term of a geometric sequence can be found using $\ell = ar^{n-1}$, where n is the number of terms, a is the first term, r is the common ratio and ℓ is the last term. Here we have $= -1024(-\frac{1}{2})^{11} = +\frac{1024}{2048} = \frac{1}{2}$

Answer: $\frac{1}{2}$

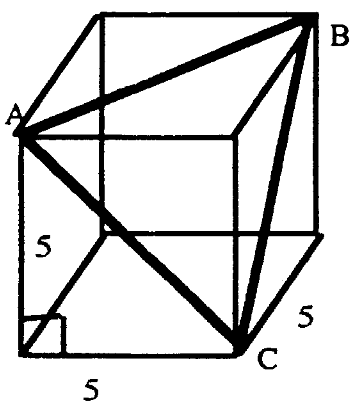
S95J2. **Method One:** Listing

Let the four people be denoted S, D, A, B. The possible arrangements are:
ABSD ABDS ASDB ADSB SDAB DSAB BASD BADS BSDA
BDSA SDBA DSBA . Thus there are twelve permutations.

This method can be tedious and you may miss some arrangements.

Method Two: Think of Sarah and David as one entity. Thus there are three entities being permuted. There are 3! ways of permuting three entities. For each of these permutations, S and D can be switched around giving $3! \cdot 2 = 12$ permutations. **Answer:** 12

S95J3. 9000 voted. The breakdown was Smith: 2700 votes, Smythe 2250 votes and Schmidt 4050 votes. Thus Schmidt received 1350 more votes than Smith. **Answer:** 1350



S95J4. Each side of the equilateral triangle is a diagonal of a face of the cube. Each of these diagonals has length $5\sqrt{2}$. The area of an equilateral triangle can be computed using the formula $A = \frac{s^2}{4} \cdot \sqrt{3}$. Thus $A = \frac{(5\sqrt{2})^2}{4}\sqrt{3} = \frac{25}{2}\sqrt{3} \approx (12.5)(1.732) \approx 21.650$ which rounds to 21.7 **Answer:** 21.7

S95J5. $\frac{1}{1995} + \frac{1994 \cdot 1996}{1995} - 1996 = \frac{1 + 1994(1996) - 1996(1995)}{1995}$
$= \frac{1 + 1996(-1)}{1995} = -1$ **Answer:** -1

S95J6. $3x^2 - x - 2 = (3x+2)(x-1)$ Thus, the two denominators are $3x+2$ and $x-1$ and $b = 2$ and $d = -1$. This means $\frac{9x+1}{3x^2 - x - 2} = \frac{a}{3x+2} + \frac{c}{x-1}$. Combine the right hand side to get $\frac{9x+1}{3x^2 - x - 2} = \frac{ax - a + 3cx + 2c}{3x^2 - x - 2}$. Focus on the numerators to get $9x+1 = (a+3c)x + (-a+2c)$.

Equate coefficients to get the following system of equations:
This gives $c = 2$ and $a = 3$. Thus $a-b+c+d = 3-2+2-1=2$

$$\begin{cases} a + 3c = 9 \\ -a + 2c = 1 \end{cases}$$

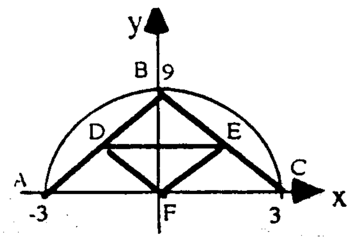
Answer: 2

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

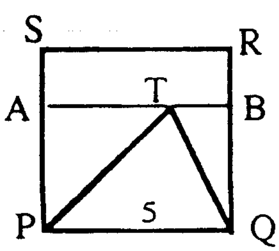
JUNIOR DIVISION **CONTEST NUMBER TWO** **SPRING 1995**

Solutions

S95J7. The four disjoint triangles formed have equal areas. Since the area of ΔABC is 27, the area of $\Delta DEF = \frac{27}{4}$ **Answer:** $\frac{27}{4}$



S95J8. The length of each side of the square is 5. In order for ΔPTQ to have area 10, the length of the altitude from T must be 4. This will happen



if T is chosen on $\overline{AB} \| \overline{PQ}$, $\frac{4}{5}$ the way up the rectangle. To have an area **LESS** than 10, any point in rectangle ABQP will suffice. Thus the probability the area is 10 or less is the ratio of the areas of rectangles ABPQ and PQRS $= \frac{4}{5} = 0.8$ **Answer:** $\frac{4}{5}$ or 0.8

S95J9. The product of the two numbers is 144. Thus $ab = 3^2 2^4$. Since the two numbers are relatively prime and "a" is as small as possible, a must be 9 and b must be 16. **Answer:** 16

S95J10. The equation of the circle is $x^2 + y^2 = 9$. Substituting $y = 2x$ gives $x^2 + 4x^2 = 9$ or $5x^2 = 9$ so that $x = \frac{3}{5}\sqrt{5}$ and $y = \frac{6}{5}\sqrt{5}$ **Answer:** $(\frac{3}{5}\sqrt{5}, \frac{6}{5}\sqrt{5})$

S95J11. This problem looks more difficult than it really is. As long as E is *any* point on the face opposite ABCD, the altitude of the pyramid is 5. Thus the volume is given by $V = \frac{1}{3}Bh = \frac{1}{3}(25)(5) = \frac{125}{3}$ or $41\frac{2}{3}$ **Answer:** $\frac{125}{3}$ or $41\frac{2}{3}$

S95J12. *The area of an equilateral triangle is found using the formula* $A = \frac{s^2}{4}\sqrt{3}$.

The original triangle, ΔABC has area $\frac{256}{4}\sqrt{3} = 64\sqrt{3}$.

The area of $\Delta DEF = \frac{1}{4}$ of the area of $\Delta ABC = 16\sqrt{3}$.

After the second stage, the area left is $\frac{1}{4}$ of this area or $4\sqrt{3}$.

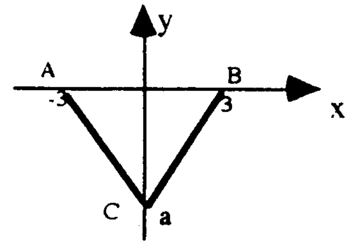
After the third stage, the area left is $\frac{1}{4}$ of this area or $\sqrt{3}$.

After the fourth stage, the area left is $\frac{1}{4}$ of this area or $\frac{\sqrt{3}}{4}$.

Answer: $\frac{\sqrt{3}}{4}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
JUNIOR DIVISION **CONTEST NUMBER THREE** **SPRING 1995**

Solutions



S95J13. $CB = AB = 6 = \sqrt{9+a^2}$ which means $9 + a^2 = 36$ so that $a = -3\sqrt{3}$. **Answer:** $-3\sqrt{3}$.

S95J14. The average speed for the round trip can be found using the harmonic mean. (See F94J15.) Let x = the average speed returning home. The average speed for the entire trip is $\frac{2}{\frac{1}{40}+\frac{1}{x}} = 48$. Solving this yields $x = 60$.

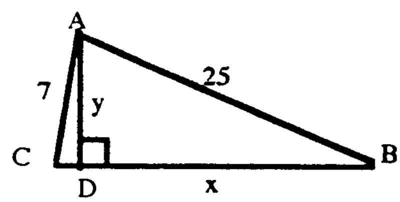
Answer: 60 mph

S95J15. Since $20^3 = 8000$, the three numbers should be close to 20. In fact, trial and error should yield that 7429 has 17, 19, and 23 for factors. Thus the product of the smallest and largest is $17 \cdot 23 = 391$. **Answer:** 391

S95J16. There are 12 triangles one unit per side, 6 triangles two units per side, and 2 triangles three units per side giving a total of 20 triangles.

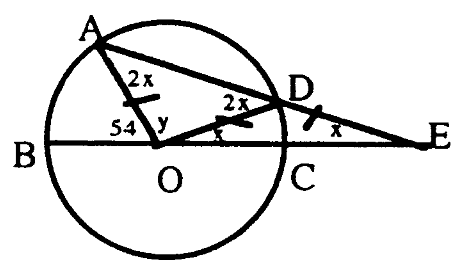
Answer: 20

S95J17. Let $x = DB$, $y = AD$, $25 - x = CD$.



Using the Pythagorean Theorem twice, we get: $y^2 = 49 - (25 - x)^2$ and $y^2 = 25^2 - x^2$. This means that $49 - (25-x)^2 = 25^2 - x^2$ Simplifying gives $x = \frac{1201}{50}$ which to the nearest integer is 24. **Answer:** 24

S95J18. Since D is on the the ⊥ bisector of $\overline{OE}$, OD = DE.(*Theorem*: Any point on the ⊥ bisector of a segment is equidistant from the endpoints of the segment.) Since $\overline{OD}$ is a radius, OD = AO meaning that ΔAOD and ΔODE are isosceles triangles. Letting $m\angle OED = x$, we get $m\angle EOD = x$, and $m\angle OAD = m\angle ODA = 2x$. This gives the equations $4x + y = 180$ and $x + y = 126$. Solving, we get $x = 18$.



Answer: 18

May 20, 1995

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1995 NYCIML contests that you requested on the application form.

The following are the corrected or alternative answers for the enclosed contests.. The original answers were inaccurate.

	Question	Correct answer
Senior A	S95S16	$3\sqrt{3}$ or $\frac{3\sqrt{3}}{2}$ if C=P or $\frac{3\sqrt{3}}{4}$ if D= P
	S95S25	1/6 or 7/36 (if d = 0)
	S95S27	16
Senior B	S95B28	1/6 or 7/36 (if d = 0)
Junior	S95J12	$\frac{\sqrt{3}}{4}$ or $\frac{\sqrt{3}}{16}$ (if the process was repeated an extra time)

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML

~~only one~~