NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER ONE

PART I: 10 Minutes SPRING, 1994

S94B1 The average weight of the 12 girls in a class is 120. The average weight of the 18 boys in the class is 150. What is the average weight of all the students in the class?

S94B2 Two circles are concentric. A chord of the larger circle is tangent to the smaller circle and has length 10. Find the area of the region between the two circles.

PART II: 10 Minutes SPRING, 1994

S94B3 Compute the number of ways 6 students can be lined up, if Arthur must stand in front of Jeffrey.

S94B4 If $(1 - i)^{10}$ is expressed as $a + bi$, find the ordered pair (a,b).

PART III: 10 Minutes SPRING, 1994

S94B5 Solve for x: $x^2 + |x| - 6 = 0$.

S94B6 In triangle ABC, $\tan A = 1/2$ and $\tan B = 1/5$. Find the value of $\tan C$.

ANSWERS

1. 138
2. 25π
3. 360
4. $-32i$
5. ± 2
6. $-7/9$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION   CONTEST NUMBER TWO

PART I: TIME: 10 MINUTES   SPRING, 1994

S94B7 If $8^x = (1/32)^y$, express y in terms of x.

S94B8 If $(320)_{\text{base } b} + (430)_{\text{base } b} = (1300)_{\text{base } b}$, find the value of b.

PART II: TIME: 10 MINUTES   SPRING, 1994

S94B9 Find the smallest positive integer N>1 such that $1 + 2 + 3 + 4 + \ldots + N$ is a perfect square.

S94B10 Cindy takes twice as long as Mary to do a job and David takes three times as long as Mary to do the same job. If the three of them work together, it takes them 2 hours to do the job. How long would it take Mary to do the job alone?

PART III: TIME: 10 MINUTES   SPRING, 1994

S94B11 If $3x + 4y = 7$ is perpendicular to $5x + ay = 9$, find the value of a.

S94B12 Billiard balls are numbered with the integers 1 through 15. If three are pocketed on one shot, find the probability that the sum of the three balls is odd.

SOLUTIONS

7. $y = -3x/5$
8. 5
9. 36
10. 11/3
11. −15/4
12. 224/455

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER THREE SPRING, 1994

PART I: TIME: 10 MINUTES

S94B13 If x is a real number, find the minimum value of $2x^2 - 3x$.

S94B14 ABCD is a rectangle (not a square) which is inscribed in a square with side 10 such that each point of the rectangle is on a different side of the square. Find the perimeter of the rectangle.

PART II: TIME: 10 MINUTES

S94B15 If x-1, x+2, and 3x+7 are the first three terms of an arithmetic progression in the order given, compute x.

S94B16 Find the smallest positive integer x for which $2250x = y^4$, where y is a positive integer.

PART III: TIME: 10 MINUTES

S94B17 If x = 1/2 and y = 1/3, find the value of arctanx + arctany.

S94B18 In triangle ABC, AB=5, BC=4 and AC=3. CD is an altitude of the triangle, CM is a median. Find the length of DM.

SOLUTIONS

13. -9/8
14. $20\sqrt{2}$
15. -1
16. 360
17. 45°
18. 7/10

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FOUR SPRING, 1994

PART I: TIME: 10 MINUTES

S94B19 Express $\sqrt{(925)^2 - (924)^2}$ as a positive integer.

S94B20 A fair die is rolled five times. Find the probability of rolling a 1 or a 2 on at least half the rolls.

PART II: TIME: 10 MINUTES

S94B21 The length of a rectangle is 6 times the width, and a diagonal is 5. Find the area of the rectangle.

S94B22 The first three terms of a geometric progression are $x + 1$, $2x$, and $2x + 3$ in that order. Compute all possible values of x.

PART III: TIME: 10 MINUTES

S94B23 Each interior angle of a regular polygon contains 174°. How many sides does this polygon have?

S94B24 Find the volume of a tetrahedron whose edge has length 2. (A tetrahedron is a pyramid whose base and three sides are congruent equilateral triangles.)

SOLUTIONS

19. 43
20. 17/81
21. 150/37
22. 3, -1/2
23. 60
24. $(1/3)\sqrt{8}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FIVE   SPRING, 1994

PART I: TIME: 10 MINUTES

S94B25 What is the only two digit number which is three times the sum of the digits?

S94B26 Find all real solutions to the equation
$x^4 + 4x^3 + 6x^2 + 4x + 1 = 16$

PART II: TIME: 10 MINUTES

S94B27 Find the length of the diagonal of an isosceles trapezoid with sides 5, 5, 7, and 15.

S94B28 Find the remainder if 7^{1994} is divided by 16.

PART III: TIME: 10 MINUTES

S94B29 If $\sqrt{x - 1} = 2$, find the value of $(x - 1)^3$.

S94B30 How many equations of the form $x^2 + bx + c = 0$ have real roots, with b and c positive integers less than or equal to 6? (b and c are not necessarily different)

SOLTUIONS

25. 27	27. $\sqrt{130}$	29. 64
26. 1, -3	28. 1	30. 19

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER ONE
SOLUTIONS

S94B1 The total weight of the girls is 12(120) = 1440. The total weight of the boys is 18(150) = 2700. The average weight is (1440 + 2700)/30 = 138.

S94B2 Let R be the radius of the larger circle and r the radius of the smaller circle. The desired area is $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$. But $5^2 + r^2 = R^2$ or $R^2 - r^2 = 25$. Therefore, the A = 25π.

S94B3 There are 6! = 720 ways that 6 students can be lined up. Of these, one half, or 360 will have Arthur in front of Jeffrey.

S94B4 $(1 - i)^2 = 1 - 2i + 1 = -2i$

$(1 - i)^{10} = (-2i)^5 = -32i^4i = -32i$

S94B5 $x^2 = |x|^2$. $|x|^2 + |x| - 6 = 0$.

$(|x| + 3)(|x| - 2) = 0$. $|x| = -3$ is impossible,

$|x| - 2 = 0$ or $x = \pm 2$.

S94B6 $\tan C = \tan[180 - (A + B)]$

$\tan(A + B) = (1/2 + 1/5)/(1 -(1/2)(1/5)) = 7/9$

$\tan[180 - (A + B)] = (0 - 7/9)/(1 + 0\cdot(7/9)) = -7/9$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER TWO SPRING, 1994

S94B7 $(2^3)^x = (2^{-5})^y$. Thus, $3x = -5y$ or $y = -3x/5$.

S94B8 $3b^2 + 2b + 4b^2 + 3b = b^3 + 3b^2$ or $b^3 - 4b^2 - 5b = 0$ or $b(b-5)(b+1) = 0$. $b = 5$ is the only possible base.

S94B9 By trial and error $1 + 2 + 3 + \ldots + 8 = 36$ is the smallest N.

S94B10 Let x = time necessary for Mary to do the job. Then $2/x + 2/2x + 2/3x = 1$ or $x = 11/3$.

S94B11 The slopes of the two lines, $-3/4$ and $-5/a$, are negative reciprocals. $-3/4 = -1/-5/a = a/5$. $4a = -15$ or $a = -15/4$.

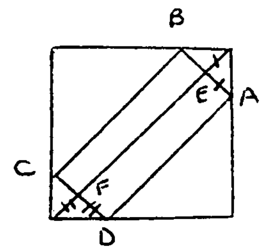
S94B12 The number of ways 3 can be chosen is ${}_{15}C_3 = 455$. For the sum to be odd, either they are all odd or exactly one is odd. ${}_8C_3 + {}_8C_1 \cdot {}_7C_2 = 56 + 8 \cdot 21 = 244$.
Thus, $P = 224/455$.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS NUMBER THREE SPRING, 1994

S94B13 The graph of $y = 2x^2 - 3x$ is a parabola with axis of symmetry $x = -b/2a = 3/4$. At $x = 3/4$, $y = -9/8$.

S94B14 According to the diagram, EA + AD + DF = a diagonal of the square = $10\sqrt{2}$. $P = 2\cdot(10\sqrt{2}) = 20\sqrt{2}$.

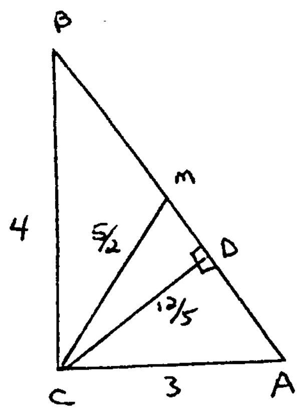


S94B15 $3x + 7 - (x + 2) = x + 2 - (x - 1)$
$2x + 5 = 3$ or $x = -1$.

S94B16 $2250 = 2^1\cdot 3^2\cdot 5^3$. To be a fourth power, all primes must be raised to the fourth power. $x = 2^3\cdot 3^2\cdot 5 = 360$.

S94B17 Let M = Arctan(1/2) and N = Arctan(1/3). Then tanM = 1/2 and tanN = 1/3. $\text{Tan}(M + N) = (1/2 + 1/3)/(1 - (1/2)(1/3)) = 1$. $M + N = \text{Arctan}1 = 45^0$.

S94B18 Since triangle ABC is a right triangle, CM = (1/2)(5) = 5/2. CD·AB = AC·BC, CD = 12/5.
$(12/5)^2 + (DM)^2 = (5/2)^2$ or $144/25 + (DM)^2 = 25/4$ which yields $DM = \sqrt{49/100} = 7/10$.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS NUMBER FOUR SPRING, 1994

S94B19 $\sqrt{(925)^2 - (924)^2} = \sqrt{(925 + 924)(925 - 924)} = \sqrt{1849} =$ 43

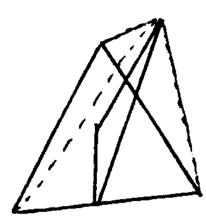
S94B20 The probability is the sum of getting 3, 4, and 5 successes.
$P = {}_5C_3(1/3)^3(2/3)^2 + {}_5C_4(1/3)^4(2/3)^1 + (1/3)^5 = 10\cdot(4/232) + 5\cdot(2/243) + 1/243 = 51/243 = 17/81$

S94B21 $(6x)^2 + (x)^2 = 25$ or $37x^2 = 25$ or $x^2 = 25/37$
$A = 6x\cdot x = 6x^2 = 6\cdot(25/37) = 150/37$

S94B22 The ratio of second to first equals the ratio of third to second. Thus, $2x/(x+1) = (2x+3)/2x$ or
$4x^2 = 2x^2 + 5x + 3$ or $2x^2 - 5x - 3 = 0$ which yields
$x = 3$ and $x = -1/2$.

S94B23 An exterior angle equals $360/N = 6$ and therefore, $N = 60$.
Or, $180(N-2)/N = 174$, $174N = 180N - 360$, $N = 6$.

S94B24 The volume = $(1/3)\cdot$Area of base$\cdot$height.
Area of base = $\sqrt{3}$
The height is one leg of a right triangle in which the other leg is the apothem $(\sqrt{3}/3)$ and the hypotenuse is the altitude of the equilateral triangle $(\sqrt{3})$. Thus,
$(\sqrt{3}/3)^2 + h^2 = (\sqrt{3})^2$, $h^2 = 8/3$, $h = \sqrt{8}/\sqrt{3}$
$V = (1/3)\cdot\sqrt{3}\cdot(\sqrt{8}/\sqrt{3}) = (1/3)\cdot\sqrt{8}$.



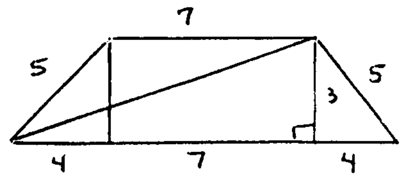
NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION SOLUTIONS NUMBER FIVE SPRING, 1994

S94B25 $10t + u = 3(t + u)$ or $7t = 2u$. The only single digits which will fit are $t = 2$ and $u = 7$.

S94B26 $(x+1)^4 = 16$. Thus, $x+1=2$ or $x+1= -2$ which yields solutions of $x=1$ or $x= -3$. (The other two roots are imaginary.)

S94B27 $3^2 + 11^2 = d^2$ or $d=\sqrt{130}$.



S94B28 $7^2 = 49 \equiv 1 \pmod{16}$ (That is, leaves a remainder of 1 when divided by 16.)
$(7^2)^{997} \equiv 1^{997} = 1 \pmod{16}$

S94B29 $\sqrt{x-1} = 2$, $x-1 = 4$, $(x-1)^3 = 64$.

S94B30 Using the discriminant, $b^2 - 4ac \geq 0$, start with b = 1, 2, 3, 4, 5, and 6 and find all possible values of c.

b	c
1	none
2	1
3	1, 2
4	1, 2, 3, 4
5	1, 2, 3, 4, 5, 6
6	1, 2, 3, 4, 5, 6

Thus, there are a total of 19 possible values.

May 25, 1994

Dear Math Team Coach,

Enclosed is your copy of the Spring, 1994 NYCIML contests that you requested on the application form.

The following are the corrected or alternative answers for the enclosed contests.. The original answers were inaccurate.

	Question	Correct answer
Senior A	S94S1 was eliminated.	It should have read smallest "positive integer".
Senior B	S94B3	120 or 360
	S94B4	(0, -32)
	S94B9	8
Junior	S94J9 was eliminated.	It was an impossible triangle.
	S94J12	(25, 77)

Have a great summer!

Sincerely yours,

Richard Geller

Secretary, NYCIML