New York City
Interscholastic Mathematics League

Senior B Division Contest Number One NYCIML Fall, 1985
Part I Time: 10 Minutes

F85B1. Compute the numerical value of $x-(x^x-x)^x$ if $x = 2$.

F85B2. In a class of 35 students, 20 students own calculators, 11 own slide rules, and 10 own neither. How many students own both calculators and slide rules?

Senior B Division Contest Number One NYCIML Fall, 1985
Part II Time: 10 Minutes

F85B3. In triangle ABC, the degree-measure of angle C is 90. Compute the numerical value of (cot A)(cot B).

F85B4. Five quarts of a solution of acid and water is 40% water. If five more quarts of water are added to this solution, what is the percentage of water in the new solution?

Senior B Division Contest Number One NYCIML Fall, 1985
Part III Time: 10 Minutes

F85B5. Find 110% of 110.

F85B6. The notation 400! means $400\cdot399\cdot398\cdot\ .\ .\ .\ \cdot3\cdot2\cdot1$. When written out in decimal notation, how many zeroes are there at the right of the number 400! ?

ANSWERS

1. -2	3. 1	5. 121
2. 6	4. 70 or 70%	6. 99

New York City Interscholastic Mathematics League

Senior B Division Contest Number Two NYCIML Fall, 1985
Part I Time: 10 Minutes

F85B7. A store makes a profit of 20% on an item, based on its selling price. What percent profit does the store make, based on cost?

F85B8. If i denotes the imaginary unit, find the ordered pair (a,b) of real numbers such that:

$$\frac{1}{1+i} + \frac{2}{a+bi} = \frac{1}{1-i}.$$

Senior B Division Contest Number Two NYCIML Fall, 1985
Part II Time: 10 Minutes

F85B9. Compute $\sqrt{2^{10} + 2^{10} + 2^{10} + 2^{10}}$.

F85B10. Two circles are concentric, and a chord of the larger circle is tangent to the smaller circle. If this chord is 12 units long, the area of the region between the two circles can be expressed as $k\pi$ square units. Compute the value of the rational number k.

Senior B Division Contest Number Two NYCIML Fall, 1985
Part III Time: 10 Minutes

F85B11. How many interior diagonals can be drawn in a regular polygon of 100 sides?

F85B12. In triangle ABC, AB = AC. In this triangle, there are points D on BA, E on CA, and F on AB (and between D and A), such that CB = CD = ED = EF = FA. Find the degree-measure of angle A.

ANSWERS

7. 25 or 25%

8. (0,-2): ordered pair required

9. 2^6 or 64

10. 36

11. 4850

12. 20 or 20°

New York City
Interscholastic Mathematics League

Senior B Division Contest Number Three NYCIML Fall, 1985
Part I Time: 10 Minutes

F85B13. A driver travels 70 miles in 2.5 hours. By how many miles per hour must he increase his speed to make this trip in 3/4 hours less time?

F85B14. Find all real numbers x such that $2^x + 2^{x-1} = 48$.

Senior B Division Contest Number Three NYCIML Fall, 1985
Part II Time: 10 Minutes

F85B15. Find the smallest natural number which, when divided by 2, 3, or 4 leaves a remainder of 1, and is also a multiple of 5.

F85B16. Find all real numbers x such that $x^2 - (1+\sqrt{2})x + \sqrt{2} = 0$.

Senior B Division Contest Number Three NYCIML Fall, 1985
Part III Time: 10 Minutes

F85B17. When a quantity of water freezes, it gains 1/11 of its volume. When a quantity of ice melts, what fraction of its volume will it lose?

F85B18. A regular polygon has 54 distinct interior diagonals. How many sides does the polygon have?

ANSWERS

13. 12 or 12 miles per hour.	15. 25	17. 1/12
14. 5	16. 1, $\sqrt{2}$	18. 12

Senior B Division Contest Number Four NYCIML Fall, 1985
Part I Time: 10 Minutes

F85B19. How many degrees are there in the angle formed by the hands of a clock at 7:20?

F85B20. In triangle ABC, the lines containing the bisector of angle B and the bisector of an exterior angle at C intersect at Q. A line through Q drawn parallel to BC intersects AC at E and AB at D. If BD = 8 and EC = 6, find ED.

Senior B Division Contest Number Four NYCIML Fall, 1985
Part II Time: 10 Minutes

F85B21. Four houses each have four floors. On each floor are four apartments, with four doors in each apartment. On each door are four hinges. There are no other hinges in the house. How many hinges are in the house?

F85B22. A clock gains five seconds per hour. Twenty-four hours after the clock is set correctly, it reads 5:58. What is the correct time?

Senior B Division Contest Number Four NYCIML Fall, 1985
Part III Time: 10 Minutes

F85B23. A girl has 1/3 as many blue stickers as red stickers, and 1/6 as many red stickers as green stickers. What fraction of her sticker collection is green stickers?

F85B24. In triangle ABC, AB = AC = 13, and BC=10. Point P is on BC and CP:PB = 2:3. If X and Y are the feet of the perpendiculars from P to AB and AC respectively, compute the sum PX + PY.

ANSWERS

19. 100 or 100° 21. 4^5 or 2^{10} or 1024 23. 9/11

20. 2 22. 5:56 24. 120/13

New York City Interscholastic Mathematics League

Senior B Division Contest Number Five NYCIML Fall, 1985
Part I Time: 10 Minutes

F85B25. A toy boat costs 3/4 of its price plus 15 cents. How much does the boat cost?

F85B26. If log(log[log(log x)]) = 0, where the base of each logarithm is 10, then $x = 10^k$. Find the positive integer k.

Senior B Division Contest Number Five NYCIML Fall, 1985
Part II Time: 10 Minutes

F85B27. One-third of a number, plus twice half of the number, equals 36. Find the number.

F85B28. In triangle ABC, AB = AC = 17. Point E is a trisection point of BC nearer B, and AE = 15. Compute the length of BC.

Senior B Division Contest Number Five NYCIML Fall, 1985
Part III Time: 10 Minutes

F85B29. A tank of oil is 1/6 full. After 75 gallons of oil are removed, it is 1/7 full. How many gallons does the tank hold?

F85B30. Find the value of tan x if sin x + cos x = 1/5 and $\pi/2 < x < \pi$.

ANSWERS

25. 60 or 60 cents	27. 27	29. 3150
26. 10^{10}	28. $12\sqrt{2}$	30. -4/3

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

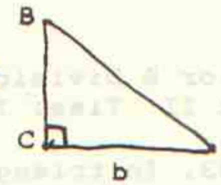
Senior B Division | Contest Number One | Fall, 1985

SOLUTIONS

F85B1. We have $2 - (2^2 - 2)^2 = 2-(4-2)^2 = 2-2^2 = 2-4 = -2$.

F85B2. We can disregard the ten students who own neither device. Using absolute value for the number of elements in a set, we have:
$|A \cup B| = |A| + |B| - |A \cap B|$.
Here, if A = students with slide rules), and B = students with calculators), we need to find $|A \cap B|$. Call this quantity x. Then $25 = 20 + 11 - x$, so x = 6

F85B3. In the diagram, cot A = a/b, and cot B = tan A = b/a, so their product is 1.



F85B4. The original solution contained (40%)(5) = 2 quarts of water, so the new solution contains 2+5 = 7 quarts of water. Since there are now ten quarts altogether, the percentage of water is 7/10 = 70%.

F85B5. We want (1.1)(110) = (1)(110) + (.1)(110) = 110 + 11 = 121.

F85B6. We want to find the highest power of ten which divided 400!. Since $10 = 5 \cdot 2$, it is sufficient to count powers of 5. There are 400/5 = 80 multiples of 5 in the product which forms 400!. In addition, there are 400/25 = 16 multiples of 25, which contribute 'extra' factors of 5. Finally, since 400/125 = 3 + 1/5, there are 3 extra powers of five from multiples of 125. And there are plenty of factors of 2--more than needed--to make 80 + 16 + 3 = 99 factors of ten in the product 400!.

NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior B Division — Contest Number Two — Fall, 1985

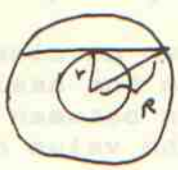
SOLUTIONS

F85B7. If S is the selling price, C the cost, and P the <u>amount</u> of profit, we have $S = C + P$. Here, $P = (1/5)S$, so $S = 5P = C + P$, and $C = 4P$. The percent profit based on cost is $P/C = 1/4 = 25\%$.

F85B8. We have $\frac{1}{1+i} - \frac{1}{1-i} = \frac{-2i}{2} = -i = \frac{-2}{a+bi}$, or $b - ai = -2$, and $(a,b) = (0,-2)$.

F85B9. The given expression equals $\sqrt{4 \cdot 2^{10}} = \sqrt{2^2 \cdot 2^{10}} = \sqrt{2^{12}} = 2^6 = 64$.

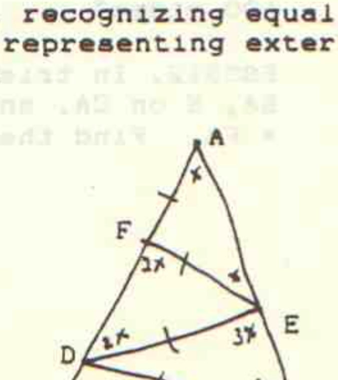
F85B10. If the radii of the circles are R and r (see diagram), then their areas are πR^2 and πr^2. The area of the region we need is $\pi(R^2 - r^2)$. Since a tangent is perpendicular to a radius at the point of contact, we can use the Pythagorean theorem, and $R^2 - r^2 = 36$. This gives 36π for the area of the region. Note that this result is independent of the actual measurements of the two radii.



F85B11. The polygon has 100 vertices. These can be connected by $100 \cdot 99/2 = 4950$ line segments. But 100 of these are sides of the polygon, so only 4850 of them are diagonals.

F85B12. Let the degree-measure of angle A be x. Then, recognizing equal base angles of isosceles triangles in the figure, and representing exterior angles as sums of the remote interior angles, we find:

$m\angle AEF = x$ (from triangle AFE),
$m\angle AFE = m\angle FDE = 2x$ (from triangle AFE),
$m\angle DEC = m\angle ECD = 3x$ (from triangle AED).
Since triangles ABC, CDB are similar,
$m\angle DCB = x$, and $m\angle BCA = x + 3x = 4x$.
Hence (in triangle ABC), $x + 4x + 4x = 180$, and $x = 20$.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior B Division Contest Number Three Fall, 1985

SOLUTIONS

F85B13. The driver is travelling at a rate of $70 \div 5/2 = 28$ miles per hour. He must make the 70 mile trip in $5/2 - 3/4 = 7/4$ hours. Hence he must go $70 \div 7/4 = 70 \cdot 4/7 = 40$ mile per hour. This is an increase in speed of 12 miles per hour.

F85B14. We have $2 \cdot 2^{x-1} + 2^{x-1} = 3 \cdot 2^{x-1} = 48$, so $2^{x-1} = 16$, $x-1 = 4$, and $x = 5$.

F85B15. We know that n-1 is a multiple of 2, 3, and 4. Hence n-1 is a multiple of 12. We are looking for the smallest multiple of 12 which is one less than a multiple of 5. A quick trial and error process shows that this number is 24, and n = 25.

F85B16. The quadratic formula is cumbersome in this problem. It is easier to note that the roots of the equation must multiply to $\sqrt{2}$ and add up to $1+\sqrt{2}$. Clearly, the numbers 1 and $\sqrt{2}$ satisfy both conditions, so these must be the roots of the equation.

F86B17. If W is the quantity (mass) of water, and I the volume of the same mass of ice, then $I = (12/11)W$. Hence $W = (11/12)I$, and ice loses 1/12 of its volume.

F86B18. See problem F85B11 . If the polygon has n sides, it will have $(n/2)(n-1) - n$ diagonals. Here, $(n/2)(n-1) - n = 54$, or $n^2-3n-108 = (n+9)(n-12) = 0$, and $n = -9, 12$. We reject the negative root.

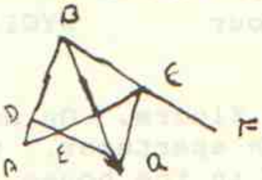
NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior B Division   Contest Number Four   Fall, 1985

SOLUTIONS

F85B19. There are 30^o between each numeral on the clock. From its position at 7:00, the hour hand has moved $(1/3)\cdot 30^o = 10^o$. Hence the angle required is $3\cdot 30 + 10 = 100^o$.

F85B20. The key to solving this problem is a well-labelled diagram. Since BQ and CQ are angle bisectors, and BCF||DEQ we have $\angle DBQ = \angle CBQ = \angle BQD$, and $\angle ECQ = \angle FCQ = \angle CQE$. Therefore DBQ and ECQ are isosceles triangles. Hence DQ = 8 and EQ = 6, so ED = 2.

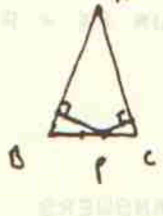


F85B21. The number of hinges is $4x4x4x4x4 = 4^5 = 2^{10} = 1024$.

F85B22. In 24 hours, the clock has gained 24x5 = 120 seconds = 2 minutes. Hence the correct time is 5:56.

F85B23. If A, B, and C are the number of blue, red, and green stickers respectively, then 3A = B, 6B = C, so C = 18A. Altogether, there are A+B+C = A + 3A + 18A = 22A stickers in the collection, so the proportion of green stickers is 18/22 = 9/11.

F85B24. From the Pythagorean theorem, the length of the altitude from A to BC is 12, so the area of the triangle is 60. We can also compute this area by adding the areas of triangles ABP, APC. This sum is $(1/2)AB\cdot PX + (1/2)AC\cdot PY = (1/2)\cdot 13\cdot (PX+PY) = 60$, so $PX+PY = 120/13$. Note that the exact position of point P along BC is irrelevant.



NEW YORK CITY INTERSCHOLASTIC MATHEMATICS LEAGUE

Senior B Division — Contest Number Five — Fall, 1985

SOLUTIONS

F85B25. If C is the cost of the boat, then $(3/4)C + 15 = C$, $C/4 = 15$, and $C = 60$.

F85B26. We use the fact that $\log_{10} x = L$ means that $10^L = x$. We have $\log[\log(\log x)] = 10^0 = 1$. Then $\log(\log x) = 10^1 = 10$, so $\log x = 10^{10}$.

Finally, $x = 10^{10^{10}}$, and $k = 10^{10}$.

F85B27. If the number is x, then $x/3 + 2\cdot x/2 = x/3 + x = 4x/3 = 36$, and $x = 3/4 \cdot 36 = 27$.

F85B28. Let $BE = x$, and draw altitude AT. Then $BC = 3x$, so $BT = 3x/2$, and $ET = BT - BE = x/2$. We have:

$AT^2 = AB^2 - BT^2 = AE^2 - ET^2$, so
$289 - 9x^2/4 = 225 - x^2/4$, or
$64 = 8x^2/4 = 2x^2$, $x^2 = 32$, and $x = 4\sqrt{2}$. Thus $BC = 12\sqrt{2}$.

F85B29. If the tank holds x gallons, then $x/6 - 75 = x/7$, and $7x - 75\cdot 6\cdot 7 = 6x$, so $x = 75\cdot 6\cdot 7 = 3150$.

F85B30. Squaring both sides of the equation, we get $\sin^2 x + \cos^2 x + 2\sin x \cos x = 1/25$. Thus, $\sin 2x = 2\sin x \cos x = -24/25$, or $\sin 2x = -24/25$, and $\cos 2x = 7/25$ or $-7/25$. We use the identity:

$\tan x = \sin x/\cos x = (\sin 2x)/(1+\cos 2x)$, and find that $\tan x = -3/4$ or $-4/3$. However, if $\tan x = -3/4$, $\sin x = 3/5$ and $\cos x = -4/5$ (in quadrant II), so that $\sin x + \cos x = -1/5$. This extraneous root came from the original squaring we did.

January 5, 1986

Dear Math Team Coach,

Enclosed is your copy of the Fall, 1985 NYCIML contests that you requested on the application form.

The following are corrections for the enclosed contests:

	Question	
Senior A		
	F85S3	Answer is 169/10
	F85S14	Answer is 133
	F85S23	Answer $\pi/8$ and $3\pi/8$
	F85S24	The question is impossible and was eliminated from the contest.
Senior B	F85B21	The question is poorly worded and either 256 or 1024 were accepted.

Have a good year.

Sincerely yours,

Richard Geller

Secretary, NYCIML