NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER ONE FALL, 1990

Part I: 10 Minutes

F90B1. Compute the number that yields the same result when it is added to 1.2 as when it is multiplied by 1.2?

F90B2. What is the units digit of the large number 388 . 787?

Part II: 10 Minutes NYCIML CONTEST ONE FALL, 1990

F90B3. Compute the length of the diagonal of an isosceles trapezoid with sides 5, 7, 7, 13.

F90B4. If the number 3,165,284,9A7,634 is divisible by 9, find A.

Part III: 10 Minutes NYCIML CONTEST ONE FALL, 1990

F90B5. The sum of all but one of the angles of a convex polygon is 1930°. How many sides does the polygon have?

F90B6. Solve for x:

$$|x|^2 + |x| - 42 = 0$$

ANSWERS

1.	6	3.	$\sqrt{114}$	5.	13
2.	3	4.	5	6.	{6,-6} or equivalent

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER TWO FALL, 1990

Part I: 10 Minutes

F90B7. Compute the positive mean proportional between $6 + 3\sqrt{2}$ and $6 - 3\sqrt{2}$.

F90B8. Compute the area of a regular hexagon with a side of 8.

Part II: 10 Minutes NYCIML CONTEST TWO FALL, 1990

F90B9. In how many ways can 6 different charms be arranged on a circular charm bracelet with no discernible starting point?

F90B10. 50! represents 50x49x48x...x3x2x1. How many consecutive zeroes appear in the rightmost digits of this number?

Part III: 10 Minutes NYCIML CONTEST TWO FALL, 1990

F90B11. Triangle ABC has vertices A(0,0), B(4,0), C(0,3) on the rectangular coordinate system. Compute the length of the altitude from A to side BC.

F90B12. Find all values of x in the interval $0^\circ \leq x \leq 90^\circ$ which satisfy $\sin 2x - \cos x \geq 0$.

ANSWERS

7. $3\sqrt{2}$ or $\sqrt{18}$

8. $96\sqrt{3}$

9. 60

10. 12

11. 2.4 or 12/5 or equivalent

12. $30^\circ \leq x \leq 90^\circ$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER THREE FALL, 1990

Part I: 10 Minutes

F90B13. Parallelogram ABCD is inscribed in a circle. If AB = 4 and BC = 6, compute the length of the radius of the circle.

F90B14. A man drives to a distant city and returns on the same route. In order to return home for an appointment, he calculates that he must average 40 mph. each way. However, his rate on the outbound journey is only 30 mph. Compute the average rate he must travel on the return trip in order to get home on time.

Part II: 10 Minutes NYCIML CONTEST THREE FALL, 1990

F90B15. A cubic block which is 8"x8"x8" is painted, then cut into one inch cubes. Compute the number of cubes that have at least one face painted?

F90B16. John averaged 80 on his first 4 tests. His mark on the fifth test was 16 points lower than the average of all five tests. Compute his mark on the fifth test.

Part III: 10 Minutes NYCIML CONTEST THREE FALL, 1990

F90B17. $(x - 2)^6$ is expanded and arranged in decreasing powers of x. Compute the coefficient of the sixth term.

F90B18. Compute the volume of the solid generated if a square with side 6 is revolved around a diagonal as its axis.

ANSWERS

13.	$\sqrt{13}$	15.	296	17.	-192
14.	60	16.	60	18.	$36\pi\sqrt{2}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FOUR FALL, 1990

Part I: 10 Minutes

F90B19. Find the 1991st digit to the right of the decimal point of the decimal equivalent of 1/14.

F90B20. A fair die is rolled 6 times. Compute the probability of rolling less than 3 exactly 5 times.

Part II: 10 Minutes NYCIML CONTEST FOUR FALL, 1990

F90B21. Compute the radius of a sphere if its volume is numerically equal to its surface area.

F90B22. Working alone, Mary can paint a room in 10 hours. After she has been working for 4 hours, John comes, and together, they complete the job in 2 additional hours. How long would it take John to paint the room alone?

Part III: 10 Minutes NYCIML CONTEST FOUR FALL, 1990

F90B23. If 20 people are in a room, and each one shakes hands with everyone else, how many handshakes take place?

F90B24. A bus company which charges $1 per ride has 500 riders per day. It finds that for every nickel it raises the fare, it loses 10 riders. How much should it charge for maximum revenue?

ANSWERS

19.	2	21.	3	23.	190
20.	4/243	22.	5 hours	24.	$1.75

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FIVE FALL, 1990

Part I: 10 Minutes

F90B25. David is 5 times as old as his daughter, but 21 years from now, he will only be twice as old as she will be then. How old is David now?

F90B26. Two congruent 30°-60°-90° triangles are placed so that they partially overlap and their hypotenuses coincide. If this hypotenuse is 6, compute the area common to both triangles.

Part II: 10 Minutes NYCIML CONTEST FIVE FALL, 1990

F90B27. A room is in the shape of a cube, 10 feet on an edge. A fly starts in a corner of the floor and walks to the corner of the ceiling diagonally opposite. Compute the shortest distance he must walk.

F90B28. Find the number of sets of 2 or more consecutive positive integers whose sum is 100.

Part III: 10 Minutes NYCIML CONTEST FIVE FALL, 1990

F90B29. At a snack bar the cost of 7 sandwiches, 5 drinks and one dessert is \$32. The cost of 10 sandwiches, 7 drinks and one dessert is \$45. Find the cost of a lunch consisting of 1 sandwich, 1 drink and 1 dessert.

F90B30. Find the value of $\cos 72^\circ - \cos 36^\circ$

ANSWERS

25. 35

26. $3\sqrt{3}$ or equivalent

27. $10\sqrt{5}$ or equivalent

28. 2

29. \$6

30. -1/2

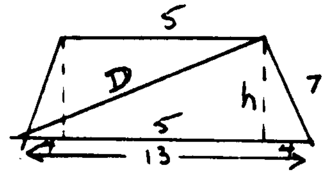
NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER ONE FALL, 1990

SOLUTIONS

F90B1. $x + 1.2 = 1.2x$
$10x + 12 = 12x$
$12 = 2x$
$6 = x$

F90B2. The units digits of 3 to a power works in cycles: 3, 9, 7, 1, ...
Since 88 is a multiple of 4, 3^{88} ends in 1.
The cycle of 7 powers is 7, 9, 3, 1, ... Since 87 is 3 more than a multiple of 4, it ends in 3. Since 1 X 3 = 3, the large number has a units digit of 3.

F90B3. The altitude is $\sqrt{49 - 16} = \sqrt{33}$.
$D^2 = (\sqrt{33})^2 + 9^2$
$D = \sqrt{114}$



F90B4. The sum of the digits must be divisible by 9. 58 + A is divisible by 9. A = 5.

F90B5. The sum of the angles must be a multiple of 180°. Therefore, 1980° must be the sum. 1980 = (n - 2) 180. n = 13.

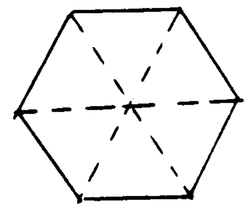
F90B6. $x^2 + x - 42 = 0$.
$x = -7$ and $x = 6$
However, -7 will not work since $|-7| \neq -7$. -6 <u>will</u> work since $|-6| = 6$.
(6, -6)

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION CONTEST NUMBER TWO FALL, 1990

SOLUTIONS

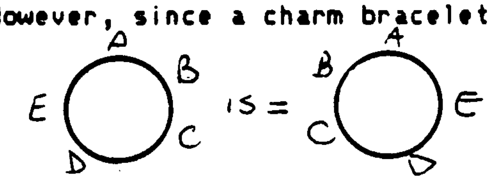
F90B7. $\dfrac{6 + 3\sqrt{2}}{x} = \dfrac{x}{6 - 3\sqrt{2}}$ $x^2 = 18$
$x = 18$

F90B8. If the radii are drawn, 6 congruent equilateral triangles are formed.



$A = 6\left(\dfrac{8^2\sqrt{3}}{4}\right) = 96\sqrt{3}$

F90B9. The first one can be placed anywhere. There are then 5! ways the others can be arranged. However, since a charm bracelet can be turned over, that is



we must divide the result by 2. $\dfrac{5!}{2} = 60$

F90B10. A zero will be produced by a factor of 2 and a factor of 5. Since 10 integers up to 50 have factors of 5, and there are more than enough evens to cover them (any even has a factor of 2), that makes 10 zeroes. But 25 and 50 each have 2 5's. Therefore, there are 12 zeroes.

F90B11. The area of this right triangle is 6. Since BC = 5,

$6 = \dfrac{1}{2} \cdot 5 \cdot x$ $\boxed{x = 12/5}$

F90B12. $\sin 2x - \cos x \geq 0$
$2\sin x \cos x - \cos x \geq 0.$
$\cos x\,(2\sin x - 1) \geq 0$

Since, in the first quadrant, $\cos x$ is always ≥ 0, $2\sin x - 1$ also must be ≥ 0
$2\sin x - 1 \geq 0$
$\sin x \geq 1/2$ $x \geq 30^\circ$
so $30^\circ \leq x \leq 90^\circ$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION   CONTEST NUMBER THREE   FALL, 1990

SOLUTIONS

F90B13. A parallelogram inscribed in a circle must be a rectangle and the diagonal is the diameter.

$d = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$
$r = \sqrt{13}$

F90B14. Since $T = \frac{D}{R}$, total time expected is $T = \frac{D}{40} + \frac{D}{40} = \frac{D}{20}$

However, time on outbound trip is D/30
let x = time on return trip.

$60x\left(\frac{D}{30} + \frac{D}{x} = \frac{D}{20}\right)$ $2Dx + 60D = 3Dx$
$2x + 60 = 3x$
$60 = x$

F90B15. The painted blocks are obviously on the outside of the cube. If these are stripped away, a 6"x6"x6" block of unpainted cubes remains.
Number of painted blocks = $8^3 - 6^3 = 512 - 216 = 296$

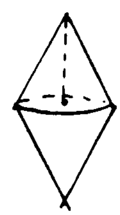
F90B16. $x = \frac{1}{5}(320 + x) - 16$
$5x = 320 + x - 80$
$x = 60$

F90B17. Sixth term of $(a + b)^6 = {}_6C_1 a^1 b^5$

in this case, ${}_6C_1 x^1 (-2)^5 = 6 \cdot x \cdot (-32)$

coefficient = -192

F90B18. The solid generated is 2 cones with their circular bases coinciding. The radius and the height of each cone is one-half the diagonal.



Cone: $V = \frac{1}{3}\pi r^2 h$
Volume of solid = $2\left(\frac{1}{3}\pi\right)(3\sqrt{2})^2\left(3\sqrt{2}\right) = 36\pi\sqrt{2}$

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

SENIOR B DIVISION CONTEST NUMBER FOUR FALL, 1990

SOLUTIONS

F9019. $\frac{1}{14} = 0.0\overline{714285}714285$, running in cycles of 6 after the first zero. Therefore, we want the 1990th from the 7. Since 1990 yields a remainder of 4 when divided by 6, the fourth digit in the cycle is 2.

F9020. Using the Bernoulli experiment, since the probability of rolling less than 3 is 1/3,

probability of 5 successes is ${}_6C_5\left(\frac{1}{3}\right)^5 \cdot \left(\frac{2}{3}\right)^1 =$

$6 \cdot \left(\frac{1}{243}\right) \cdot \left(\frac{2}{3}\right) = \left(\frac{4}{243}\right)$

F9021. $\frac{4}{3}\pi r^3 = 4\pi r^2$

$\frac{4}{3}r = 4 \qquad r = 3$

F9022. Let x = time necessary for John to do the job alone.

$\frac{4}{10} + \frac{2}{10} + \frac{2}{x} = 1 \qquad \frac{2}{x} = \frac{4}{10} \qquad x = 5$

F9023. Since 2 people are involved in a handshake, the number is ${}_{20}C_2 = 190$.

F9024. If P is the number of nickels it raises the fare and R is the revenue,

$$R = (100 + 5P)(500 - 10P)$$

$$= 50{,}000 + 1500P - 50P^2$$

graphing this, an inverted parabola, the maximum occurs at

$P = \frac{-b}{2a} = \frac{-1500}{2(-50)} = 15$

Therefore, the fare should be \$1.00 + 15(.05) = \$1.75.

(It is much easier using calculus.)

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE
SENIOR B DIVISION   CONTEST NUMBER FIVE   FALL, 1990

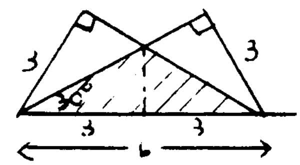
SOLUTIONS

F90B25. Let x = daughter's age.   5x = Dave's age.

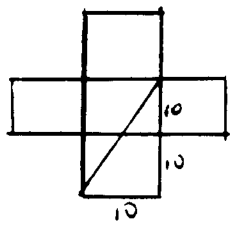
$5x + 21 = 2(x + 21)$
$3x = 21$
$x = 7$ Dave = 35

F90B26. The altitude of the desired triangle is $\frac{3}{\sqrt{3}}$ or $\sqrt{3}$.

The area is $\frac{1}{2} 6\sqrt{3} = 3\sqrt{3}$



F90B27. If the room is opend up, with the floor in the middle and the walls on the side, it would look like this:



The shortest distance would be the hypotenuse.

$S = \sqrt{10^2 + 20^2}$
$S = \sqrt{500}$ or $10\sqrt{5}$

F90B28. Using x, x+1, x+2, etc. as the consecutive integers, the following would have to have integral solutions:

a) $2x+1 = 100$	g) $8x+28 = 100$	
b) $3x+3 = 100$	h) $9x+36 = 100$	Only d and g have
c) $4x+6 = 100$	i) $10x+45 = 100$	integral solutions,
d) $5x+10 = 100$	j) $11x+55 = 100$	and there are
e) $6x+15 = 100$	k) $12x+66 = 100$	obviously none
f) $7x+21 = 100$	l) $13x+78 = 100$	bigger.

F90B29. $7S + 5D + T = 32$

$10S + 7D + T = 45$

Multiplying top by 3, bottom by 2 and subtracting:

$21S + 15D + 3T = 96$
$\underline{20S + 14D + 2T = 90}$
$S + D + T = 6$

F90B30. Using the double angle formulas,

$$\cos 72 = 2\cos^2 36 - 1$$
$$\cos 36 = 1 - 2\sin^2 18$$

Adding

$$\cos 72 + \cos 36 = 2(\cos^2 36 - \sin^2 18)$$

But, $\sin 18 = \cos 72$

$$\cos 72 + \cos 36 = 2(\cos^2 36 - \cos^2 72)$$
$$= 2(\cos 36 + \cos 72)(\cos 36 - \cos 72)$$
$$1 = 2(\cos 36 - \cos 72)$$
$$1/2 = \cos 36 - \cos 72$$

and $$-1/2 = \cos 72 - \cos 36$$