# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
## SENIOR B DIVISION CONTEST NUMBER ONE

**PART I: TIME: 10 MINUTES** FALL, 1994

F94B1 Solve for x: $2^{x^2} \cdot 2^{4x+5} = 2$

F94B2 If $a^3 - b^3 = 30$, $a - b = 3$, and $a \cdot b = 2$, find the value of $a^2 + b^2$.

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**PART II: TIME: 10 MINUTES** FALL, 1994

F94B3 Find the units digit of $1993^{1993}$.

F94B4 If $\log_8 \cos x = -\frac{1}{2}$, find the value of $\cos 2x$.

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**PART III: TIME: 10 MINUTES** FALL, 1994

F94B5 Find the area of the circle which is circumscribed around a triangle with sides 7, 24, and 25.

F94B6 Find the sum of all positive odd 5 digit numbers containing the digits 1, 2, 3, 4, and 5 with no repetition.

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**ANSWERS**

1. $-2$
2. 8
3. 3
4. $-3/4$
5. $625\pi/4$
6. 2,399,976

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER TWO

PART I: TIME: 10 MINUTES FALL, 1994

F94B7 Find the value of $(-1/8)^{-2/3}$ in simplest form.

F94B8 Compute the number of positive integer factors of 3600.

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PART II: TIME: 10 MINUTES FALL, 1994

F94B9 At a restaurant, 4 sandwiches, 7 drinks, and 10 desserts cost \$15.80, while 3 sandwiches, 5 drinks, and 7 desserts cost \$11.05. Find the cost of a meal consisting of 1 sandwich, 1 drink, and 1 dessert.

F94B10 Solve for all values of x: $3x - 16\sqrt{x} + 5 = 0$.

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PART III: TIME: 10 MINUTES FALL, 1994

F94B11 Find the value of $(2i + 2)^6$ in simplest form. $(i = \sqrt{-1})$

F94B12 In a class of 24, 10 are picked to serve on a committee. Find the probability that both Jennifer and James are picked.

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ANSWERS

7. 4
8. 45
9. \$1.55
10. {1/9, 25}
11. −512i
12. 15/92

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER THREE

PART I: TIME: 10 MINUTES FALL, 1994

F94B13 The first three terms of a geometric progression are x, x+3, and 2x+6. Compute the fourth term.

F94B14 Find the area of the region bounded by $x^2 - y^2 = 0$ and $x = 2$.

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PART II: TIME: 10 MINUTES FALL, 1994

F94B15 Compute the value of $(\log 6)\div(\log 1/6)$.

F94B16 A fast food restaurant serves chicken nuggets in quantities of 7 or 10, so that, for example, 24 can be bought exactly (2 sevens and a 10) but 25 cannot. What is the largest number that cannot be bought exactly?

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PART III: TIME: 10 MINUTES FALL, 1994

F94B17 How many diagonals can be drawn in a convex polygon with 30 sides?

F94B18 Find the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$ .

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ANSWERS

| | | |
|---|---|---|
| 13. 24 | 15. -1 | 17. 405 |
| 14. 4 | 16. 53 | 18. $(1 + \sqrt{5})/2$ |

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FOUR

PART I: TIME: 10 MINUTES FALL, 1994

F94B19 Express (x,y) as an ordered pair if $x^2 - 4x + y^2 + 12y = -40$.

F94B20 How many positive integer perfect squares are factors of $x = 2^6 \cdot 3^7 \cdot 5^8 \cdot 7^2 \cdot 11^3 \cdot 13^1$?

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PART II: TIME: 10 MINUTES FALL, 1994

F94B21 How many real values of x, $0^\circ \le x \le 360^\circ$ satisfy the equation $\sin x + \sin 2x = 0$?

F94B22 In the country of Fredonia, a base other than 10 is used for the monetary system. When a man paid for a watch costing 430 fredons with a 1000 fredon bill, he got 240 fredons change. What base is used in Fredonia?

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PART III: TIME: 10 MINUTES FALL, 1994

F94B23 A square and an equilateral triangle have equal perimeters. The area of the triangle is $36\sqrt{3}$. Find the area of the square.

F94B24 How many times between noon and midnight are the hands of the clock perpendicular to each other?

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ANSWERS

19. (2, -6)
20. 320
21. 5
22. 7
23. 81
24. 22

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FIVE

PART I: TIME: 10 MINUTES FALL, 1994

F94B25 What is the radius of a circle in which a chord of length 12 is 4 units from the center?

F94B26 If m and n are roots of $2x^2 + 3x + 7 = 0$, find the value of $m^3 \cdot n + n^3 \cdot m$.

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PART II: TIME: 10 MINUTES FALL, 1994

F94B27 Express as a fraction in lowest terms: 0.2311111...

F94B28 Bill has 10 different books on a shelf: 4 Math, 3 English, and 3 Science. In how many ways can he arrange them if books on the same subject must be placed together?

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PART III: TIME: 10 MINUTES FALL, 1994

F94B29 The square of $a + b\sqrt{3}$ is $84 - 30\sqrt{3}$. Find all ordered pairs of integers (a,b).

F94B30 Find the area of a triangle with sides 13, 14, and 15.

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**ANSWERS**

25. $\sqrt{52}$
26. $-133/8$
27. 52/225
28. 5184
29. (3, -5), (-3, 5)
30. 84

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SOLUTIONS CONTEST NUMBER ONE FALL, 1994

F94B1 $2^{(x^2 + 4x + 5)} = 2^1$. Thus, $x^2 + 4x + 5 = 1$ or $x^2 + 4x + 4 = 0$ or $x = -2$.

F94B2 $a^3 - b^3 = (a - b)(a^2 + ab + b^2) = 30$

$3(a^2 + b^2 + 2) = 30$

$a^2 + b^2 = 8$

F94B3 The units digits of powers of 1993 appear in cycles of 4, namely, 3, 9, 7, and 1. Since $1993 \equiv 1 \bmod 4$, the units digit is 3.

F94B4 $8^{-\frac{1}{2}} = \cos x$ or $\cos x = 1/2\sqrt{2}$. $\cos 2x = 2\cos^2 x - 1 = 2(1/2\sqrt{2})^2 - 1 = -3/4$.

F94B5 Since it is a right triangle, one half of the hypotenuse is the radius, thus, $A = \pi(25/2)^2 = 625\pi/4$.

F94B6 There are 4! or 24 numbers which end in 5. The units column adds up to 120, each other column adds up to 60. The sum is 666,720. Likewise, the sum of the numbers ending in 3 is 799,992, and ending in 1 is 933,264. The sum is 2,399,976.

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
## SOLUTIONS CONTEST NUMBER TWO FALL, 1994

F94B7 $(-\frac{1}{8})^{-\frac{2}{3}} = (-8)^{\frac{2}{3}} = (-2)^2 = 4$

F94B8 $3600 = 2^4 \cdot 5^2 \cdot 3^2$. The number of factors is $(e_1+1)(e_2+1)\cdots(e_n+1)$ where $e_n$ are the exponents of the prime factors. $N = 5 \cdot 3 \cdot 3 = 45$.

F94B9 $4s + 7d + 10t = 1580$ (1)
$3s + 5d + 7t = 1105$ (2)
Multiplying (2) by 3, we obtain $9s + 15d + 21t = 3315$ (3)
Multiplying (1) by 2, we obtain $8s + 14d + 20t = 3160$ (4)
(3) - (4) yields, $s + d + t =$ \$1.55.

F94B10 Let $y = \sqrt{x}$. Thus, $3y^2 - 16y + 5 = 0$. $(3y-1)(y-5) = 0$. $y = 1/3$ or $y = 5$. Thus, $x = 1/9$ or $x=25$.

F94B11 $(2i + 2)^2 = 4i^2 + 8i + 4 = 8i$
$(2i + 2)^6 = (8i)^3 = 512i^3 = -512i$

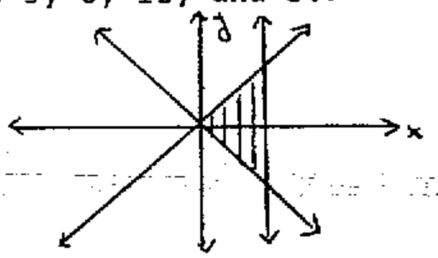
F94B12 The number of committees possible is ${}_{24}C_{10}$. The number of committees with James and Jennifer on them is ${}_{22}C_8$. Thus, $({}_{22}C_8)/({}_{24}C_{10}) = 15/92$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SOLUTIONS CONTEST NUMBER THREE FALL, 1994

F94B13 $x/(x+3) = (x+3)/(2x+6) = (x+3)/2(x+3) = 1/2$
$2x = x + 3$ or $x = 3$. The terms are 3, 6, 12, and 24.

F94B14 The region is bounded by the lines $y = x$, $y = -x$ and $x = 2$.
$A = \frac{1}{2}\cdot 4\cdot 2 = 4$.




F94B15 $(\log 6)/(\log 1 - \log 6) = (\log 6)/(-\log 6) = -1$.

F94B16 Using the units digit as a key, any number which ends in 7 can be bought, simply by adding the correct number of tens. Likewise, any number which ends in 4 can be bought, from 14 up. Since 3 is the last units digit that 7 produces $(9\cdot 7 = 63)$, 53 is the largest number that cannot be bought exactly.

F94B17 Since any vertex will not have a diagonal drawn to itself and the two adjacent vertices, the number of diagonals that can be drawn is $D = N(N-3)/2$. If $N = 30$, $D = 405$.

F94B18 Letting $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$, $x = \sqrt{1 + x}$.
$x^2 = 1 + x$, $x^2 - x - 1 = 0$ or $x = (1 \pm \sqrt{5})/2$, but only the positive answer is possible. Thus, $x = (1 + \sqrt{5})/2$.

# NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

## SOLUTIONS CONTEST NUMBER FOUR FALL, 1994

F94B19 Completing the squares, $(x-2)^2 + (y+6)^2 = 0$. Thus, the solution is $(2, -6)$.

F94B20 The exponents of each prime must be even. There are 4 choices (0, 2, 4, 6) for the exponent of 2, 4 for 3, 5 for 5, 2 for 7, 2 for 11 and 1 for 13. Thus, $4 \cdot 4 \cdot 5 \cdot 2 \cdot 2 \cdot 1 = 320$.

F94B21 $\sin x + 2\sin x\cos x = 0$. $\sin x(1 + 2\cos x) = 0$. Thus, $x = 0^\circ, 180^\circ, 360^\circ$ or $x = 120^\circ, 240^\circ$.

F94B22 $b^3 - (4b^2 + 3b) = 2b^2 + 4b$

$b^3 - 6b^2 - 7b = 0$ or $b(b-7)(b+1) = 0$. Therefore, $b = 7$ is the only possible base.

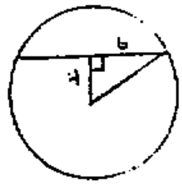
F94B23 Finding a side of the triangle, $s^2\sqrt{3}/4 = 36\sqrt{3}$ or $s = 12$. $P = 36$. A side of the square is 9, and the area is 81.

F94B24 The hands will be perpendicular twice each hour except that 3 P.M. and 9 P.M. serve as the second occurrence for the previous hour and the first occurrence for the next hour. Therefore, there are 22 times.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SOLUTIONS CONTEST NUMBER FIVE FALL, 1994

F94B25 By the Pythagorean Theorem, $4^2 + 6^2 = r^2$ or $r = \sqrt{52}$.




F94B26 $m^3 \cdot n + n^3 \cdot m = mn(m^2 + n^2) = mn[(m + n)^2 - 2mn]$

$m \cdot n = 7/2$ and $m + n = -3/2$. Thus,

$(7/2)[(-3/2)^2 - 2\cdot(7/2)] = (7/2)[(9/4) - 7] =$

$(7/2)(-19/4) = -133/8$.

F94B27 $x = 0.2311111\ldots$

$10x = 2.3111111\ldots$

$9x = 2.08$ or $x = 2.08/9 = 208/900 = 52/225$.

F94B28 There are 4! ways of arranging the Math books, 3! for the English, and 3! for the Science. Also, there are 3! ways of arranging the subjects. $4!\cdot3!\cdot3!\cdot3! = 5184$.

F94B29 $(a + b\sqrt{3})^2 = a^2 + 2ab\sqrt{3} + 3b^2$. Equating the coefficients, $2ab = -30$ and $a^2 + 3b^2 = 84$. Thus, $ab = -15$ or $(3, -5)$ and $(-3, 5)$.

F94B30 Either use Hero's formula $A = \sqrt{s(s-a)(s-b)(s-c)} =$

$\sqrt{21\cdot8\cdot7\cdot6} = \sqrt{7\cdot3\cdot4\cdot2\cdot7\cdot3\cdot2} = 7\cdot3\cdot2\cdot2 = 84$ OR

$h^2 = 13^2 - x^2$

$h^2 = 15^2 - (14 - x)^2$

$13^2 - x^2 = 15^2 - 14^2 + 28x - x^2$

$140 = 28x$ or $x = 5$

$h = 12$ and thus, $A = \frac{1}{2}\cdot12\cdot14 = 84$