NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER ONE

PART I: TIME: 10 MINUTES FALL, 1994

F94S1 How many ordered pairs of integers (x,y) satisfy $x^2 - y^2 = 24$?

F94S2 Let [x] be the greatest integer less than or equal to x. In the rectangular region in the coordinate plane given by $-5 < x < 5$ and $-5 < y < 5$, compute the number of points of intersection for the graphs of $y = 2[x]$ and $y = 2x - 2$.

PART II: TIME: 10 MINUTES FALL, 1994

F94S3 An equilateral triangle of perimeter P is constructed and the midpoints of the sides are joined to create an interior triangle. The midpoints of the inner triangle are joined to form another triangle and this process is repeated infinitely in each new triangle that is formed. Including the original triangle, express in simplest form in terms of P the sum of the lengths of all the line segments created by this process.

F94S4 Within the set B of 5-digit binary numbers (numbers whose digits are either 0 or 1 including those with leading 0's) define d(m,n) to be the number of places in which the digits of m and n differ. If m = 00000, compute the number of elements of B so that $d(m,n)<4$.

PART III: TIME: 10 MINUTES FALL, 1994

F94S5 Compute:

$$\frac{\cos^3(15°) - \sin^3(15°)}{\cos^3(15°) - \cos^2(15°)\sin(15°) - \cos(15°)\sin^2(15°) + \sin^3(15°)}$$

F94S6 Let a and b be, respectively, the positive values of the x and y intercepts of the ellipse given by the equation $9x^2 + 16y^2 = 144$. A circle whose radius is the average of a and b is drawn with center at the origin and a polygon is formed by joining the points of intersection of the ellipse and the circle with line segments. Compute the area within the region bounded by the polygon.

<u>ANSWERS</u>

1. 8
2. 0
3. 2P
4. 26
5. $5\sqrt{3}/6$
6. $12\sqrt{195}/7$

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER TWO

PART I: TIME: 10 MINUTES FALL, 1994

F94S7 Find the set of all integers x that satisfy $3^{x+1} + 6^x = 297$.

F94S8 A 6″ by 8″ rectangular piece of paper is folded so that a pair of opposite vertices coincide. Compute the length of the crease ("folded line segment").

PART II: TIME: 10 MINUTES FALL, 1994

F94S9 Among all three digit positive integers, compute the probability of finding a positive integer with exactly three distinct positive integer factors?

F94S10 Identify the domain and range of $[x]^2 + [y]^2 = 9$ that has no domain values in common with the domain of $x^2 + y^2 = 9$. ([a] denotes the greatest integer less than or equal to a.)

PART III: TIME: 10 MINUTES FALL, 1994

F94S11 In triangle ABC, median $\overline{CD}$ is drawn and a circle is inscribed in triangle ACD. If AC = CD = 2 and AB = 4, compute the ratio of the area of the circle to the area of triangle ABC.

F94S12 Let the sequence $\{a_k\}$ be defined as follows:

$$a_1 = 1 \text{ and } a_k = \begin{cases} 2a_{k-1}, & k \text{ even} \\ 2 - a_{k-1}, & k \text{ odd} \end{cases}$$

Compute the value of $\dfrac{a_{99} - a_{97}}{a_{95} - a_{93}}$.

ANSWERS

7. {3}
8. 7.5
9. 7/900
10. $3 \leq x < 4$ and $0 \leq y < 1$
11. $\pi\sqrt{3}/18$
12. 4

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER THREE

PART I: TIME: 10 MINUTES FALL, 1994

F94S13 If f(xy) = f(x)+f(y) for all x,y >0 and f(2)=1, compute f(1/4).

F94S14 Two candles are 35 cm tall. After being lit, one candle lasts 5 hours and the other lasts 7 hours. If both candles are lit at exactly 7pm, compute the time, to the nearest minute, that one candle will be twice as tall as the other?

PART II: TIME: 10 MINUTES FALL, 1994

F94S15 Let triangle ABC be drawn in the coordinate plane so that the vertices are A(0,0), B(6,0), and C(3,4). Compute the coordinates (x,y) of the point P in the interior of the triangle that is equidistant from the three vertices.

F94S16 Each of the 1500 students at Mathzfun High School were assigned an integer from 1 to 1500 and were seated in a single row along the football field in numerical order. Over the loudspeaker the principal gave the following sequence of 1500 instructions:

a) All students stand.
b) Every second student begining with #2, sit.
c) Every third student beginning with #3, reverse your position.
d) Every fourth student beginning with #4, reverse your position.

and so on so that the nth instruction requires every nth student beginning with #n to reverse the stand/sit position. After all 1500 instructions were given, prizes were given to those students who were standing. Compute the number of prizes given.

PART III: TIME: 10 MINUTES FALL, 1994

F94S17 Compute the sum of the coefficients of $(10x + y)^5$

F94S18 Let $a_n = a_{n-2} - a_{n-1}$ with $a_1 = 1$ and $a_2 = -1$
and let $b_n = b_{n-2} + b_{n-1}$ with $b_1 = 1$ and $b_2 = -1$.
Compute $(a_{1994} - b_{1994}) + (a_{1993} - b_{1993})$.

ANSWERS

13. -2
14. 10:53 pm
15. (3, 7/8)
16. 38
17. 161,051
18. 0

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FOUR

PART I: TIME: 10 MINUTES FALL, 1994

F94S19 You want to distribute 150 items among 13 people so that each person receives a different number of items, but no one receives more than 15. Compute the least number of items that will remain undistributed?

F94S20 A triangle has sides of lengths $x+1$, $x+2$, and $x+3$ where x is an integer. The altitude drawn to the side of length $x+2$ has a length of x. Compute the value of x.

PART II: TIME: 10 MINUTES FALL, 1994

F94S21 A dart board is constructed by 5 concentric circles such that the innermost radius is 3 inches and the distances between the circles are all 1 inch. From the inside to the outside, a dart that lands in each disjoint region receives 9, 7, 5, 3, and 1 point respectively. If 2 darts are thrown and land at random on the dart board, compute the probability, as a fraction in simplest form, that the total point score is less than 7.

F94S22 In triangle ABC, the medians to $\overline{AC}$ and $\overline{BC}$ are perpendicular. If $AC = 2x$ and $BC = 2y$, express AB^2 in terms of x and y.

PART III: TIME: 10 MINUTES FALL, 1994

F94S23 If the roots of $x^2 + bx + c = 0$ are the reciprocals of the roots of $x^2 + x - 1 = 0$, find the ordered pair (b,c).

F94S24 Five cards are marked differently with the letters a, b, c, d, and e. Five envelopes are marked differently with the letters A, B, C, D, and E. A card is placed in each envelope. In how many ways can the cards be placed in the envelopes so that no envelope contains a card marked with its lowercase counterpart?

ANSWERS

19. 33
20. 12
21. 810/2401
22. $4(x^2 + y^2)/5$
23. $(-1,-1)$
24. 44

NEW YORK CITY INTERSCHOLASTIC MATH LEAGUE
SENIOR A DIVISION CONTEST NUMBER FIVE

PART I: TIME: 10 MINUTES FALL, 1994

F94S25 Given the equations
$\sqrt{2}\,x^2 + \sqrt{6}\,x + \sqrt{5} = 0$ and $\sqrt{5}\,x^2 - \sqrt{15}\,x + \sqrt{2} = 0$,
write a quadratic equation in the form $x^2 + bx + c = 0$ whose roots have a sum equal to the sum of the four roots of these equations and whose product is equal to the product of the four roots of these equations.

F94S26 For $x > 0$, express cot(Arcsin x + Arccos x) as a rational expression in terms of x.

PART II: TIME: 10 MINUTES FALL, 1994

F94S27 Compute the sum of the digits of $11^5 - 11^4$.

F94S28 In square ABCD, point P is located in the interior so that $m\angle PAB = m\angle PBA = 30°$. If PD = 1, compute the area of the square.

PART III: TIME: 10 MINUTES FALL, 1994

F94S29 Let $f(x) = \sqrt{x^2 - 6x + 9}$ and $g(x) = 6 - f(x)$. Compute the number of square units in the area of the region enclosed by the graphs of $y = f(x)$ and $y = g(x)$.

F94S30 In triangle ABC, AB = 5, BC = 6, and AC = 7. Let D be a point on $\overline{AB}$ and E be a point on $\overline{BC}$ so that segment $\overline{DE}$ divides the perimeter of triangle ABC in half and creates interior regions equal in area. Compute the smallest possible length of $\overline{DB}$.

ANSWERS

25. $x^2 + 1 = 0$
26. 0
27. 16
28. $(12 + 3\sqrt{3})/13$
29. 18
30. $(9 - \sqrt{21})/2$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR DIVISION A - SOLUTIONS

CONTEST NUMBER ONE - FALL, 1994

F94S1

$x^2 - y^2 = (x+y)(x-y)$ and both of these factors must be integers if x and y are both integers. They must also be of the same sign.
This leads to 16 possible pairs of simultaneous equations to solve of the form :
$x + y = a$
$x - y = b$ where a and b are cofactors of 24, including negative integers.
Only 8 of these (those with the factor pairs of ±12,±2 and ±6,±4 in all possible configurations) will yield integer solutions:
(7,5), (-7,-5), (7,-5), (-7,5), (5,1), (-5,-1), (5,-1), and (-5,1).

F94S2

The range of $y = 2[x]$ in this region is the set of integers {-4,-2,0,2,4}.
The graph of $y = 2x - 2$ passes through the points (-1,-4), (0,-2), (1,0), (2,2), and (3,4) none of which are on the graph of $y = 2[x]$.

F94S3

The perimeter of each created inner triangle is 1/2 of its predecessor.
Therefore, the sum is $P + P/2 + P/4 + P/8 + \ldots = P(1 + 1/2 + 1/4 + \ldots)$
This infinite geometric series has a sum of 2, giving the sum of the line segments to be 2P.

F94S4

There is only one 5-digit binary number,n, so that d(00000,n)=5. Namely, 11111. For d(00000,n) = 4, there are ${}_5C_1 = 5$ numbers containing four 1's and one 0. Of the entire set of 32 elements of B, the remaining 26 will have at most three 1's making $d(00000,n) \le 3$.

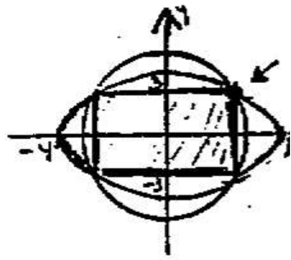
F94S5

Factoring the numerator and denominator yields:

$$\frac{(\cos(15°) - \sin(15°))(\cos^2(15°) + \cos(15°)\sin(15°) + \sin^2(15°))}{(\cos(15°) - \sin(15°))(\cos^2(15°) - \sin^2(15°))}$$

Cancelling the common factor and applying the standard trig identities for the double angle, we get:

$$\frac{1 + \frac{1}{2}\sin(30°)}{\cos(30°)} = \frac{1 + \frac{1}{4}}{\frac{\sqrt{3}}{2}} = \frac{\frac{5}{4}}{\frac{\sqrt{3}}{2}} = \frac{10}{4\sqrt{3}} = \frac{5\sqrt{3}}{6}$$



F94S6

The intercepts of the ellipse are a = 4 and b = 3. The radius of the circle is 7/2. The symmetry of the graphs forces this polygon to be a rectangle and we can compute the area by finding the area of the rectangle lying in quadrant I and multiplying by 4. For this we need the coordinates of the point of intersection of the circle and the ellipse in quadrant I. The area we seek is the product of these coordinates.
The system of equations to solve is: $9x^2 + 16y^2 = 144$ and $x^2 + y^2 = 49/4$.
Multiplying the second equation by 16 and subtracting the first we find that $7x^2 = 52$ or $x^2 = 52/7$
Using this in the second equation we find that $y^2 = 49/4 - 52/7 = 135/28$.
Therefore, $x^2y^2 = (52/7)(135/28) = 1755/49$ and the area is
$xy = \sqrt{1755}/7 = 3\sqrt{195}/7$ and the area of the entire region is $12\sqrt{195}/7$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR DIVISION A - SOLUTIONS

CONTEST NUMBER TWO - FALL, 1994

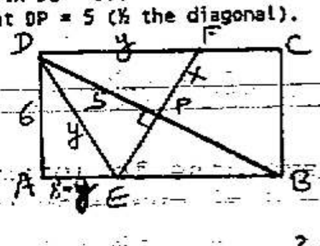
F94S7

$3^{x+1} + 6^x = 3^x(3 + 2^x)$. If x is an integer, then both of these factors are integers. 297 has the following possible pairs of cofactors: 1,297 ; 3,99 ; 9,33 ; 27,11. Clearly, if $x = 3$, then $3^x = 27$ and $3+2^x = 11$. No other value of x would produce a pair of cofactors.

Alternative Solution:
$f(x) = 3^{x+1} + 6^x$ is clearly an increasing function. With this in mind, should an integer value of x yield 297, it must be the only such value. By checking f(x) over the integers starting from $x = 0$, we see that
$f(0) = 4$, $f(1) = 15$, $f(2) = 63$, and $f(3) = 297$.

F94S8

Call the rectangle ABCD so that $AB = CD = 8$ and $BC = DA = 6$. Fold the rectangle so that B coincides with D. Let E and F be the endpoints of the fold line on AB and CD, respectively.
Segment EF is the perpendicular bisector of diagonal BD. Call the point of intersection of BD and EF, P. Also, draw DE. By the symmetry produced by the line of reflection EF, triangle EDF is isosceles with DE = DF.
Now, let $x = PF$ and $y = DE = DF$. Again, by symmetry, $AE = 8-y$. We also know that $DP = 5$ (½ the diagonal).
Using the Pythagorean theorem on right triangle DAE, we have
$6^2 + (8-y)^2 = y^2$ giving $y = 100/16 = 25/4$
Using the Pythagorean Theorem on right triangle DPF, we have
$x^2 + 5^2 = (25/4)^2$ giving $x = 15/4$
The entire fold line is $2x = 15/2 = 7.5$



F94S9

The only numbers with exactly 3 distinct factors are squares of prime numbers. That is, for a prime p, p^2 has the factors 1, p, and p^2.* The integers whose squares have three digits are in the set {10,11, ..., 31}. Of these, the primes are 11, 13, 17, 19, 23, 29, and 31.
Since there are 900 three digit numbers, the probability is 7/900.

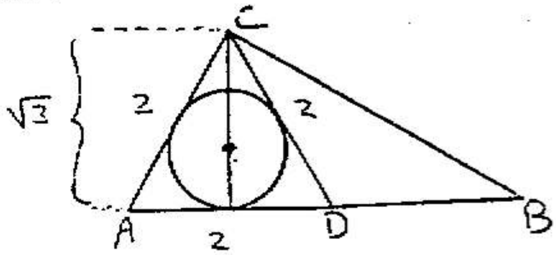
* The number of factors of an integer is found by looking at the prime power factorization of the integer and forming the product of the integers formed by adding 1 to each power. That is, if $N = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$,
then the number of factors is $(a_1+1)(a_2+1)\ldots(a_k+1)$.
Since 3 is a prime number, the only way to have 3 distinct factors is to have p^2.

F94S10

As in the circle $x^2 + y^2 = 9$, the smallest domain value of $[x]^2 + [y]^2 = 9$ is $x = -3$. (For $x < -3$, the equation would yield $[y]^2 < 0$.) The largest possible value for $[x]^2$ is 9 which can occur for $3 \le x < 4$. Since 3 is also in the domain of the circle, we have as our domain $3 < x < 4$. The range corresponding to this domain must satisfy $[y] = 0$.
This will occur for $0 \le y < 1$.
Note the graph of $[x]^2 + [y]^2 = 9$ consists of four disjoint semi-open square regions:
$-3 \le x < -2$ $0 \le y < 1$ & $0 \le x < 1$ $3 \le y < 4$ & $0 \le x < 1$ $-3 \le y < -2$ & $3 \le x < 4$ $0 \le y < 1$

F94S11

Triangle ACD is equilateral since AD = ½AB = 2. Also, the area of triangle ABC is twice the area of triangle ACD since a median creates two triangles of equal area. The center of a circle inscribed in a triangle is found at the intersection of the angle bisectors. In an equilateral triangle the angle bisectors are also medians which meet 1/3 along each median from the side it is drawn to. Since the median of an equilateral triangle is also an altitude, the radius of the inscribed circle is 1/3 the length of the altitude or $\sqrt{3}/3$. The area of the circle is $\pi/3$. The area of triangle ACD is $\sqrt{3}$. The ratio we seek is $\pi/3$ / $2\sqrt{3}$ or $\pi\sqrt{3}/18$.



F94S12

The beginning of the sequence is
1, 2, 0, 0, 2, 4, -2, -4, 6, 12, -10, -20, 22, 44, -42,...
The odd terms form the sequence:
1, 0, 2, -2, 6, -10, 22, -42, ...
The differences of consecutive odd terms form the sequence:
$a_3 - a_1 = -1$, $a_5 - a_3 = 2$, $a_7 - a_5 = -4$, $a_9 - a_7 = 8$ etc.

In general $a_{2k+1} - a_{2k-1} = (-1)^k 2^{k-1}$

Therefore, $a_{99} - a_{97} = (-1)^{49} 2^{48}$ and $a_{95} - a_{93} = (-1)^{47} 2^{46}$

and the quotient is $2^2 = 4$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR DIVISION A - SOLUTIONS

CONTEST NUMBER THREE - FALL, 1994

F94S13

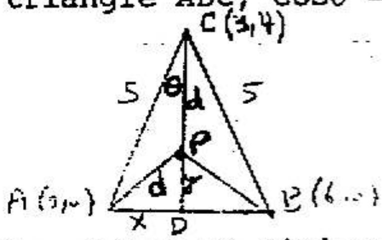
$f(x) = f(x \cdot 1) = f(x) + f(1) \longrightarrow f(1) = 0.$
$f(1) = f(x \cdot 1/x) = f(x) + f(1/x) = 0 \longrightarrow f(1/x) = -f(x).$
$f(1/4) = -f(4) = -f(2 \cdot 2) = -2f(2) = -2 \cdot 1 = -2.$
Note that f(x) is in reality $\log_2 x$.

F94S14

The "fast" candle burns at a rate of 7 cm/hr and the "slow" candle burns at a rate of 5 cm/hr. After h hours, the fast candle is 35 - 7h cm tall and the slow candle is 35 - 5h cm tall. The equation to solve is 2(35 - 7h) = 35 - 5h giving h = 35/9 = 3.89 hrs = 3 hrs 53 min. The time at which this occurs is, therefore, 10:53 pm.

F94S15

Triangle ABC is isosceles with AC = BC = 5. Point P will lie on the altitude CD to AB. Let d = AP = BP = CP.
Let θ be the measure of angle ACD. From right triangle ADC, cosθ = 4/5.
From triangle APC, the law of cosines gives
$d^2 = 5^2 + d^2 - 2(5)(d)(4/5)$
$0 = 25 - 8d$ giving $d = 25/8$.
Therefore, x = AD = 3
and y = CD - CP = 4 - d = 4 - 25/8 = 7/8.



F94S16

The final position of student #n depends upon the number of divisors of the integer n. For the student to be standing at the end of the exercise, the number of divisors must be odd. Only the perfect squares have exactly an odd number of divisors. $1444 = 38^2$ is the largest perfect square less than 1500. Therefore, there are 38 winners.

F94S17

Using the binomial expansion we have that $(10x+y)^5 =$

$${}_5C_0\, 10^5 x^5 y^0 + {}_5C_1\, 10^4 x^4 y^1 + {}_5C_2\, 10^3 x^3 y^2 + {}_5C_3\, 10^2 x^2 y^3 + {}_5C_4\, 10^1 x^1 y^4 + {}_5C_5\, 10^0 x^0 y^5$$

The sum of the coefficients are therefore:
1x100,000 + 5x10,000 + 10x1,000 + 10x100 +5x10 + 1x1 = 161,051.
Alternative Solution:
In any expansion $(ax + by)^n$, the sum of the coefficients can easily be found by substituting x=y=1 and evaluating.
In this problem, we would have $(10+1)^5 = 11^5 = 161,051$.

F94S18

The sequence a_n is 1, -1, 2, -3, 5, -8, 13, -21, 34, -55, ...
The sequence b_n is 1, -1, 0, -1, -1, -2, -3, -5, -8, -13, ...
The sequence
$a_n - b_n$ is 0, 0, 2, -2, 6, -6, 16, -16, 42, -42, ...

For n even, $(a_n - b_n) + (a_{n-1} - b_{n-1}) = 0$.

This can be proven by noting that $a_n = (-1)^{n+1} F_n$ where F_n is the n^{th} Fibonacci number and $b_n = -F_{n-3}$ for $n \geq 4$.
For n even,
$$\begin{aligned}(a_n - b_n) + (a_{n-1} - b_{n-1}) &= (-F_n + F_{n-3}) + (F_{n-1} + F_{n-4}) \\ &= -F_{n-1} - F_{n-2} + F_{n-3} + F_{n-1} + F_{n-4} \\ &= -F_{n-2} + (F_{n-3} + F_{n-4}) \\ &= -F_{n-2} + F_{n-2} = 0.\end{aligned}$$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR DIVISION A - SOLUTIONS

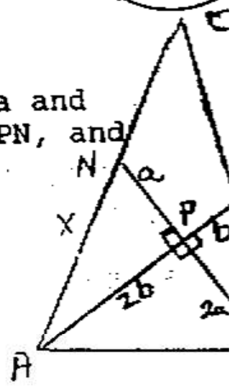
CONTEST NUMBER FOUR - FALL, 1994

F94S19
At the very least you could give the following amounts: 1,2,3,...,13. You would have distributed 91 items leaving you with 59. If you give two more to each person, you would have distributed 91+26=117, leaving you with 33. Since the thirteenth person would now have the maximum of 15 items, this is the maximum distribution.

F94S20
Heron's Formula for the area of a triangle is $\sqrt{s(s-a)(s-b)(s-c)}$ where a, b, and c are the side lengths and $s = \frac{1}{2}(a+b+c)$, the semiperimeter.
The perimeter of our triangle is 3x+6 and $s = 3(x/2 + 1)$
Applying this to our triangle and equating it with the usual area formula we have: $\sqrt{3(x/2 + 1)(x/2 + 2)(x/2 + 1)(x/2)} = (x/2)(x + 2)$
For simplicity, let y = x/2 and square both sides.
$3y(y+1)^2(y+2) = 4y^2(y+1)^2$ or $3(y+2) = 4y$ giving y = 6 and, hence, x = 12.

F94S21
The probability of landing in a region is given by the area of the disjoint region divided by the area of the entire board.
Therefore, $Prob(1) = (7^2 - 6^2)\pi/7^2\pi = 13/49$, $Prob(3) = (6^2 - 5^2)\pi/7^2\pi = 11/49$, and $Prob(5) = (5^2 - 4^2)\pi/7^2\pi = 9/49$.
The only ways to achieve a total score less than 7 is to throw 1-1, 1-3, 3-1, 1-5, 5-1, or 3-3.
The total probability is, therefore,
$Prob(1)^2 + 2Prob(1)Prob(3) + 2Prob(1)Prob(5) + Prob(3)^2$
or $(13/49)^2 + 2(13/49)(11/49) + 2(13/49)(9/49) + (11/49)^2 = 810/2401$

F94S22
Call the medians $\overline{AM}$ and $\overline{BN}$ and their point of intersection, P.
We have AN = x and BM = y. Let PN = a and PM = b, giving BP = 2a and AP = 2b. Using the Pythagorean Theorem on right triangles APB, APN, and BPM, we get the following three equations:
$AB^2 = 4a^2 + 4b^2$, $x^2 = a^2 + 4b^2$, $y^2 = 4a^2 + b^2$
Adding the latter two equations
$x^2 + y^2 = 5a^2 + 5b^2$
Therefore, $AB^2 = 4a^2 + 4b^2 = 4(x^2 + y^2)/5$



F94S23

The reciprocals are $\frac{2}{-1+\sqrt{5}}$ *and* $\frac{2}{-1-\sqrt{5}}$ and when the denominators are rationalized, they become $\frac{1+\sqrt{5}}{2}$ *and* $\frac{1-\sqrt{5}}{2}$. The sum of the roots is 1 and the product of the roots is -1. Since a = 1, then b = -1 and c = -1 giving the quadratic $x^2 - x - 1 = 0$. Note that the positive root of the original quadratic is the golden mean and that the reciprocals are also the opposites of the original roots.

F94S24
There are 4 possibilities for envelope A. Suppose b is placed in envelope A. If a is then placed in envelope B, only 2 possible configurations:
Ab Ba Ce Dc Ed and Ab Ba Cd De Ec.
If c is placed in envelope B, there are only 3 possible configurations:
Ab Bc Ca De Ed , Ab Bc Cd De Ea , and Ab Bc Ce Da Ed.
There are also 3 configurations for d in evelope B and for e in envelope B.
Thus, for b in envelope A, there are 11 configurations.
Therefore, there are 11x4 = 44 configurations for the desired outcome.

NEW YORK CITY INTERSCHOLASTIC

SENIOR DIVISION A - SOLUTIONS

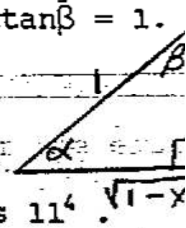
CONTEST NUMBER FIVE - FALL, 1994

F94S25

The sum of the roots of the first equation is $-\sqrt{6}/\sqrt{2} = -\sqrt{3}$ and the sum of the roots of the second equation is $\sqrt{15} / \sqrt{5} = +\sqrt{3}$. The sum of all four roots is 0.
The product of the roots of the first equation is $\sqrt{5} / \sqrt{2}$ and the product of the roots of the second equation is $\sqrt{2} / \sqrt{5}$. The product of all four roots would be 1.
In the derived equation $ax^2 + bx + c = 0$, if we let $a = 1$, then $b=0$ and $c=1$ giving the equation $x^2 + 1 = 0$.

F94S26

Let $\alpha = \arcsin x$ and $\beta = \arccos x$.
A right triangle with acute angles α and β is representative of the given.
Therefore, $\tan \alpha = x/\sqrt{1-x^2}$ and $\tan \beta = \sqrt{1-x^2} / x$. Clearly, $\tan\alpha\tan\beta = 1$.
Since $\tan(\alpha+\beta) = (\tan \alpha + \tan \beta)/(1 - \tan \alpha \tan \beta)$,
$\cot(\alpha+\beta) = (1 - \tan \alpha \tan \beta)/(\tan \alpha + \tan \beta) = 0$.



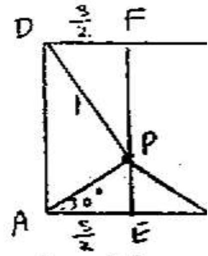
F94S27

$11^5 - 11^4 = 11^4 (11-1) = 11^4 \times 10$ whose sum of digits is the same as 11^4.

$11^n = (10 + 1)^n = {}_nC_0\, 10^n + {}_nC_1 10^{n-1} + \ldots + {}_nC_{n-1}\, 10^1 + {}_nC_n\, 10^0$.
$11^4 = (10 + 1)^4 = {}_4C_0\, 10^4 + {}_4C_1 10^3 + {}_4C_2\, 10^2 + {}_4C_3\, 10^1 + {}_4C_4\, 10^0$.
$= 14,641$ and the sum of the digits is 16.

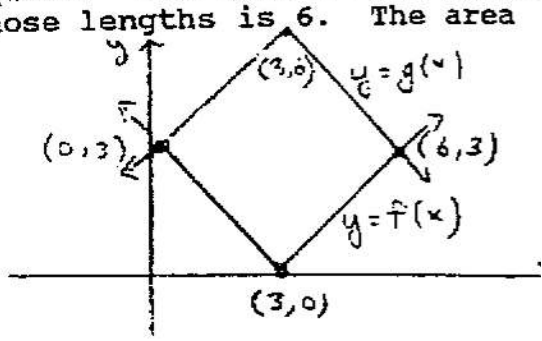
F94S28

Let s be the length of the side of the square. Draw $\overline{FPE}$ // $\overline{AD}$ with F on $\overline{CD}$ and E on $\overline{AB}$. We get, $AE = s/2$ and $PE = s/2\sqrt{3}$.
Therefore, $FP = s - s/2\sqrt{3}$. Using the Pythagorean Theorem on right triangle DFP, we get $s^2 = 3/(4 - \sqrt{3}) = (12 + 3\sqrt{3})/13$.



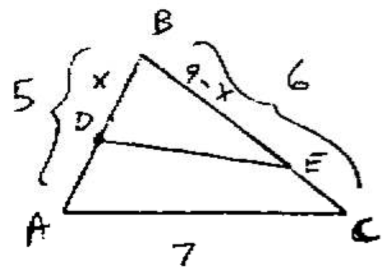
F94S29

f(x) when simplified is $|x - 3|$. Therefore, the graphs are piecewise linear graphs enclosing a region whose border is a square. The area of the square is 1/2 the product of the diagonals each of whose lengths is 6. The area of the region is $(1/2)(6)(6) = 18$.



F94S30

The perimeter of triangle ABC is 18.
$DB+BE = \frac{1}{2}(18) = 9$.
Let $DB = x$. Therefore, $BE = 9 - x$.
The area of triangle ABC is $\frac{1}{2}(AB)(BC)\sin B$
and the area of triangle DBE is $\frac{1}{2}(x)(9 - x)\sin B$ (which we also require to be the area of region ACED.)
Therefore, $30 = 2x(9 - x)$ or $x^2 - 9x + 15 = 0$.
The solutions are $x = (9 \pm \sqrt{21})/2$.
The smaller value is $(9 - \sqrt{21})/2$.



January 25, 1995

Dear Math Team Coach,

Enclosed is your copy of the Fall, 1994 NYCIML contests that you requested on the application form.

The following are the corrected or alternative answers for the enclosed contests.. The original answers were inaccurate.

	Question	Correct answer
Junior	F94J7 was eliminated	It should have read "greatest" common divisor
Senior A	F94S30 was eliminated	The answer was impossible for the given triangle.

Have a great spring term!

Sincerely yours,
Richard Geller
Secretary, NYCIML

Only one