PART I: 10 Minutes NYCIML Contest One Fall 1994

F94J1. If the least common multiple of two positive integers, a and b, is 18 and the greatest common factor of a and b is 3, compute the product ab.

F94J2. A rectanglular region is drawn by connecting the four points A(0,0), B(6,0), C(6,3) and D(0,3). A solid is formed by rotating this rectangular region 360^o about the x axis. Compute the volume of the solid formed.

PART II: 10 Minutes NYCIML Contest One Fall 1994

F94J3. D is the midpoint of $\overline{BC}$ in equilateral $\triangle ABC$. Circle O with diameter $\overline{AC}$ is constructed. If the area of circle O is 16π, compute the length of $\overline{AD}$ to the nearest tenth.

F94J4. Bill can do a job working alone in four hours. Hillary can do the same job alone in two hours. If the two work together undistracted, in how many minutes will the job be done?

PART III: 10 Minutes NYCIML Contest One Fall 1994

F94J5. Compute the sum of the first thirty terms of the sequence 7, 11, 15, 19,. . .

F94J6. When $\frac{(1993!)(1995!) - (1994!)^2}{(1994!)(1995!)}$ is reduced, the simplest way to express the result is $\frac{a}{b}$, where a and b are relatively prime positive integers. compute the value of a.

Answers

1. 54	3. 6.9	5. 1950
2. 54π	4. 80	6. 1

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER TWO** **FALL 1994**

PART I: 10 Minutes **NYCIML Contest Two** **Fall 1994**

F94J7. If the product of two positive integers is 2883 and their least common multiple is 93, compute their least common divisor.

F94J8. Compute the value of $\sqrt{(7012344)(7012346)+1}$.

PART II: 10 Minutes **NYCIML Contest Two** **Fall 1994**

F94J9. A triangular region is drawn by connecting the three points A(0,0), B(3,6), and C(6,0). A solid is formed by rotating this region 360^{o} about the x-axis. Compute the volume of the solid formed.

F94J10. A number reading the same from left to right as right to left is called a "palindromic number." For example, 1135311 is a palindromic number. An integer x is randomly chosen so that $0 < x < 10000$. The probability that x is a palindromic number is $\frac{a}{b}$ where a and b are relatively prime integers. Compute the value of a+b.

PART III: 10 Minutes **NYCIML Contest Two** **Fall 1994**

F94J11. $\sqrt[3]{8}$, $\sqrt[3]{x}$, and $\sqrt[3]{64}$ form an arithmetic sequence. Compute the value of x.

F94J12. If $\frac{(x!)(x+2)! - (x!)\ (x+1)!}{(x-1)!(x+1)!} = 156$, compute the value of x.

Answers

7. 31	9. 72π	11. 27
8. 7012345	10. 103	12. 12

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER THREE** **FALL 1994**

PART I: 10 Minutes **NYCIML Contest Three** **Fall 1994**

F94J13. Compute the value of $\frac{2!}{1!}-\frac{3!}{2!}+\frac{4!}{3!}-\cdots+\frac{100!}{99!}$.

F94J14. The product of two positive integers is 3600. Their greatest common divisor is 20 and their least common multiple is $c^d e^f g^h$, where $c<e<g$. If c, d, e, f, g, and h are positive integers and c, e, and g are prime numbers compute the ordered triple(d,f,h).

PART II: 10 Minutes **NYCIML Contest Three** **Fall 1994**

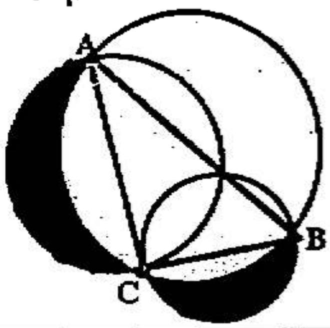
F94J15. Going to work, Ms. Zhang averaged 30 mph. Returning home over the same route, she averaged 60 mph. Compute her average speed for the total trip to and from work that day.

F94J16. A triangular region is drawn by connecting the three points A(0,0), B(6,3) and C(0,3). A solid is formed by rotating this region 360° about the x-axis. Compute the volume of the solid generated .

PART III: 10 Minutes **NYCIML Contest Three** **Fall 1994**

F94J17. Roslyn began driving when the mileage on her car's odometer was 75802. If the final mileage on her car's odometer was five miles past the fourth palindromic number she passed that day, compute the number of miles in her trip.

F94J18. $\overline{AC}$, $\overline{BC}$, and $\overline{AB}$ are diameters of the three circles in the diagram. If AC = 8 and BC = 6, compute the shaded area.



Answers

13. 51	**15.** 40	**17.** 370
14. (2,2,1)	**16.** 36π	**18.** 24

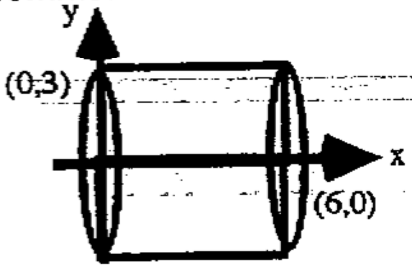
NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

JUNIOR DIVISION **CONTEST NUMBER ONE** **FALL 1994**

Solutions

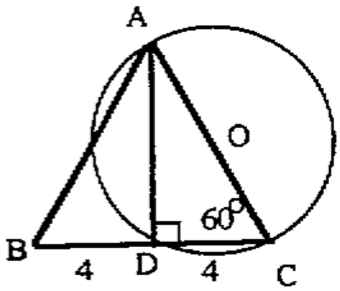
F94J1. Method One: Obviously, the two numbers satisfying these conditions are 18 and 3 or 9 and 6. In either case, the product of the two numbers is 54.

Method Two: Theorem: The product of two positive integers equals the product of their least common multiple and their greatest common factor. Thus the desired product is $3 \cdot 18 = 54$. **Answer:** 54



F94J2. As the diagram indicates the resulting solid is a circular cylinder with radius 3 and height 6. Thus the volume generated is $\pi(3)^2(6) = 54\pi$.

Answer: 54π



F94J3. The diameter of the circle has length 8. Since the median to a side of an equilateral triangle is also an altitude, we can use the Pythagorean Theorem or the fact that ΔADC is a 30-60-90 triangle in order to calculate AD. Either way, $AD = 4\sqrt{3} \approx 4(1.732)$ or 6.9 to the nearest tenth. **Answer:** 6.9

F94J4. Bill works at the rate $\frac{1}{4}$ of the job each hour. Hillary works at the rate $\frac{1}{2}$ of the job each hour. If they work x hours, the portion of the job is $\frac{3}{4}x$. The whole job is to be done so we get the equation $\frac{3}{4}x = 1$ or $x = \frac{4}{3}$ hours which is 80 minutes. **Answer:** 80 minutes

F94J5. The sum of the first n terms of an arithmetic sequence is $\frac{n}{2}(a+\ell)$ where n is the number of terms, a is the first term and ℓ is the last term. The last term can be found using $\ell = a + (n-1)d$, where d is the common difference. Here we have $\ell = 7 + 29(4) = 123$ and the sum = $\frac{30}{2} \cdot (7 + 123) = 15(130) = 1950$ **Answer:** 1950

F94J6. $$\frac{(1993!)(1995!) - (1994!)^2}{(1994!)(1995!)} = \frac{(1993!)(1995!)}{(1994!)(1995!)} - \frac{(1994!)^2}{(1994!)(1995!)}$$

$$= \frac{1}{1994} - \frac{1}{1995} = \frac{1}{(1994)(1995)}$$ **Answer:** 1

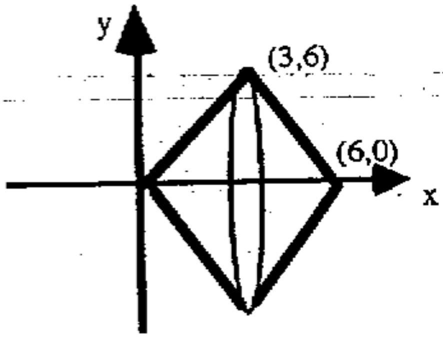
***Please note*:** Concepts used today will be repeated later this year.

JUNIOR DIVISION   CONTEST NUMBER TWO   FALL 1994

Solutions

F94J7. Use Theorem: The product of two numbers is equal to the product of their least common multiple and their greatest common factor. Thus the greatest common divisor is $\frac{2883}{93} = 31$.   **Answer:** 31

F94J8. The radicand is in the form $(x-1)(x+1) + 1 = x^2 - 1 + 1 = x^2$. Thus the needed result is merely $\sqrt{x^2} = x = 7012345$ **Answer:** 7012345



F94J9. As the diagram indicates, the resulting solid can be divided into two congruent right circular cones. The volume of one such cone is $\frac{1}{3}\pi(6^2)3 = 36\pi$. The needed volume is therefore 72π.

Answer: 72π

F94J10.

# of digits in integer	# of palindromic numbers
1	9
2	9•1
3	9•10•1=90
4	9•10•1•1=90

Thus there are a total of 198 palindromic integers on the given interval. The probability that a number is palindromic is therefore $\frac{198}{9999} = \frac{2}{101}$

Answer: $a+b = 103$

F94J11. The arithmetic sequence is 2, $\sqrt[3]{x}$, 4. This must be the same as 2,3,4 so that $\sqrt[3]{x} = 3$ and $x = 27$.   **Answer:** 27

F94J12. $\frac{(x!)(x+2)! - (x!)\,(x+1)!}{(x-1)!(x+1)!} = 156 \rightarrow x(x+2) - x - 156 = 0$ so that $x^2 + x - 156 = 0$ and $(x+13)(x-12) = 0$ so that $x = 12$.   **Answer:** 12

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

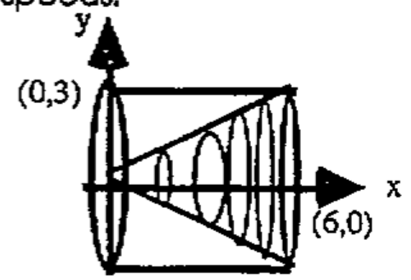
JUNIOR DIVISION **CONTEST NUMBER THREE** **FALL 1994**

Solutions

F94J13. $\frac{2!}{1!} - \frac{3!}{2!} + \frac{4!}{3!} - \ldots + \frac{100!}{99!} = 2 - 3 + 4 - 5 + 6 - \ldots + 100 =$ $(2+4+6+8+\ldots+100) - (3+5+7+\ldots+99) = \frac{50}{2}(102) - \frac{49}{2}(102) = 51(50) - 49(51) =$ $51(50-49) = 51$ <u>Answer:</u> 51

F94J14. The least common multiple is $\frac{3600}{20} = 180 = 2^2 3^2 5^1$ <u>Answer:</u> (2,2,1)

F94J15. Let "d" represent her distance to work Since she averaged 30 mph, her time coming to work was $\frac{d}{30}$. Likewise, her time coming home was $\frac{d}{60}$. Her average speed is equal to $\frac{\text{total distance traveled}}{\text{total time traveling}} = \frac{2d}{\frac{d}{30} + \frac{d}{60}}$. This simplifies to $\frac{120d}{3d} = 40$ miles per hour. Note that in general, average speed is found by taking the HARMONIC mean of the two speeds. <u>Answer:</u> 40mph



F94J16. The volume is computed by subtracting the volume of the right circular cone from the volume of the right circular cylinder: $V = 54\pi - \frac{1}{3}\pi(54) = \frac{2}{3}(54\pi) = 36\pi$.

<u>Answer:</u> 36π.

F94J17. The first four palindromic numbers after 75802 are 75857, 75957, 76067 and 76167. Therefore, her final mileage was 76172. This means that she drove 76172–75802 = 370 miles. <u>Answer:</u> 370

F94J18. Let X = the area of the semicircle below $\overline{BC} = 4.5\pi$.

Let Y = the area of the semicircle to the left of $\overline{AC} = 8\pi$.

Since ΔABC is a right triangle, (two of its sides are tangents to circles), AB=10.

Let Z = the area of the semicircle below $\overline{AB} = 12.5\pi$.

Note that the sum of the first two semicircular areas is equal to the third semicircular area. (X+Y=Z) This is the heart of the solution! The shaded area = X + Y − (Z − the area of ΔABC) = the area of ΔABC = 24. Thus the shaded area is equal to the area of the triangle!!

<u>Answer:</u> 24

January 25, 1995

Dear Math Team Coach,

Enclosed is your copy of the Fall, 1994 NYCIML contests that you requested on the application form.

The following are the corrected or alternative answers for the enclosed contests.. The original answers were inaccurate.

	Question	Correct answer
Junior	F94J7 was eliminated	It should have read "greatest" common divisor
Senior A	F94S30 was eliminated	The answer was impossible for the given triangle.

Have a great spring term!

Sincerely yours,
Richard Geller
Secretary, NYCIML

Only one